Math 125 Final Exam

Wednesday, May 8th, 2019, 8:00–10:00am

Directions. Fill out your name, signature and student ID number on the lines below **right now**, before starting the exam.

You must show all your work and carefully justify your methods to obtain full credit. Answers must be neat, organized and unambiguous. You must state and justify the methods used.

Simplify your answers. Any fraction should be written in lowest terms. **No cell-phone, no calculator, no cheat sheet nor scratch paper are allowed**. Remember, USC considers cheating as a very serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told at the end of the exam.

Full name (please print): ______________________________

Signature: ______________________________

Student ID: ______________________________

☐ G. Dreyer 11am ☐ G. Dreyer 12pm ☐ G. Kim 9am
☐ G. Kim 1pm ☐ P. Tokorcheck 10am ☐ P. Tokorcheck 11am

1 (20 pts)	5 (12 pts)
2 (18 pts)	6 (15 pts)
3 (10 pts)	7 (20 pts)
4 (15 pts)	8 (15 pts)

115 points total

Problem 1. Limits!!

(a) Find, if it exists, the value of the limit

$$\lim_{x\to 1} \frac{\sqrt{2+x+x^2}-2}{x-1}.$$

(b) Find, if it exists, the value of the limit

$$\lim_{x\to 0} \frac{\sin(3x^2)}{4x\,\sin(2x)}.$$

(c) Find, if it exists, the value of the limit

$$f(x) = \lim_{x \to +\infty} \frac{e^{2x}}{e^x + 3e^{2x}}.$$

(d) Find, if it exists, the value of the limit

$$f(x) = \lim_{x \to -\infty} \frac{e^{2x}}{e^x + 3e^{2x}}.$$

Problem 2. Antiderivatives and integrals!!

(a) Find the expression of the antiderivative

$$F(x) = \int \frac{e^{2x}}{e^{2x} - 1} \ln^3(e^{2x} - 1)\, dx.$$

(b) Find the expression of the antiderivative

$$F(x) = \int \frac{x+1}{x-2}\, dx.$$

(c) Find the value of the integral

$$I = \int_0^{\frac{\pi}{4}} \sec^4(x)\, dx.$$

Hint: $\sec^2(x) = 1 + \tan^2(x)$

Problem 3. Find the derivative of the following function **using the definition**. No credit will be given for using derivative rules.

$$f(x) = \frac{1}{1+x^2}$$

Problem 4. Let $s(x)$ be the following function

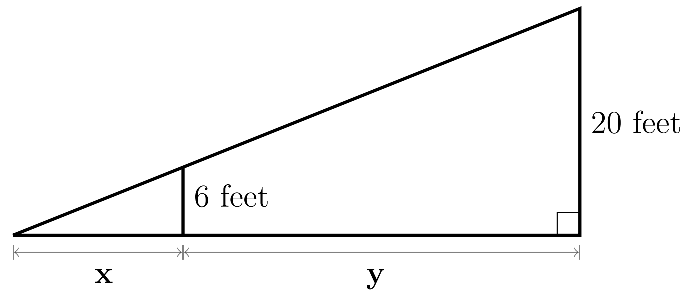
$$s(x) = \int_1^x \frac{1}{\sqrt{t^2+3}}\, dt.$$

(a) Show that $s(x)$ is one-to-one on $\mathbb{R}$.

(b) Find $(s^{-1})'(0)$.

(c) Use a line tangent to the graph of $s^{-1}(x)$ to approximate the value of $s^{-1}(0.1)$.

Problem 5. A 6-foot-tall man walks at the speed of 7 ft/sec straight towards a streetlight that is 20 feet high.



(a) At what rate is the length of his shadow changing?

(b) At what rate is the tip of his shadow moving along the ground?

Problem 6. A factory wants to build cylindrical office waste bins with a hemispherical cap on the top and a disk at the bottom. Each bin is made of 6π square feet of material. What is the maximal capacity of a bin? You must carefully justify your methods.

Area of a cylinder = (circumference)(height) $= 2\pi rh$

Volume of a cylinder = (area)(height) $= \pi r^2 h$

Area of a sphere $= 4\pi r^2$

Volume of a sphere $= \dfrac{4}{3}\pi r^3$

Problem 7. Consider the function $f(x) = \dfrac{(x^4+1)^{\frac{1}{4}}}{1-x}$ on the domain $(-\infty, 1) \cup (1, +\infty)$.

(a) Investigate for the existence of horizontal and vertical asymptotes of the graph of f. Your answer must be supported by the careful calculation of relevant limits.

Hint: $(x^4)^{\frac{1}{4}} = |x|$

(b) $f'(x) = \dfrac{(x+1)(x^2-x+1)}{(1-x)^2(x^4+1)^{\frac{3}{4}}}$. Note that (x^2-x+1) is always positive.

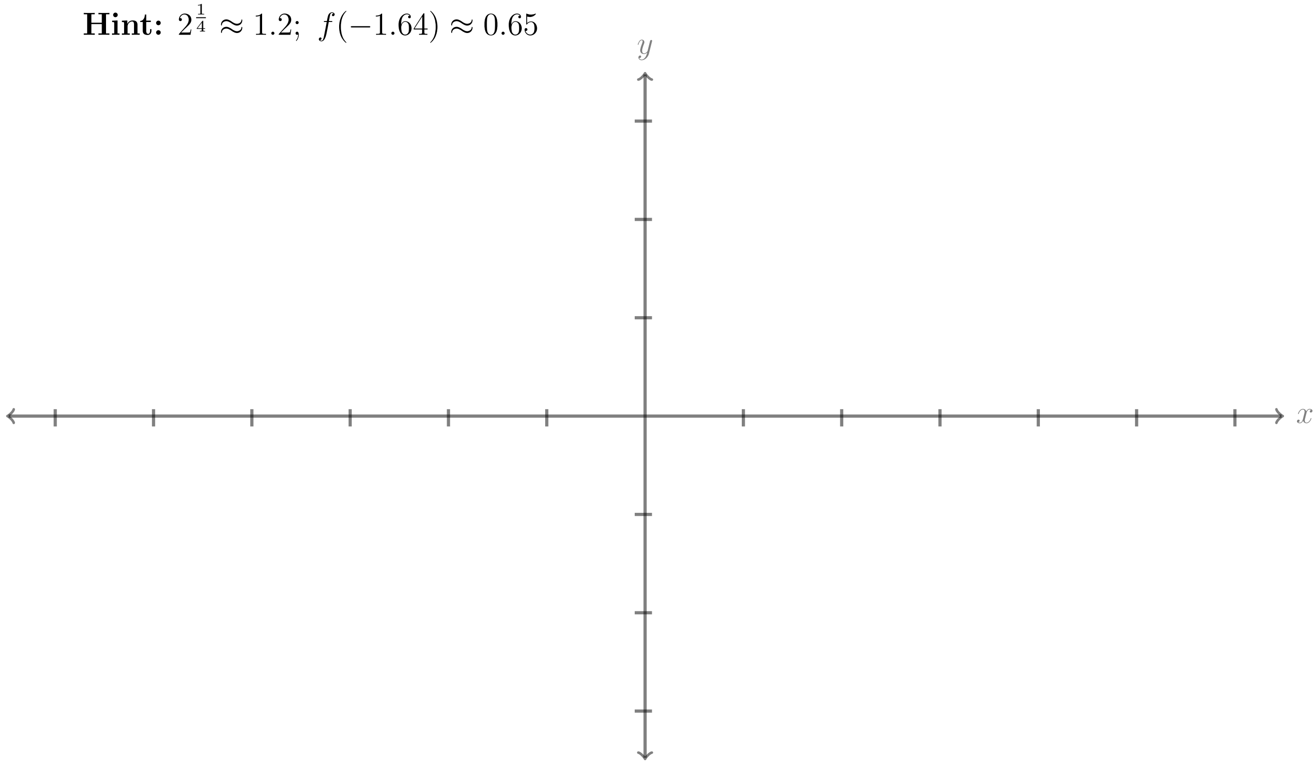
Study the sign of f', then determine the intervals of of increase, and of decrease of f. Indicate the values of local extrema, if any.

(c) $f''(x) = \dfrac{(x + 1.64)}{(1 - x)} M(x)$, where $M(x) > 0$.

Study the sign of f'', then determine the intervals where f is concave up, and where it is concave down. List all inflection points, if any.

(d) Based on all the information gathered in the previous questions, sketch the graph of f as accurately as possible. Include all relevant facts as well as some remarkable points.

Hint: $2^{\frac{1}{4}} \approx 1.2$; $f(-1.64) \approx 0.65$



Problem 8. Consider the following function, defined on $[-2, 0]$, by

$$f(x) = \begin{cases} x^2 + 4x, & -2 \leq x \leq -1 \\ 2x^2 - 4x - 9, & -1 < x \leq 0 \end{cases}.$$

(a) Is this function guaranteed to have a global (absolute) maximum and global minimum on $[-2, 0]$? Carefully justify your answer.

(b) Find the point $x = c$ that satisfies the **Mean Value Theorem for Derivatives** on $[-2, 0]$. If this function has no such point, explain why.

(c) Find the global **maximum** and global **mimimum** of $f(x)$, if they exist, and also where each occurs.

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