Math 125 Final Exam

Wednesday, December 11th, 2019, 2:00–4:00pm

Directions. Fill out your name, signature and student ID number on the lines below **right now**, before starting the exam.

You must show all your work and carefully justify your methods to obtain full credit. Answers must be neat, organized and unambiguous. You must state and justify the methods used.

Simplify your answers. Any fraction should be written in lowest terms. **No cell-phone, no calculator, no cheat sheet nor scratch paper are allowed**. Remember, USC considers cheating as a very serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told at the end of the exam.

Full name (please print): ______________________

Signature: ______________________

Student ID: ______________________

- ☐ G. Dreyer 9am
- ☐ Q. Feng 12pm
- ☐ G. Reyes 1pm
- ☐ H. Yue 9am
- ☐ G. Dreyer 10am
- ☐ N. Haydn 11am
- ☐ F. Tabing 12pm
- ☐ H. Yue 10am
- ☐ N. Emerson 2pm
- ☐ G. Reyes 11am
- ☐ J. Taylor 12pm

1 (18 pts)	5 (23 pts)
2 (16 pts)	6 (20 pts)
3 (15 pts)	7 (16 pts)
4 (16 pts)	8 (16 pts)

Total: / 140 points

Problem 1. Consider the function $f(x) = 3x^{\frac{5}{3}} + 60x^{\frac{2}{3}}$.

a) Show that $f'(x) = \dfrac{5(x+8)}{\sqrt[3]{x}}$. Then find and classify all local extrema of f.

b) Find the values of the global extrema of f on the domain $[-1, 1]$.

c) Justify briefly, but precisely, why f has global extrema on the interval $[-1, 1]$.

Problem 2. Let x, y, z be the lengths of the sides of a right triangle, where z is the length of the hypothenuse. Assume that x decreases at a rate of $\sqrt{3}$ ft/min, and y increases at a rate of 4 ft/min. At time $t = 0$, we know that $x = 2$ ft and $y = 2\sqrt{3}$ ft.

a) Find the rate of change of z at time $t = 0$. Your answer must be fully simplified.

b) Let θ be the angle between the sides x and z. Find the value of θ at time $t = 0$.

c) Find the rate of change of θ at time $t = 0$. Your answer must be fully simplified.

Problem 3. Consider the integrals $I = \int_0^2 (1 + x^2)\, dx$ and $J = \int_0^2 e^{x^2}\, dx$.

<u>*Note:*</u> *the value of the integral J cannot be evaluated explicitly.*

a) Write I as the limit of a right Riemann sum.

b) Compute the limit of the Riemann sum. **<u>Hint:</u>** $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

c) Given that $e^{x^2} \geq 1 + x^2$ for all values of x, determine which value is larger: J or 4.59?

Problem 4. Consider the function $f(x) = \ln(1 - x) + e^{-x}$, for $x < 1$.

a) Show that f is one-to-one on the interval $(-\infty, 1)$.

b) Determine the domain of f^{-1}. You must justify your findings.

c) Find an approximate value for $f^{-1}(0.92)$. Your final answer should be given as a decimal value.

Problem 5. Consider the function $f(x) = \dfrac{\sqrt{4-x^2}}{x+1}$ on the domain $[-2,-1) \cup (-1,2]$. You may freely use any of the following facts.

i) $f'(x) = \dfrac{-x-4}{(x+1)^2\sqrt{4-x^2}}$

ii) $f''(x) = \dfrac{(x+1.84)(1.38-x)K(x)}{x+1}$, where $K(x) > 0$

iii) $f(1.38) = 0.6$

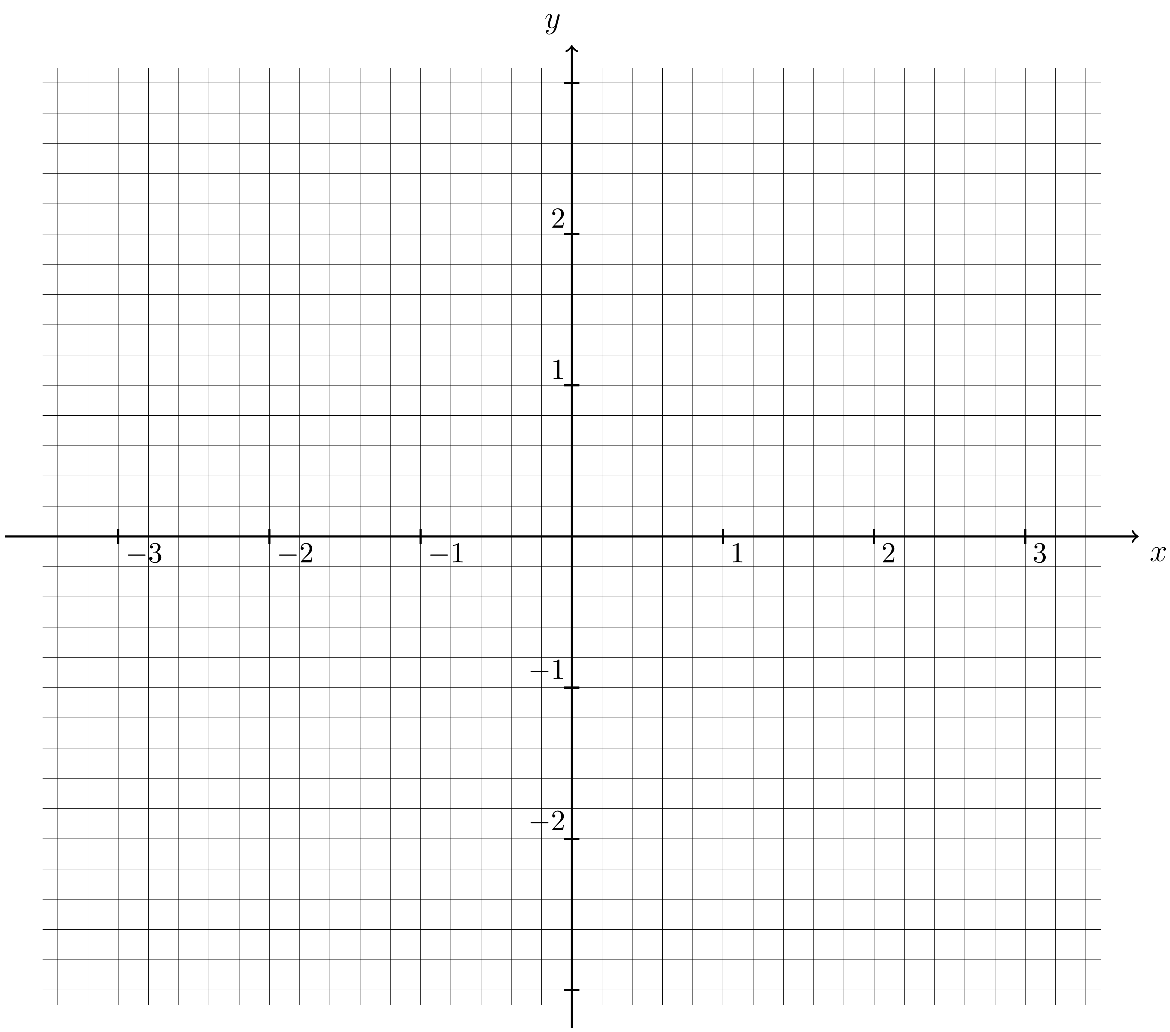
iv) $f(-1.84) = -0.93$

a) Study the sign of $f'(x)$. Determine the intervals where f is increasing, and the intervals where it is decreasing. Indicate the values of the local extrema, if any. You must justify your findings.

b) Study the sign of $f''(x)$. Determine the intervals where f is concave up, and the intervals where it is concave down. List the inflection points, if any. You must justify your findings.

c) Investigate for the existence of vertical/horizontal asymptotes. Your findings must be supported by the careful calculation of relevant limits. (Each vertical asymptote must be supported by two limits.)

d) Based on all the information gathered in the previous questions, sketch the graph of f as accurately as possible. Include and clearly label all relevant points and asymptotes.



Problem 6. Consider the function f defined by

$$f(x) = \begin{cases} \dfrac{\sin(5x^2)}{x} + 8, & \text{if } x < 0 \\ (a-b)x + 2a, & \text{if } x \geq 0 \end{cases} .$$

a) Determine the value of the constant a for which f is continuous at $x = 0$. You must carefully justify your answer.

b) Determine the values of the constants a and b for which f is differentiable at $x = 0$. You must carefully justify your answer.

Problem 7. One wants to calculate the value of the limit

$$\ell = \lim_{x \to 0^+} \frac{\sqrt[3]{5x+64} - 4}{\sqrt[5]{x}}.$$

a) Show that $\sqrt[3]{5x+64} < 4 + \frac{5}{48}x$ for all $x > 0$. You must justify your methods.

b) Use the inequality of part a) to compute the value of ℓ. You must justify your methods.

Problem 8. Consider the function F defined on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by the integral

$$F(x) = \int_1^{2\sin(x)} \sqrt{4 - t^2}\, dt.$$

a) Calculate $F'(x)$. You must justify your findings.

b) Use the result of part a) to find an explicit expression for $F(x)$. You must justify your findings. <u>**Hint:**</u> $\cos^2(x) = \dfrac{1}{2}\cos(2x) + \dfrac{1}{2}$

c) Use the result of part b) to determine the value of $\displaystyle\int_1^2 \sqrt{4 - t^2}\, dt$.

Scratch-scratch zone...

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