

Question 1. Circle only the true statements.

- a) The demand curve relates the quantity of units that the public is willing to buy to the price of the unit.
- b) The supply curve is the number of units that the manufactures should make to maximise profit.
- c) The graph of $y = (x - 2)^2$ is equivalent to the graph of $y = x^2$, but shifted two units to the left.
- d) The profit function is given by adding the cost and revenue function.

Question 2. If the marginal cost of producing q units is negative and the marginal revenue of producing q units is positive. What can you say about the marginal profit?

- a) The marginal profit is positive and so we want to decrease production.
- b) The marginal profit is positive and so we want to increase production.
- c) The marginal profit is negative and so we want to increase production.
- d) The marginal profit is negative and so we want to decrease production.
- e) We don't have enough data to answer.

Question 3. Let $T(r)$ be the total time in months in order to pay off a \$1000 credit card loan using minimum monthly payments, if the annual interest rate is $r\%$. What is the practical meaning of $T'(r)$?

- a) $T'(r)$ is equal to the minimum monthly payment.
- b) $T'(r)$ is the rate of change of the annual interest rate with respect to time.
- c) If the annual interest rate were to increase by 1%, the total time required to pay off the credit card loan would increase by about $T'(r)$ months.
- d) If an extra dollar were to be charged to the credit card, the total time required to pay off the credit card loan would increase by about $T'(r)$ months.
- e) None of the above.

Question 4. Select the correct derivative $f'(x)$ of the function $f(x) = x^x$.

- a) $x \cdot x^{x-1}$
- b) $\ln(x) \cdot x^x$
- c) $(\ln(x) + 1) \cdot x^x$
- d) x^{x-1}
- e) None of the above.

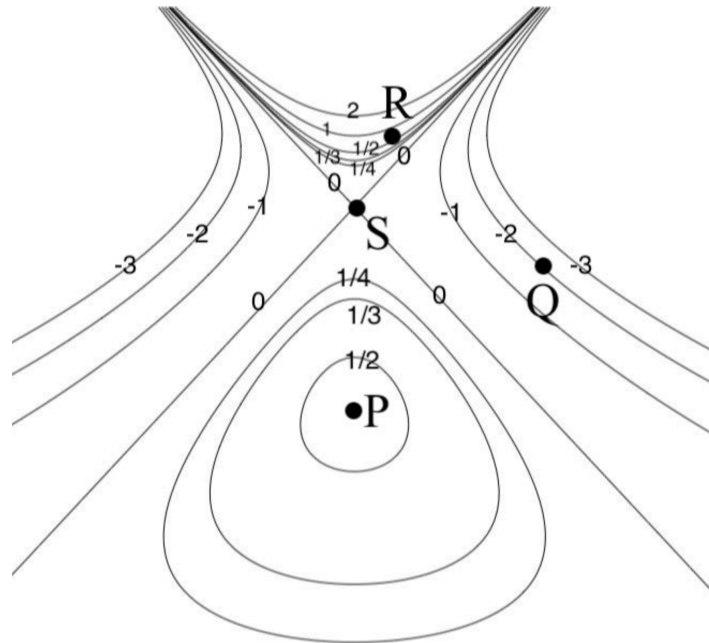
Question 5. What is the 6th derivative of $f(x) = \pi e^{kx} + \ln(x) + 7$.

- a) 0.
- b) 1
- c) $\pi k^6 e^{kx} + (-1)^5 \frac{120}{x^6}$
- d) $\pi k^3 e^{kx} + (-1)^3 \frac{720}{x^6}$

Question 6. If $f'(x) = 0$ and $f''(x) > 0$, what can you say about x ? Circle all that apply.

- a) x is a critical point for f .
- b) x is a critical point for f' .
- c) x is a local minima for f .
- d) x is a local maxima for f .
- e) None of the above.

Question 7. The contour map of $f(x, y)$, where x is the horizontal and y is the vertical axis is shown below:



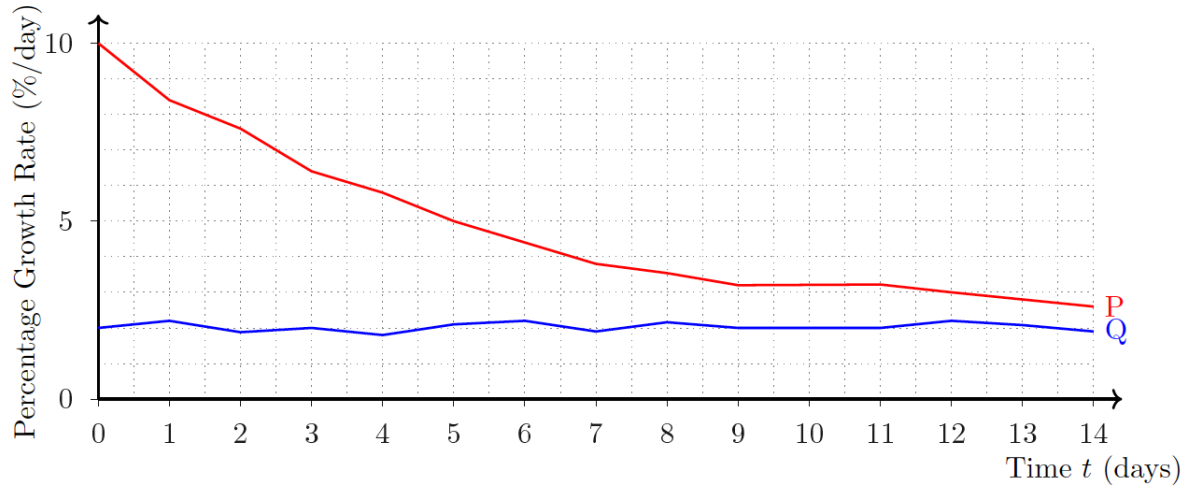
Which of the following statements are true?

- $f_x(P) \approx 0$.
- $f_y(R) < 0$
- $f_y(Q) < 0$
- $f_{xx}(S) < 0$
- $f_{yy}(S) < 0$

Question 8. Let $f(x, y) = e^x y^2 + xy$. Select all answers that represent $\int_R e^x y^2 + xy dA$ with $R = [0, 3] \times [2, 5]$.

- $\int_0^3 \int_2^5 (e^x y^2 + xy) dy dx$.
- $\int_0^3 \int_2^5 (e^x y^2 + xy) dx dy$.
- $\int_2^5 \int_0^3 (e^x y^2 + xy) dy dx$.
- $\int_2^5 \int_0^3 (e^x y^2 + xy) dx dy$.

Question 9. Shown below is the percentage growth rate for the total number of COVID-19 cases as a function of time in days since April 1, 2020 in two different provinces P and Q.



Which of the following equations could be a good approximation for the total number of cases $N(t)$ in province Q? In all answers, assume C is a constant.

- a) $N(t) \simeq 0.02$.
- b) $N(t) \simeq 0.02t + C$.
- c) $N(t) \simeq -0.02t + C$.
- d) $N(t) \simeq Ce^{0.02t}$.
- e) $N(t) \simeq Ce^{-0.02t}$.

Question 10. Which of the following is greatest for a good being sold at its equilibrium price?

- a) The equilibrium price.
- b) The average price producers were willing to produce the good at.
- c) The average total gain from trade (also known as the average total surplus).
- d) It cannot be determined from the information given.

Question 11. Circle all correct differentiation rules.

- a) $(cf(x))' = f'(x)$.
- b) $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$.
- c) $\frac{d}{dx}x^n = nx^{n-1}$.
- d) $\frac{d}{dx}\frac{f}{g}(x) = \frac{f'(x)g'(x)-f(x)g''(x)}{g^2(x)}$

Question 12. Husky Pup Pet Store sells two competing brands of dog food: Fido Food and Canine Cuisine. Let $F(x, y)$ be the number of bags of Fido Food sold per week, where x is the price (in dollars) of each bag of Fido Food and y is the price (in dollars) of each bag of Canine Cuisine. Please select all statements are both correct and well-justified.

- F_y is positive, because an increase in the price of Canine Cuisine will result in a decrease in the demand for Canine Cuisine and an increase in the demand for Fido food.
- F_y is positive, because an increase in the price of Canine Cuisine will result in an increase in the price of Fido Food.
- F_y is negative, because if the demand for Canine Cuisine increases, the demand for Fido food will decrease.
- F_y is negative, because an increase in the price for Fido Food will result in a decrease in the demand for Fido Food.
- None of the above.

Question 13 (6 points). A volcano is erupting and the rate at which ash is released into the atmosphere is being measured, with measurements taken once every minute. Call this rate $r(t)$, where t is measured in seconds after the initial eruption and $r(t)$ is measured in tonnes per second.

- (a) (1 point) Write an integral which represents the total amount of ash released in the first five minutes after eruption.
- (b) (3 points) Here are values of $r(t)$ at each minute after the eruption.

t	0	60	120	180	240	300
$r(t)$	0	2	4.5	7	5.5	8

Give an estimate for the total amount of ash released after five minutes.

- (c) (2 points) Volcano observatories send out a Volcano Observatory Notice for Aviation (VONA) to warn pilots about high levels of ash emissions. The observatory monitoring this region will issue a VONA if the amount of ash released exceeds 1500 tonnes in a day. If 450 tonnes of ash were released into the atmosphere during a smaller, nearby eruption earlier the same day, will the observatory issue a VONA during the first five minutes of the current eruption?

Question 14. Compute the following integrals:

a) $\int_1^5 e^{5x} + x^4 + \frac{5}{x} dx.$

b) $\int x^3 e^{x^2} dx.$

c) $\int_1^4 \ln(4x) dx.$

Question 15. The following table details the anticipated profit generated by different types of Biomass Power generators based on the amount of power that will be used. Numbers in Parenthesis are negative - so $(105.30) = -105.30$.

Designed Capacity (kW)	Power Demand (kW)			
	3000	5000	8000	10000
3000	4,470,740.00	4,470,740.00	4,470,740.00	4,470,740.00
5000	1,481,316.30	6,154,717.00	6,154,717.00	6,154,717.00
8000	(2,213,380.58)	2,460,019.42	9,964,220.00	9,964,220.00
10000	(3,897,126.95)	776,273.05	8,280,473.05	12,938,480.00

Estimate the profit from a Biomass power plant designed to generate up to 7000 kW if 4500 kW of power is demanded by the electric utility.

Question 16. An eight-year old girl named Carmen decides to start raising money for her future USC tuition by selling lemonade. Based on Carmen's experience with selling lemonade on her front lawn, she finds that for every dollar she increases the price, the number of glasses of lemonade she sells decreases by 6. She also knows that if she sets the lemonade at \$5, she will sell 12 glasses of lemonade.

- What is the demand equation for this situation? Carefully explain your reasoning.
- Carmen's costs are 50 cents per glass of lemonade. Write down her profit as a function of x , the number of glasses of lemonade sold.
- How many glasses of lemonade does Carmen have to sell in order to maximize her profit? Show all work and verify that this quantity maximizes profit. If you use a test, state the name of the test you used.

Question 17. A mining company takes cores in a region of Elko county Nevada to measure the density of gold ore in a region near a prior gold mine. They summarize their findings in the following table:

$\rho(x, y)$	x=0m	x=50m	x=100m	x=150m	x=200m
y=0m	27	15	12	10	2
y=100m	23	20	14	10	8
y=200m	14	25	18	15	11
y=300m	10	15	23	17	12
y=400m	6	13	20	20	15
y=500m	1	8	14	23	19

- Estimate the average density of gold in this region.
- If the price of gold is \$1722.00 per oz, how much is it worth to dig up each meter of soil in this region?

Question 18. Find the average value of $f(x, y) = xe^{xy}$ on the rectangle given by letting x vary between 0 and 2 and y between 0 and 1.

Question 19. Classify the critical points of $f(x, y) = x^2 - xy^2 + y^2$.