# Math 118: Business Calculus — Spring 2019
# Final Exam

May 8, 2019

First Name: ______________________________ (as in student record)

Last Name: ______________________________ (as in student record)

USC ID: ______________ Signature: ______________________________

Please circle your instructor and lecture time:

| Hall | Haskell | Tabing |
|---|---|---|
| 11am | 9am | 1pm |
| | 12pm | 2pm |

- This exam has 11 problems, and will last 120 minutes.
- You may use any scientific **non-graphing** calculator. Any calculation done on the test that consists of something other than addition, subtraction, multiplication, division, raising a number to a power, or taking logs of a number, must be explicitly shown.
- Cell phones must be turned off and put away during the test. You may not use your cell phone as a calculator.
- You may use one 8.5 x 11 or A4 handwritten 'cheat' sheet (front and back). It must be written in pen or pencil *in your own handwriting*; typed or photocopied sheets are not permitted.
- Try to keep your solutions in the space provided for each question. You may continue solutions on other pages if you clearly indicate where to find your solution.
- Show all of your work and justify every answer to receive full credit.

**Do not write in the box below:**

| Q1 | Q2 | Q3 | Q4 | Q5 | | Partial 1 |
|---|---|---|---|---|---|---|
| /18 | /18 | /20 | /20 | /14 | | /90 |
| Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Partial 2 |
| /24 | /17 | /14 | /20 | /20 | /15 | /110 |

/200

**Question 1** (18 points)**.** Suppose the number of cases of measles $M$ (that are reported) $t$ years after 2018 is given by

$$M = 372(2.1)^t$$

(a) What can we say about how $M$ changes as time passes? *Circle all responses that are correct.*

  (a) $M$ increases by 3.1 cases every year.
  (b) $M$ increases by 2.1 cases every year.
  (c) $M$ increases by 210 cases every year.
  (d) $M$ increases by a factor of 3.1 every year.
  (e) $M$ increases by a factor of 2.1 every year.
  (f) $M$ increases by a factor of 210 every year.
  (g) $M$ increases by 3.1% every year.
  (h) $M$ increases by 2.1% every year.
  (i) $M$ increases by 110% every year.

(b) Find the (instantaneous) relative rate at which the number of cases of measles is increasing (per year).

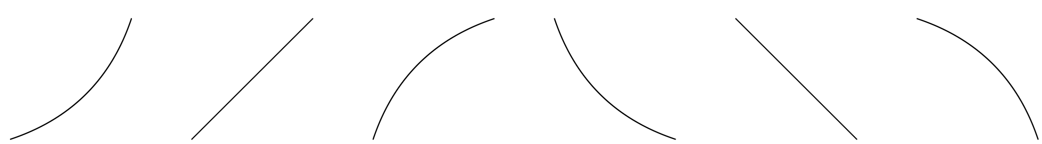
(c) How long will it take for the number of cases of measles to increase by a factor of 10? Write your answer correct to 2 decimal places.

**Question 2** (18 points)**.** Let $L$ denote the number of people (in thousands) who received liposuction each year. It is known that:

- $L = 191.3$ in 2011.
- $L$ increased at an average rate of 4.5 thousand people per year from 2011 to 2019.
- $\dfrac{dL}{dt} = 5.3$ in 2011.

(a) How many people received liposuction in 2019? (Find the exact number.)

(b) One of the graphs below shows how $L$ changed from $t = 2011$ to $t = 2019$. Which is it? Circle your answer.



A B C D E F

(c) Estimate the number of people who received liposuction in 2013.

**This problem is continued on the next page.**

**Question 2 continued**

(d) Complete the sentence below about your approximation in (c) by choosing the appropriate response for each blank. *Hint: use your answer to (b).*

My answer is ________________

(i) an overestimate
(ii) an underestimate
(iii) exactly right

because the ________________________________________

(i) tangent line at $t = 2011$
(ii) tangent line at $t = 2019$
(iii) tangent line between $t = 2011$ and $t = 2019$
(iv) secant line at $t = 2011$
(v) secant line between $t = 2011$ and $t = 2019$

lies __________ the graph of $L$ at $t = 2013$.

(i) above
(ii) below
(iii) on

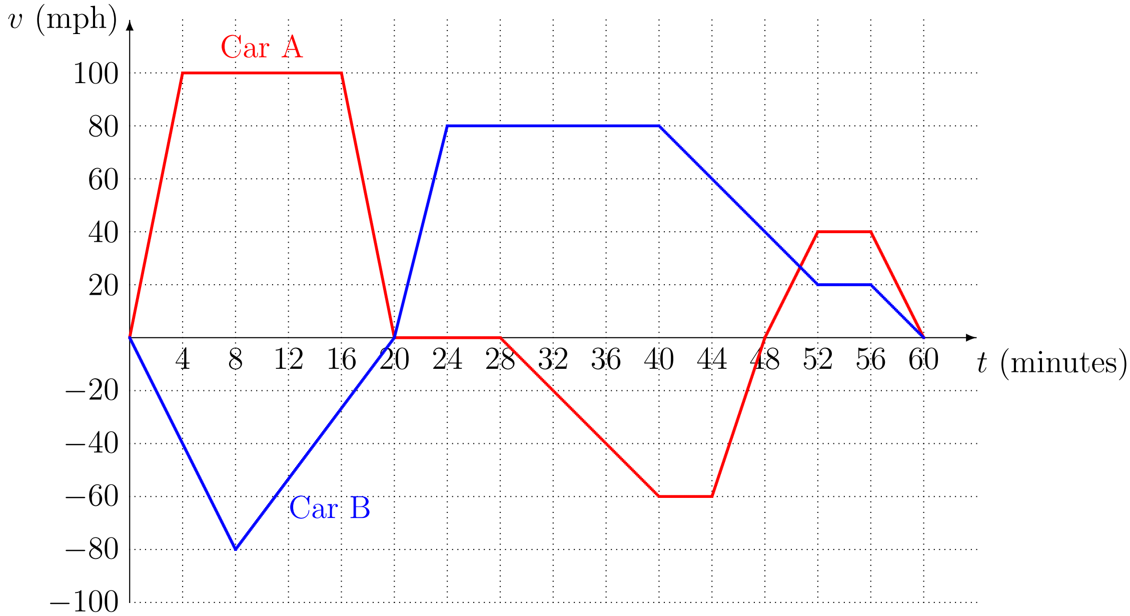
**Question 3** (20 points). Condo Larissian wishes to build a condominium complex with one condo per floor. To stay competitive in the housing market he must make the average cost per condo as small as possible. The costs associated with building the complex are the following.

- There is a fixed cost of 2.5 million dollars (or 2,500 thousand dollars) to buy the land and secure the permits.
- A complex with $n$ floors costs $30n^2$ thousand dollars to build.

(a) What is the average cost per condo when he builds a complex with $n$ floors? (Include both fixed costs and construction costs in your average.)

(b) How many floors should he build if he wants to minimize the average cost per condo? Be sure to justify that you have found a global minimum and not just a critical point. Also, notice that your answer should be an integer.

**Question 4** (20 points)**.** A bank is situated on a straight road that runs East-West. After robbing the bank, two robbers drive off in separate cars. The velocities (in mph) of the two cars are shown below, where a positive velocity indicates the car is moving eastbound and a negative velocity indicates the car is moving westbound.




(a) Is Car A speeding up, slowing down, or maintaining a constant speed in the interval $28 \leq t \leq 40$? Explain.

(b) One of the cars stopped at a convenience store to pick up a donut. Which car was this? Explain briefly.

(c) How far did Car B travel in the first 20 minutes? Remember to show your work and give units.

(d) Were the cars reunited at $t = 60$? Explain carefully.

**Question 5** (14 points)**.** The marginal cost $MC(q)$ (in dollars per units), when $q$ units have been produced so far, is given by

$$MC(q) = 0.06q^2 - 2.8q + 103$$

The fixed costs are 10,000 dollars.

(a) Approximately how much does it cost to produce the 101st unit?

(b) The manufacturer has already produced 100 units. How much will it cost to produce the next 50 units?

**This problem is continued on the next page.**

**Question 5 continued**

(c) What is the total cost of producing 150 units?

**Question 6** (24 points). Compute the following definite and indefinite integrals:

(a) $\displaystyle\int \left( \frac{2}{3t^2} - t^{-1} + 3 \cdot 2^t \right) \, dt$

(b) $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$

**This problem is continued on the next page.**

**Question 6 continued**

(c) $\int t(1.2)^t \, dt$

**Question 7** (17 points)**.** When a certain company spends $s$ thousand dollars advertising on social media and $t$ thousand dollars advertising on television, its revenue is $R = f(s,t)$. Suppose that:

$$f_s(200, 250) = 1.7 \quad \text{and} \quad f_t(200, 250) = 0.8$$

(a) Assuming the company currently spends \$200,000 advertising on social media and \$250,000 advertising on television, which of the following are true for the company? *(Circle all that apply.)*

(A) More spending on social media advertisements will increase revenue.

(B) More spending on television advertisements will increase revenue.

(C) Less spending on social media advertisements will decrease revenue.

(D) More spending on social media advertisements will increase profit.

(E) More spending on television advertisements will increase profit.

(b) In each part below, estimate which quantity is larger and by approximately how much. Circle the larger quantity.

i) $f(200, 250)$ or $f(200, 252)$

ii) $f(202, 250)$ or $f(200, 252)$

iii) $f(200, 250)$ or $f(199, 253)$

**Question 8** (14 points).

(a) Suppose $f(x,y) = e^{x^2y+xy}$. Find $f_x(x,y)$.

(b) Suppose $c = \dfrac{ab}{a^2+b^2}$. Find $\dfrac{\partial c}{\partial a}$.

(c) Suppose $P = h(r,t) = Be^{-rt}$. Find $\dfrac{\partial^2 P}{\partial r \partial t}$.

**Question 9** (20 points). A company has a monopoly on two of its products, whose demand curves are given by

$$p_1 = 600 - q_1 \qquad \text{and} \qquad p_2 = 500 - q_2.$$

Its revenue from selling the two products is

$$R = p_1 q_1 + p_2 q_2.$$

(a) Using the above demand curves, write an expression for the company's revenue $R$ as a function of $q_1$ and $q_2$.

(b) Suppose the cost of producing $q_1$ units of the first product and $q_2$ units of the second product is

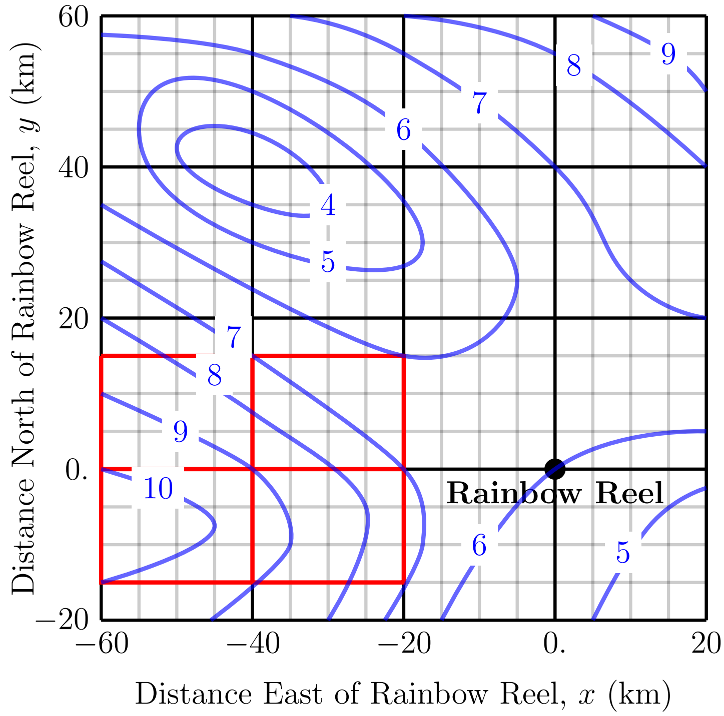
$$C(q_1, q_2) = 10000 + 200q_1 + 150q_2 + q_1 q_2.$$

Write an expression for the company's **profit** $\pi$ as a function of $q_1$ and $q_2$.

(c) Find all critical points for the company's **profit** function. Decide whether each represents a local maximum, local minimum, or neither.

(If desired you may switch from $(q_1, q_2)$ to $(x, y)$ to simplify the notation.)

**Question 10** (20 points)**.** The county of Soggy Bottom occupies a square region that runs 80 kilometers East-West and 80 kilometers North-South. The contour diagram below shows the number of centimeters of rainfall $R(x, y)$ that fell at different points in the county during the month of April. The numbers $x$ and $y$ indicate the distances (in kilometers) east and north of the town of Rainbow Reel.




(a) Is $R_y(0, 0)$ positive or negative? Explain.

(b) Mark clearly on the contour graph the point $L$ where the least rain occurred. About how much rain did it get?

(c) Mark clearly on the contour graph the point $M$ where the most rain occurred. About how much rain did it get?

**This problem is continued on the following page.**

**Question 10 continued**

(d) Estimate the total volume of water that fell during the month of April in the rectangle highlighted in red where $-60 \leq x \leq -20$ and $-15 \leq y \leq 15$. Find upper and lower estimates using $\Delta x = 20$ and $\Delta y = 15$. Clearly show how you obtained your estimates and give your answers in cubic centimeters. *Please note: There are* $10^5$ *cm in a kilometer.*

**Question 11** (15 points)**.** Evaluate (exactly) the integral

$$\iint_R \frac{x}{y} \cdot (y^2 - x^2)\, dA$$

where $R$ is the rectangle in which $-1 \le x \le 2$ and $1 \le y \le 4$.