Question 1. The **demand curve** for a certain product is given by

$$q = 1000 - 20(p - 50).$$

(a) What price p and quantity q correspond to the maximum possible **revenue**? You must justify that your solution represents a global maximum.

(b) If the marginal cost of production is

$$MC = 120 - 2q + (.03)q^2,$$

find the price p and quantity q that corresponds to maximum **profit**. You must justify that your solution represents a global maximum.

(c) If the fixed costs are equal to $C(0) = 100$, find the cost function $C(q)$ and determine whether the maximum in Part (b) represents a net profit or a net loss.

Question 2. The effect of advertising is typically to move the demand curve to the right.

Assume that, regardless of price, to gain an additional n customers a company must spend $A = (0.025)n^2$ dollars on advertising. The **demand curve** from Question 1 then becomes

$$q = (1000 + n) - 20(p - 50).$$

Let $N = R - A$ be the net value of revenue less advertising, where R is the revenue, and $A = (0.025)n^2$ is the amount spent on advertising.

(a) Write N as a function of p and n and verify that $p = 100$, $n = 1000$ is a critical point for this function.

(To simplify the notation, you may use the variables $x = p$ and $y = n$ to write $N = f(x, y)$.)

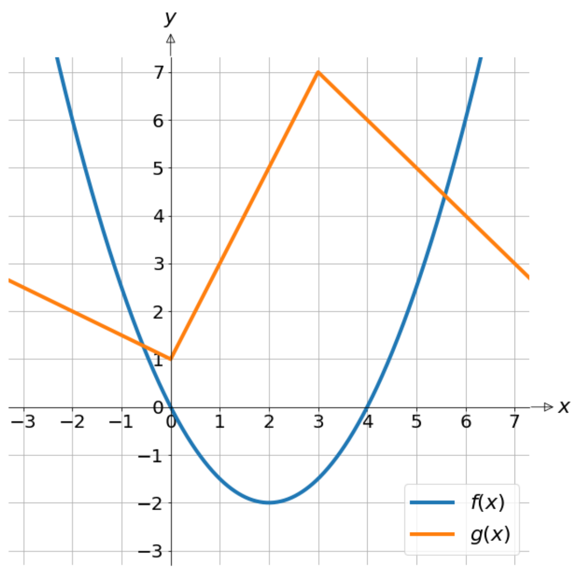
(b) Use a second derivative test to classify this critical point as a local max, local min, or neither.

Question 3. The graphs of two functions f and g are shown below, where g is a piecewise linear function and f is the quadratic function

$$f(x) = \frac{1}{2}x^2 - 2x.$$

(a) Let $v(x) = g\left((f(x))^2\right)$. Find $v'(1)$.

(b) Let $k(x) = \dfrac{e^{f(x)}}{g(x)}$. Find $k'(4)$.



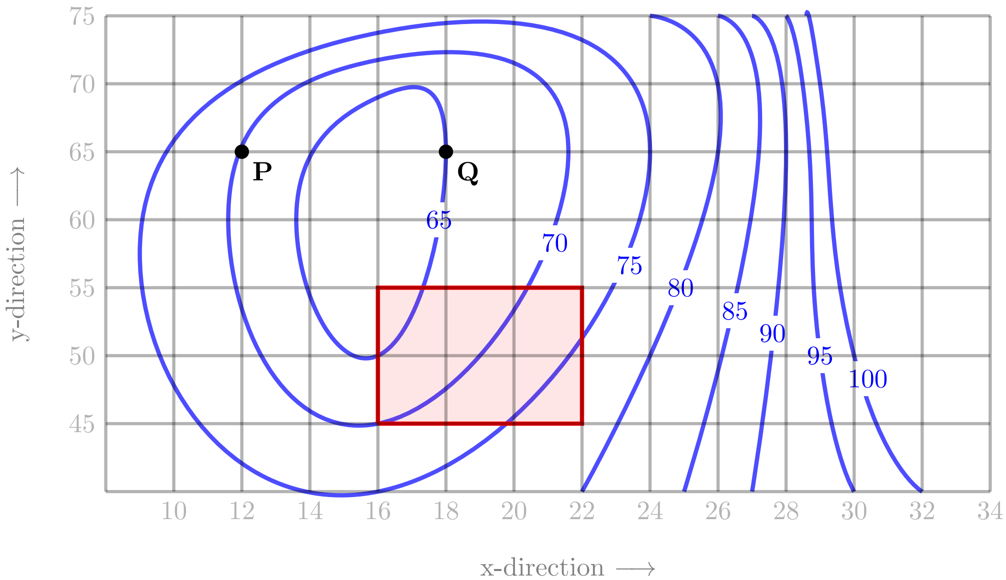
Question 4. Consider the function $f(x, y) = \dfrac{y}{6x^2 + y^3}$.

(a) Find both partial derivatives of f at the point $(-3, 2)$.

(b) Estimate the value of $f(x, y)$ when $x = -2.7$ and $y = 1.91$ by hand, using methods from this course.

(You'll earn no credit for computations done with a computer.)

Question 5. The following contour diagram shows the **altitude** A (in meters) around a fresh meteor crater, based on the (x,y)-coordinates centered at a scientist's base camp.



- Estimate the values of $\dfrac{\partial A}{\partial x}$ and $\dfrac{\partial A}{\partial Y}$ at the point $(12, 65)$ marked **P**.

- Use the derivatives in Part (a) to estimate the altitude A at cooridnates $(12.5, 64.8)$.

- At the point $(18, 65)$ marked **Q**, is $\dfrac{\partial^2 A}{\partial x^2}$ positive or negative? Explain how you know.

- Within the region highlighted in red, with $16 \leq x \leq 22$ and $45 \leq y \leq 55$, use a **Riemann sum** with six rectangles to estimate the **average altitude**.

Question 6. A long, heavy rain fills a lake until the dam that controls the lake bursts, and water begins to flood out of the dam. The rate at which water is entering/exiting the lake is modeled by

$$v(t) = -t^3 \ln(t)$$

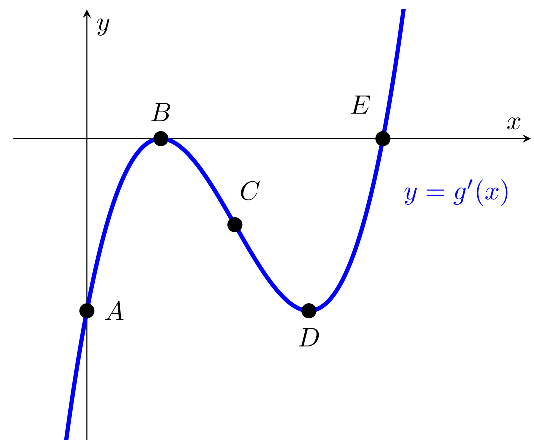
where t is the time (in days) since the storm began and v is the flow of water (in km^3 per day). If the lake held $5\,\text{km}^3$ of water before the storm, find the new volume of the lake after **36 hours**.

Question 7. For a function $g(x)$, the graph of the **derivative** $g'(x)$ is shown below.

(a) Which points in this image represent **local** maximum and minimum values of $g(x)$? Name each point and explain how you classified it.

(b) Which of the 5 labeled points represent **global** maximum and minimum values of $g(x)$ on the interval $[\,0, E\,]$?

(c) Does the function $g(x)$ have any **inflection points**? Identify their locations and explain your reasoning in terms of **concavity**.



Question 8.

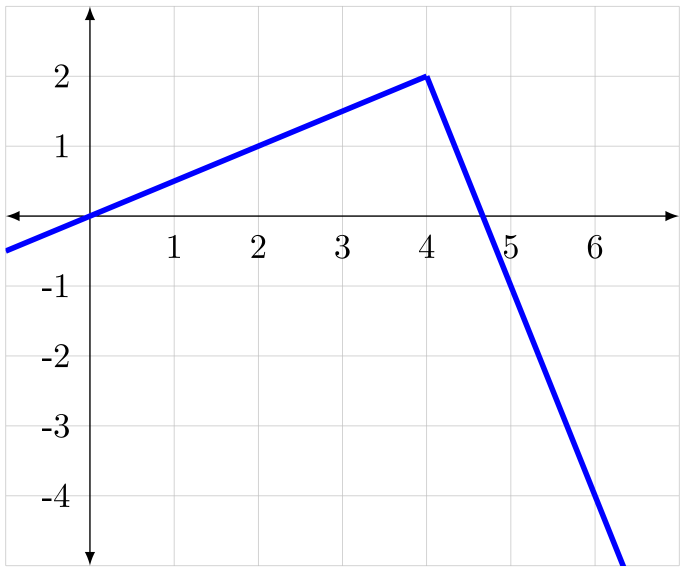
A food truck earns revenue for a company at a continuous rate of $S = 56,000t + 80,000$ dollars/year. The cost of the truck is $\$251,000$ and t is the time in years since the truck was purchased.

The company also has access to a savings account earning 7% interest per year, compounded continuously.

After **two years**, excluding all other expenses, has the truck paid for itself? That is, has the present value of the revenue passed the cost of the truck?

(You'll earn partial credit if you use a computer to compute your integrals. You'll earn full credit for computing the integral correctly by hand and showing all your steps.)

Question 9. The graph of a function $f(x)$ is shown below.



Suppose $g(x)$ is a differentiable function such that $g(0) = 2$ and $g(1) = 6$. Use this information along with the graph of $f(x)$ to evaluate the following integral.

$$\int_0^1 f(g(x)) \cdot g'(x)dx$$

Question 10. Consider the integral

$$I = \int_0^{\sqrt{2}} \int_{y^2}^{2} y^5 e^{x^2}\, dx\, dy.$$

Sketch the domain $\mathcal{R}$ of integration of I in the xy–plane. Then determine the value of I.

(You must carefully explain your methods. You'll earn no credit for computations done with a computer.)