Math 118: Business Calculus — Fall 2020

Final Exam - Version 1

Gradescope Entry Code - **N8824G**

Question 1. A developer can build a house at a cost of:

$$C(x) = 1500 - \frac{500\ln(5x)}{x}.$$

In this function, x is in thousands of square feet and C is in thousands of dollars.

(a) What is the minimum cost to build a home?

(b) If the homeowner wants their new house to be between 500 and 2000 square feet, then what are the maximum and minimum costs?

(c) Find the **average value** of this cost function across square footages ranging from 500 and 2000.

(The average *value* is not the same thing as the average cost *function* C(x)/x.)

Question 2. Consider the function $f(x, y) = \dfrac{x}{6x^2 + y^3}$.

(a) Find both partial derivatives of f at the point $(2, -3)$.

(b) Estimate the value of $f(x, y)$ when $x = 2.09$ and $y = -2.8$ by hand, using methods from this course.

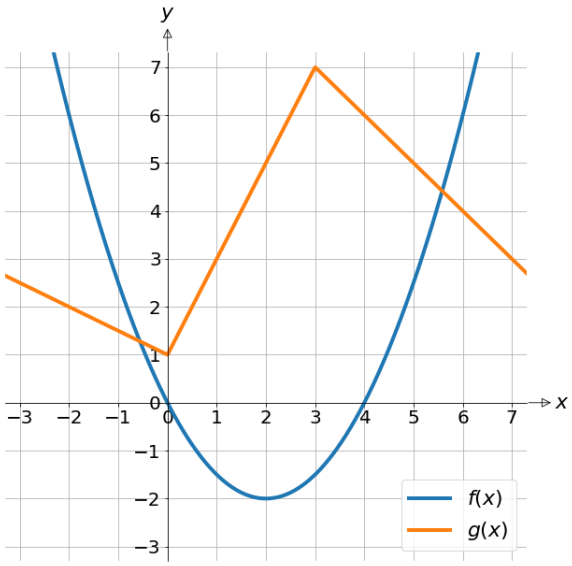
(You'll earn no credit for computations done with a computer.)

Question 3. The graphs of two functions f and g are shown below, where g is a piecewise linear function and f is the quadratic function

$$f(x) = \frac{1}{2}x^2 - 2x.$$

(a) Let $u(x) = f(\ln(g(x)))$. Find $u'(-2)$.

(b) Let $h(x) = f(x) \cdot \sqrt{g(x)}$. Find $h'(2)$.



Question 4. A certain function $f(x, y)$ has the following derivatives:

$$\frac{\partial f}{\partial x} = (x^2 + xy - y^2 + 2x - y)e^x$$

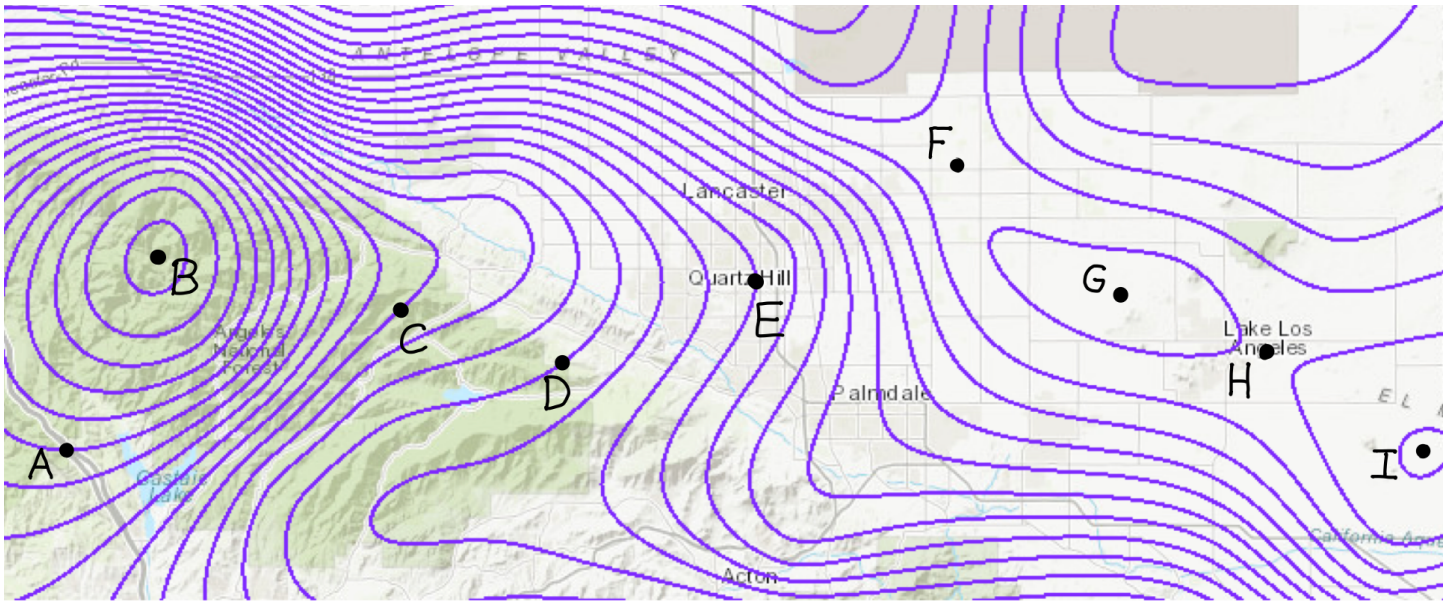
$$\frac{\partial f}{\partial y} = (x - 2y)e^x$$

Find all critical points of $f(x, y)$ and classify them using a second derivative test.

(You'll earn no credit for computations done with a computer.)

Question 5. An *isohyet map* is a contour diagram for the amount of rainfall in a given period of time. It gives the level curves of a function $R = f(x, y)$, which represents the amount of rainfall at a point (x, y).

The isohyet map below represents the estimated 24 hour rainfall (in inches) for a "95th percentile storm" around Lancaster, CA. (Source: LA County Department of Public Works)



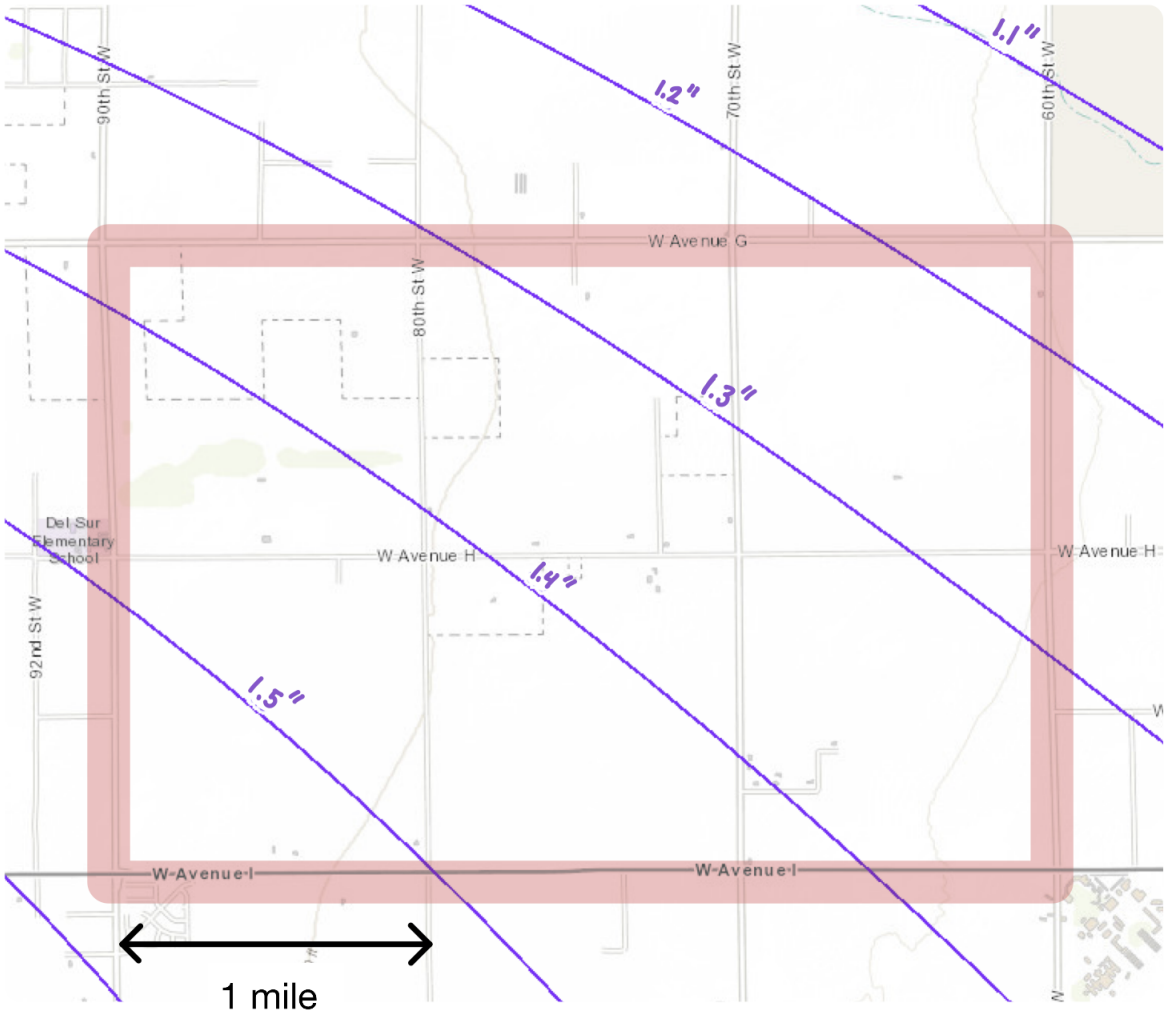
Several points have been marked on the isohyet map. The value of $f(x, y)$ (in inches) at each point is recorded below (rounded to the nearest .05 inch).

Point	A	B	C	D	E	F	G	H	I
$f(x, y)$	2.40	2.95	1.90	1.60	1.30	0.55	0.45	0.55	0.35

Using the above information, along with the contour diagram, list all points where each of the following occurs, and provide a brief explanation for each of your conclusions.

- $f_x = 0$ and $f_y \neq 0$
- $f_x \neq 0$ and $f_y = 0$
- $f_x = 0$ and $f_y = 0$
- $f(x, y)$ has a saddle point
- $f(x, y)$ has a local minimum
- $f(x, y)$ has a local maximum

Question 6. From the map in Question 4, a detailed view of the Antelope Valley is shown below:



The large street blocks are approximately 1 mile $\times$ 1 mile. Estimate the total volume of rainfall in the region between 60th and 90th streets, and between Avenue G and Avenue I, using a **Riemann sum** with six rectangles. Give your answer in cubic inches.

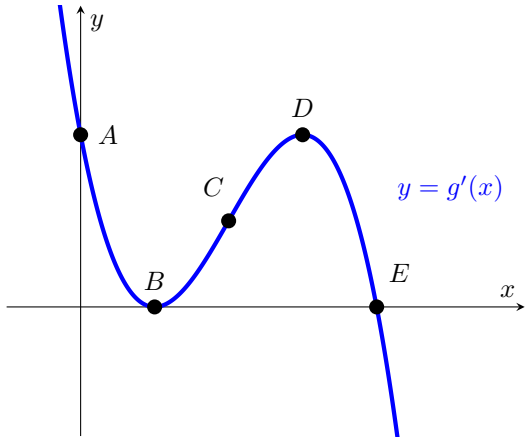
(Conversions: There are 5280 feet in 1 mile, and 12 inches in 1 foot.)

Question 7. For a function $g(x)$, the graph of the **derivative** $g'(x)$ is shown below.

(a) Which points in this image represent **local** maximum and minimum values of $g(x)$? Name each point and explain how you classified it.

(b) Which of the 5 labeled points represent **global** maximum and minimum values of $g(x)$ on the interval $[\,0, E\,]$?

(c) Does the function $g(x)$ have any **inflection points**? Identify their locations and explain your reasoning in terms of **concavity**.



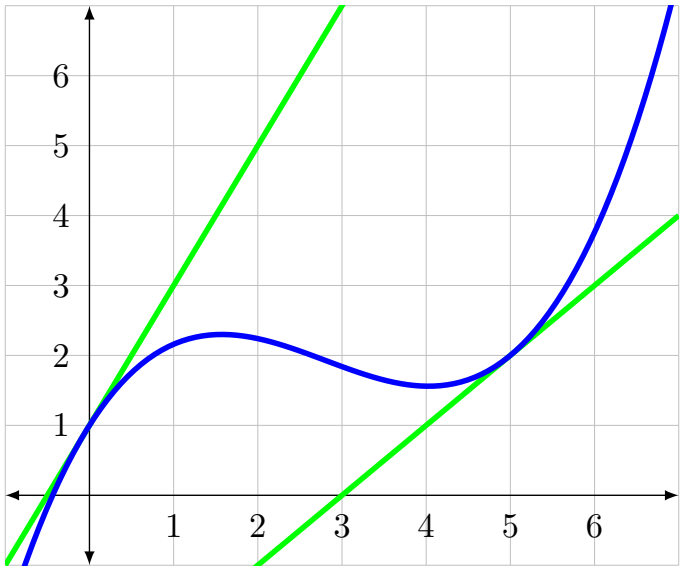
Question 8. A company is expected to earn $\$60,000$ a year, at a continuous rate, for 8 years. You have the chance to buy the rights to the earnings of the company now for $\$350,000$. Either way, you can earn an interest rate of 7%, compounded continuously, by investing money into your savings account.

(a) Should you buy the rights to the earnings of this company? Support your decision with Mathematics.

(b) Given the company's predicted future value at the end of the 8-year period, at what point will the company hold **half** of that value?

(You'll earn partial credit if you use a computer to compute your integrals. You'll earn full credit for computing the integral correctly by hand and showing all your steps.)

Question 9. The graph of a twice-differentiable function $f(x)$ is shown below in blue, along with the lines tangent to $f(x)$ at $x = 0$ and $x = 5$ shown in green.



(a) Use this graph to evaluate: $\displaystyle\int_0^5 f'(x)dx$

(b) Use this graph to evaluate: $\displaystyle\int_0^5 f''(x)dx$

(c) Use this graph to evaluate: $\displaystyle\int_0^5 x \cdot f''(x)dx$

Question 10. Consider the integral

$$I = \int_0^1 \int_x^{\sqrt{x}} e^{\frac{x}{y}}\, dy\, dx.$$

Sketch the domain $\mathcal{R}$ of integration of I in the xy–plane. Then determine the value of I.

(You must carefully explain your methods. You'll earn no credit for computations done with a computer.)
