Math 118: Business Calculus
Final Exam

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Fall 2019

11 December 2019

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First Name: _____ (as in student record)

Last Name: _____ (as in student record)

USC ID: _____   Signature: _____

Please circle your instructor and lecture time:

Feng 1pm	Gerhardt 9am 12pm	Levitt 11am 12pm	Quintero 1pm 2pm	Tabing 9am 10am	Tokorcheck 10am

- This exam has 8 problems and will last 120 mins, leaving about 15 mins per question.

- You may use any scientific **non-graphing** calculator.

- You may use one 8.5 x 11 inch handwritten formula sheet (front and back).

- Try to keep your solutions in the space provided for each question. You may continue solutions on other pages if you clearly indicate in that space where to find your solution.

- Show all of your work and justify every answer to receive full credit.

Do not write in the box below:

Q1	Q2	Q3	Q4	Partial 1
/10	/15	/25	/30	/80
Q5	Q6	Q7	Q8	Partial 2
/24	/20	/16	/20	/80

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Question 1 (10 points). One of the following graphs depicts a function with the property that $f(x) = -f'(x)$. Choose the correct graph.

(a)

(b)

(c)

(d)
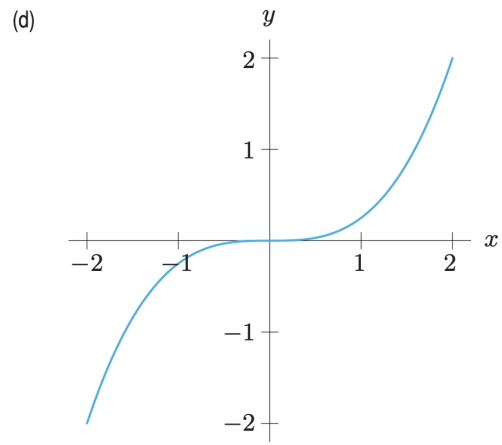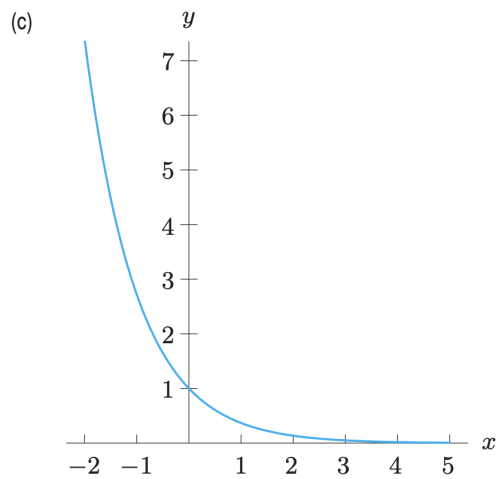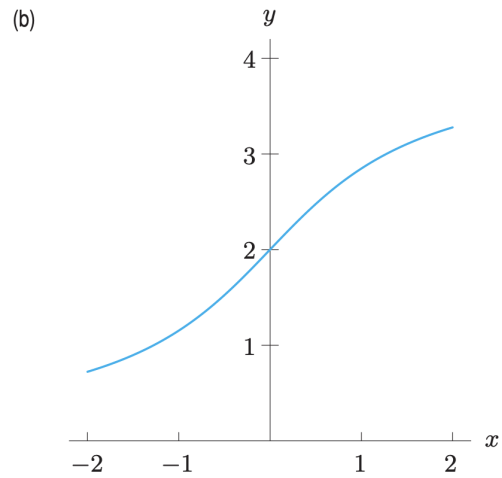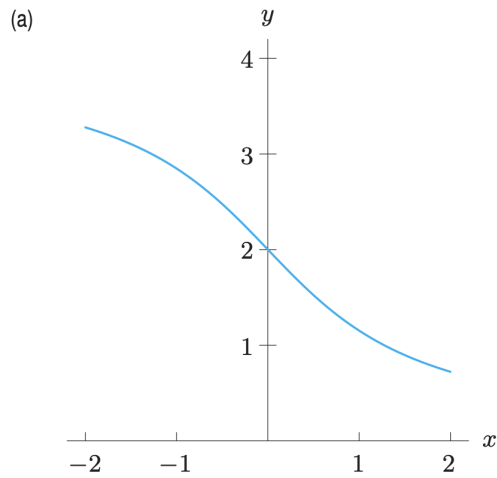
Also, please provide a brief **explanation** for your choice below.

Question 2. A company manufactures a product that costs them \$2 per unit. It is estimated that if they sell the product for p dollars each, then they will sell $2500e^{-p/20}$ units per week.

5 pts **(a)** Write a formula for the company's **profit** as a function of the price p.

10 pts **(b)** At what price should the company set the price of their product at in order to maximize their weekly profit? Verify that this price actually gives maximum profit.

Question 3. If the question part is multiple-choice, you do not need to justify your answers.

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5 pts **(a)** Given the function $f(x) = \dfrac{x}{e^x}$, which of the following formulas is true?

(i) $f'(x) + f(x) = e^x$

(ii) $f'(x) + f(x) = e^{-x}$

(iii) $f'(x) + f(x) = 1$

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7 pts **(b)** Given $R(s, t) = \dfrac{t}{s^2 - t^2}$, find $\dfrac{\partial R}{\partial t}$. You do not need to simplify your answer.

5 pts **(c)** Given the function $h(x, y) = (y - x^2)e^y$, which of the following formulas is true?

(a) $2\left(\dfrac{\partial h}{\partial y}\right) - x\left(\dfrac{\partial h}{\partial x}\right) = -2e^y$

(b) $2\left(\dfrac{\partial h}{\partial y}\right) - x\left(\dfrac{\partial h}{\partial x}\right) = 2e^y$

(c) $2\left(\dfrac{\partial h}{\partial y}\right) - x\left(\dfrac{\partial h}{\partial x}\right) = 2(1 + y)e^y$

8 pts **(d)** $g(u, v) = u\ln(uv^2)$, find $\dfrac{\partial^2 g}{\partial u \partial v}$.

Question 4. Be sure to carefully explain every step. If you use a substitution or integration by parts, make your substitutions absolutely clear to the reader.

15 pts **(a)** Find the following antiderivative:

$$\int \left[x \log_3(x) \right]^2 dx$$

15 pts **(b)** Compute the total areas enclosed by the curves $y = 2x\sqrt{2x+5}$, $y = 0$, $x = -2$, and $x = 5$. (Consider all areas to be positive.)

$$y = 2x\sqrt{2x+5}$$

graph is not to scale
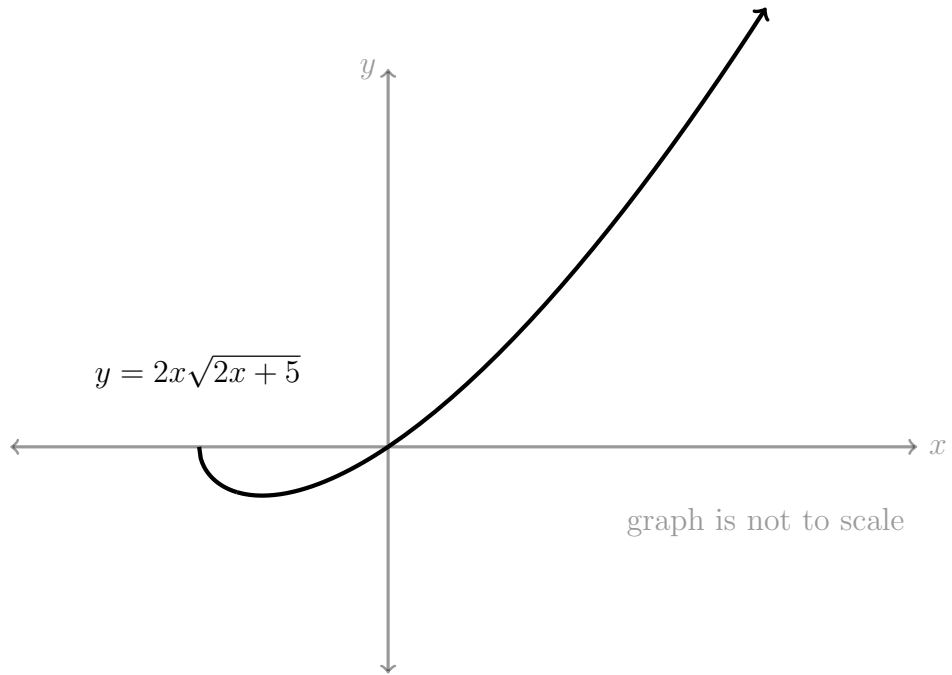
Question 5. You are considering purchasing a house as an investment property. The sale price is $748,000 and the rental income will exceed the mortgage by $200 each month. This profit will be deposited in an account earning 2% interest, compounded continuously.

10 pts **(a)** Find the **present value** of rental income stream over 5 years.

4 pts **(b)** Find the **future value** of rental income stream in 5 years.

Now, for Parts (c) and (d) assume that the property will be assessed **once per year**, and you expect the value of the house to increase by 3% per year.

6 pts **(c)** Find the new **value** of the house after 5 years.

4 pts **(c)** Find the **net profit** on this overall investment after five years.

Question 6 (20 points)**.** Find all **critical points** of the function f given by

$$f(x, y) = x^3 - 3x + y^2 e^x$$

Classify each one as a local maximum, local minimum, or saddle point. If you cannot classify a critical point, indicate why not.

Question 7 (16 points). Consider the following two functions, and assume that each is both continuous and differentiable. In each table, the values of y are listed vertically down the side, and x is listed horizontally across the top:

Table 8.2

$y\backslash x$	0	1	2	3	4
0	0	2	4	6	8
1	4	6	8	10	12
2	8	10	12	14	16
3	12	14	16	18	20
4	16	18	20	22	24

Table 8.3

$y\backslash x$	0	1	2	3	4
0	0	2	4	6	8
1	-4	-2	0	2	4
2	-8	-6	-4	-2	0
3	-12	-10	-8	-6	-4
4	-16	-14	-12	-10	-8

- If $f_x(2,3) > 0$ and $f_y(2,3) < 0$, which table could represent $f(x,y)$?

  Table 8.2   Table 8.3   Both   Neither

- If $g_{xx}(2,3) = 0$, which table could represent $g(x,y)$?

  Table 8.2   Table 8.3   Both   Neither

- If $h(x,y)$ is always **concave up** in the y-direction, which table could represent $h(x,y)$?

  Table 8.2   Table 8.3   Both   Neither

- If $p(x,y)$ has $\int_0^2 \int_1^3 p(x,y)\,dx\,dy = 36$, which table could represent $p(x,y)$?

  Table 8.2   Table 8.3   Both   Neither

Question 8 (20 points)**.** Evaluate the double integral:

$$\int_0^1 \int_0^2 \left(x^3 \, e^{x^2 y} \right) dx \, dy$$

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