MATH 226 **Final Exam** **SPRING 2018**

Last Name: ______________________ **First Name:** ______________________

Signature: ______________________ **Student ID:** ______________________

☐ D. Crombecque 9am ☐ D. Crombecque 10am ☐ J-M. Leahy 12pm
☐ G. Dreyer 11am ☐ G. Dreyer 1pm

Directions. Fill out your name, signature and student ID number on the lines above before starting the exam! Also, check the box next your professor's name.

- You must *show all your work and justify your methods* to obtain full credit. State any theorems that you use. Clearly indicate your final answers.
- Simplify your answers to a reasonable degree. Any fraction should be written in lowest terms. You need not evaluate expressions such as $\ln 5$, $e^{0.7}$ or $\sqrt{229}$.
- You may use the SINGLE sided HANDWRITTEN sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2}'' \times 11''$ paper. You may have anything written on one side of it, but it must be written in your own handwriting. No other notes or books are allowed during the test.
- No calculators or other electronic devices are allowed. *Turn off your cell phone.*
- **DO NOT WRITE OUTSIDE THE DESIGNATED BOXES.**
- Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

Problem	Points	Score	Problem	Points	Score	Problem	Points	Score
1	10		4	15		7	10	
2	15		5	20		8	25	
3	20		6	15		9	20	
Subtotal	**45**		Subtotal	**50**		Subtotal	**55**	

Total	

Problem 1. Consider the two planes

$$P_1 : x + y - z = 2 \quad \text{and} \quad P_2 : 3x - 4y + 5z = 6.$$

(a) Find the parametric equations of their line of intersection.

(b) Find the angle between the two planes.

Problem 2. Consider the two parametrized curves

$$\mathbf{r_1}(t) = \langle 1+t^2, 2-t, t^4+3t^2-4t+4\rangle \quad \text{and} \quad \mathbf{r_2}(u) = \langle u^2, 3-u, u^4+u^2-6u+8\rangle,$$

where t and u are in $\mathbb{R}$.

(a) Find the coordinates of the point of intersection P of the two curves.

(b) The curves traced out by $\mathbf{r_1}$ and $\mathbf{r_2}$ lie on a surface S. Find an equation of the tangent plane to the surface S at the point P found in part (a).

Problem 3. (a) Evaluate the following integral $\int_0^3 \int_y^{3y} e^{x^2} \, dx dy$.

Problem 3 (continued).

(b) Let E be the solid bounded from below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + (z-2)^2 = 2$. Set up, BUT DO NOT EVALUATE, a triple integral in CYLINDRICAL COORDINATES that yields the volume of the solid E.

Problem 4. Consider the surface S defined by the equation

$$\sin(yz) - x^2 z = 2.$$

(a) Find the values of $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at the point $(1, 0, -2)$ on S.

Problem 4 (continued).

(b) Approximate the value of z when $x = 1.1$ and $y = -0.3$.

(c) Consider a path $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ lying on the surface S such that $\mathbf{r}(0) = \langle 1, 0, -2 \rangle$. Assume that $\frac{dx}{dt}(0) = -5$ and $\frac{dy}{dt}(0) = 5$. Find the value of $\frac{dz}{dt}(0)$.

Problem 5. Consider the function

$$f(x, y) = \frac{xy^2}{2} + \frac{x^3}{3} - x.$$

on the domain $\mathcal{D} = \{(x, y) \in \mathbb{R}^2 : x^2 + \frac{y^2}{2} \leq 3\}$.

(a) Find the critical points of f contained in the interior of $\mathcal{D}$.

(b) Classify the critical points of f in the interior of $\mathcal{D}$ as local maxima, local minima or saddle points.

Problem 5 (continued).

(c) Find the absolute minimum and maximum values of f on $\mathcal{D}$.

Problem 6. Consider the vector field

$$\mathbf{F}(x,y,z) = \langle 2xyze^{x^2y}, z^2 + x^2ze^{x^2y}, e^{x^2y} + 2yz - 3z^2 \rangle.$$

For ALL possible smooth curves C initiating from the point $(0,-1,1)$ and ending at the point $(\sqrt{\ln(2)},1,1)$, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

Problem 7. Compute the line integral of the vector field $\mathbf{F}(x, y) = \langle xy, x^2 + x \rangle$ along the triangular curve C oriented counterclockwise with vertices $(0, 1)$, $(-1, 0)$ and $(1, 0)$.

Problem 8. Consider the surface S defined as the open cap of the sphere $x^2 + y^2 + z^2 = 2$, $z \geq 1$. The surface S is equipped with upwards orientation.

(a) Let $\mathbf{G}(x, y, z) = -z\mathbf{j} + \mathbf{k}$. Parametrize the surface S to evaluate $\iint_S \mathbf{G} \cdot d\mathbf{S}$. There is extra space provided the next page.

Problem 8 (a) (extra space).

Problem 8 (continued).

(b) Let $\mathbf{F}(x, y, z) = x\mathbf{j} + xz\mathbf{k}$. Verify that $\text{Curl}\,\mathbf{F} = \mathbf{G}$ and then verify Stoke's Theorem for the vector field $\mathbf{F}$ and the surface S described in the previous page.

Problem 9. Consider the vector field

$$\mathbf{F}(x,y,z) = \left(xy^2 + e^z\right)\mathbf{i} + \left(yz^2 + \tan(zx)\right)\mathbf{j} + \left(\ln(y^2+x^2) + zx^2\right)\mathbf{k}.$$

Use the method of your choice to evaluate $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S}$, where S is the closed boundary of the solid E bounded from below by the cone $z = \sqrt{x^2+y^2}$ and above by the sphere $x^2+y^2+z^2 = 2z$. with outward orientation. There is extra space provided on the next page.

Problem 9 (extra space).