Math 226 Final Exam
May 10th, 2017

Directions. Fill out your name, signature and student ID number on the lines below **right now**, before starting the exam! Also, check the box next to the class for which you are registered.

You must **show all your work and justify your methods** to obtain full credit. Name the theorems you are using. Write everything that you want graded on these pages and clearly indicate your answers. If you need more space, use the back of these pages and clearly indicate where the continuation may bef ound. Write as legibly as possible. Simplify your answers to a reasonable extent. Any fraction should be written in lowest terms. You need not evaluate expressions such as $\ln 5$, $e^{0.7}$, and $\sqrt{3}$. Do not use scratch paper; use the back of the previous page if additional room is needed.
NO calculators and NO use of cells phones or laptops is allowed
You may use the sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2} \times 11$ paper. You may have anything written on it (on both sides), but it must be written *in your own handwriting.*
Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

Name (please print): ______________________________

Signature: ______________________________

Student ID: ______________________________

☐ D.Crombecque 11am ☐ D Crombecque 1pm ☐ N. Ziane
☐ N. Tiruviluamala 9am ☐ N. Tiruviluamala 12pm

Do not write on this page below this line!

1 (20 pts)	5 (20 pts)
2 (15 pts)	6 (20 pts)
3 (20 pts)	7 (15 pts)
4 (15 pts)	8 (25 pts)
9 (20 pts)	

Problem 1. (1) Find an equation of the plane that passes through the points $A(2, 1, 1)$, $B(-1, -1, 10)$ and $C(1, 3, -4)$.

(2) A second plane passes through $(2, 0, 4)$ and has normal vector $< 2, -4, -3 >$. Find the angle between the planes.

(3) Find parametric equations for the line of intersection of the two planes.

Problem 2. Consider the surface

$$-z + y^2 + \sqrt{10x - 4y} = 0$$

- Find the equation of the tangent plane to the surface at the point $(4, 1, 3)$.

- Find an approximation for the z value of the point on the surface where $x = 3.9$ and $y = 1.1$

Problem 3. Find the absolute maximum and minimum values of the function

$$f(x, y) = x^2 - \frac{x}{2} + y^2 - y$$

in the region given by

$$x^2 + y^2 \leq 5$$

Problem 4. Find the (u, v) coordinates of the points on the parameterized surface

$$\mathbf{r}(u, v) = \left\langle u^2, v^3, uv \right\rangle$$

where the function

$$T(x, y, z) = 12x + y - 12z$$

achieves local minima and local maxima. Justify your work clearly.

Problem 5. Consider the solid E bounded the tetrahedron with corners at $(0,0,0),(1,0,0),(0,0,2)$, and $(0,2,2)$ and the function

$$f(x,y,z) = x^2ye^z$$

(1) Set up (BUT DO NOT EVALUATE) the triple integral $\int\int\int_E f(x,y,z)dV$ with the order of integration given by

$$dV = dy\ dz\ dx.$$

(2) Set up (BUT DO NOT EVALUATE) the triple integral $\int\int\int_E f(x,y,z)dV$ with the order of integration given by

$$dV = dz\ dy\ dx.$$

Problem 6. Let $\mathbf{F}(x, y, z) = \langle y\cos x + z + y^2, \sin x + z^2 + 2xy, x + 2yz\rangle$.

(1) Show that $\mathbf{F}$ is conservative. Then find a function f such that $\mathbf{F} = \nabla f$.

(2) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve C:

$$\mathbf{r}(t) = \langle t, \sin(2t) + 1, \cos t\rangle, \quad 0 \leq t \leq \frac{\pi}{2}.$$

Problem 7.

Using a method of your choice, evaluate the work done by the force $\mathbf{F} =< \sqrt{1+x^3}, 2xy >$

along the closed curve, oriented counterclockwise, C with is the edges of the triangle with vertices $(0,0), (1,0)$, and $(1,3)$.

Problem 8. Let S be the open paraboloid of equation $x = \frac{1}{2}(y^2 + z^2)$ with $0 \leq x \leq 2$ and oriented so that its normal vectors have negative x components.

(1) Evaluate the area of the surface S.

(2)

$$\mathbf{F}(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \langle x, y + z, z - y \rangle,$$

Using a method of your choice, compute

$$\int\int_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$$

Problem 9. Using a method of your choice, evaluate the flux $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where

$$\mathbf{F} =< z^2 x, (\frac{1}{3}y^3 + \tan z), (x^2 z + y^2) >$$

and S is the CLOSED surface oriented outwards consisting of the hemisphere $x^2 + y^2 + z^2 = 4$ with $z \geq 0$ and the disk $x^2 + y^2 \leq 4$ in the xy-plane.