The University of Southern California

MATH 226, Spring 2016

Final Exam

Last Name: ______________________________

First Name: ______________________________

Signature: ______________________________

Please indicate your instructor and lecture time below:

C. Blois	G. Reyes	J. Toulisse
11 am	10 am	9 am
12 pm	1 pm	-

Instructions:
Please show all of your work and reasoning. You may use one 8.5-by-11-inch formula sheet, written in your own handwriting on both sides. No other notes, books, calculators, electronic devices, or other memory aids are allowed for use during the test. All electronic devices, including mobile phones, must be turned off. Numerical answers should be left in calculator-ready form, unless otherwise indicated. If you need more space than what is provided, please use the back of the previous page, indicating clearly where the solution is continued. The exam lasts two hours.

Question	Points	Score
1	10	
2	12	
3	13	
4	10	
5	10	
6	15	
7	15	
8	15	
Total:	100	

[10 points] 1. Let P be a point in the domain of a smooth function f of three variables.

(a) Find the gradient of f at P if all of the following is known:

- $|\nabla f(P)| = 3$;
- The plane $x + 2y - z = 0$ is tangent to the level surface of f going through P;
- The first coordinate of $\nabla f(P)$ is positive.

(b) Let $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ be a smooth curve such that $\mathbf{r}(0) = P$ and $\mathbf{r}'(0) = \langle 1, 1, 0 \rangle$. Find

$$\frac{d}{dt}(f(\mathbf{r}(t)) \qquad \text{at} \quad t = 0.$$

(Here, f is the same function and P is the same point as in part (a).)

[12 points] 2. Find the (x, y) coordinates of all critical points of the function

$$F(x, y) = 2x^3 + 6xy^2 - 3y^3 - 150x$$

and classify each as a local maximum, a local minimum or a saddle point.

[13 points] 3. Using the method of Lagrange Multipliers, find the maximum and minimum values of

$$f(x, y) = x^2 y$$

on the circle $x^2 + y^2 = 1$.

[10 points] 4. Evaluate the iterated integral

$$\int_{-1}^{0} \left(\int_{\sqrt{-y}}^{1} e^{-x^3} \ dx \right) dy.$$

[10 points] 5. Fuzzy the bumble bee flies in the wind. At any given point (x, y, z), the wind exerts the force

$$\mathbf{F}(x, y, z) = \langle yz, xz, xy \rangle$$

on Fuzzy.

(a) Show that the force field $\mathbf{F}$ is conservative in $\mathbb{R}^3$ and find a potential function for $\mathbf{F}$.

(b) If Fuzzy flies along the helicoidal path

$$\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle, \qquad 0 \le t \le \frac{\pi}{4},$$

find the work done by the wind on Fuzzy.

[15 points] 6. Use Green's Theorem to evaluate

$$\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r},$$

where $\mathbf{F} = \langle -\frac{1}{2}y^2x^2, y^3x \rangle$ and $\mathcal{C}$ is the boundary of the region D lying inside the circle $x^2 + y^2 = 1$, above the x-axis, and to the left of the line $x = -\frac{1}{2}$, with positive (counterclockwise) orientation.

[15 points] 7. Evaluate the line integral

$$\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$$

where

$$\mathbf{F} = \langle x + y^2, y + z^2, z + x^2 \rangle$$

and the curve $\mathcal{C}$ is the intersection of the plane $x + y + z = 1$ and the coordinate planes, oriented counterclockwise if viewed from above (where "up" is the positive z-direction).

(a) Without using Stokes' Theorem (by direct computation);

(b) Using Stokes' Theorem.

(Extra space)

[15 points] 8. Let S be the boundary of the region enclosed by the positive cone $z = \sqrt{x^2+y^2}$ and the sphere $x^2+y^2+z^2=4$. Calculate the flux of the field $\mathbf{F} = \langle xz^2, yx^2, zy^2 \rangle$ outward across S,

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$