Math 226 Final Exam
December 9th, 2015

Directions. Fill out your name, signature and student ID number on the lines below **right now**, before starting the exam! Also, check the box next to the class for which you are registered.

You must **show all of your work and justify your methods** to obtain full credit. Circle your final answers. Simplify your answers (unless the instructions indicate that it is unnecessary to do so). Do not use scratch paper; use the back of the previous page if additional room is needed. No calculators are allowed, but you may use the double sided HANDWRITTEN sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2} \times 11$ paper. Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

Name (please print): ______________________________

Signature: ______________________________

Student ID: ______________________________

☐ A. Kryshchenko 10 AM	☐ A. Kryshchenko 1 PM	☐ C. Blois 1 PM
☐ N. Tiruviluamala 9 AM	☐ N. Tiruviluamala 12 PM	☐ G. Dreyer 11 AM
☐ C. Haskell 10 AM	☐ C. Haskell 11 AM	☐ D. Crombecque 12 PM

Do not write on this page below this line!

1 (10 pts)	6 (15 pts)
2 (15 pts)	7 (15 pts)
3 (15 pts)	8 (15 pts)
4 (10 pts)	9 (15 pts)
5 (15 pts)	

125 Points Total

1. (10 points) Suppose $z = f(x, y)$ where

$$\begin{aligned} f(1,3) &= 7, \\ f_x(1,3) &= 5, \\ \text{and} \qquad f_y(1,3) &= -4. \end{aligned}$$

Suppose, moreover, that

$$\begin{aligned} x &= st \\ \text{and} \qquad y &= 2s + t. \end{aligned}$$

(a) Find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ when $(s, t) = (1, 1)$.

(b) Find an approximate value of z when $s = 0.9$ and $t = 1.1$.

2. (15 points) Consider the function

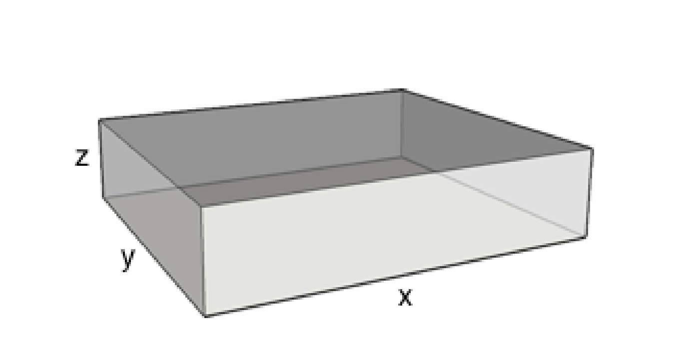
$$f(x, y) = x^3 - 4xy + 2y^2.$$

(a) Find the (x, y) coordinates of all of the critical points of f.

(b) Classify each critical point of f found in the previous part as a local maximum, a local minimum, or a saddle point.

(c) Find the absolute maximum and minimum VALUES of the same function $f(x, y) = x^3 - 4xy + 2y^2$ on the region determined by $0 \leq x \leq 1$ and $x - 1 \leq y \leq 0$.

3. (15 points) A box without a top has dimensions x inches by y inches by z inches:



The cost of the material for the base of the box is 4 cents per square inch. The cost of the material for the sides of the box is 1 cent per square inch. If the total budget is 48 cents, what are the dimensions of the box with the largest volume that can be produced?

(You may assume without justification that only an absolute maximum exists. You may also assume that the dimensions at which this maximum occurs are all strictly positive.)

4. (10 points) Let T be the triangle with vertices $(0,0)$, $(0,1)$ and $(2,1)$. Evaluate the integral

$$\iint_T e^{y^2}\, dA.$$

5. (15 points) Let E be the region in the first octant that lies below the plane $6x + 3y + z = 6$. Suppose that we are trying to find

$$m = \iiint_E x\, dV.$$

(a) Express (do not evaluate) m as a triple integral with the order of integration given by

$$dV = dz\ dx\ dy.$$

(b) Express (do not evaluate) m as a triple integral with the order of integration given by

$$dV = dx\ dy\ dz.$$

6. (15 points) Calculate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

in each case.

(a) $\mathbf{F}(x, y) = \langle -y, x \rangle$ and C is the portion of the parabola $x = y^2$ from the point $(1, -1)$ to the point $(1, 1)$.

(b) $\mathbf{F}(x, y, z) = \nabla f$, where $f(x, y, z) = xe^{y^2+z}$ and C is the curve given by $\mathbf{r}(t) = \langle \cos t, \sin t, t^3 \rangle$ from $t = 0$ to $t = \pi$.

(c) $\mathbf{F}(x, y) = \left\langle y^3 + e^{x^2}, -x^3 + \cos(y^2) \right\rangle$ and C is the circle $x^2 + y^2 = 4$ oriented counter-clockwise.

7. (15 points) Find the surface area of the part of the surface $z = 4xy$ that lies above the disk $x^2 + y^2 \leq 1$.

8. (15 points) Let S be the hemisphere given by

$$x^2 + y^2 + z^2 = 9$$

with $y \geq 0$ and oriented so that its normal vectors have positive y components. If

$$\mathbf{F}(x, y, z) = \left\langle 2z + y\sin x, 3x + ye^{z^2}, -2x \right\rangle,$$

compute the integral

$$\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}.$$

9. (15 points) A volcano occupies the solid region which is bounded between the plane $z = 0$ and the upside down paraboloid $z = 1 - x^2 - y^2$. The heat flow at the point (x, y, z) in the volcano is given by

$$\mathbf{F} = \left\langle x + \sin z, y + \cos z, e^{x^2+y^2} - z \right\rangle.$$

Calculate the flux of heat flowing out from all sides of the volcano (including its base). In other words, calculate

$$\iint_S \mathbf{F} \cdot d\mathbf{S},$$

where S is the closed boundary surface of the volcano oriented outwards.