Math 226 **FINAL EXAM** **December 18, 2013**

Last Name: ______________________ **First Name:** ______________________

Student ID Number: ____________________ **Signature:** ____________________

Circle your instructor's name:

Asok Crombecque 9am Crombecque 10am Haydn
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INSTRUCTIONS

1. Write everything that you want graded on these pages and clearly indicate your answers. If you need more space, use the back of these pages and clearly indicate where the continuation may be found. Write as legibly as possible.
2. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer.
3. You may use one letter-sized sheet of handwritten notes. No other aids such as calculators, cell phones, laptops, and textbooks are allowed.

TOTAL NUMBER OF POINTS: 200

Problem	Value	Score
1	10	
2	15	
3	20	
4	20	
5	25	
6	20	
7	20	
8	20	
9	30	
10	20	
Total	200	

1. (10 pts) Find an equation of the plane that is perpendicular to the curve

$$\mathbf{r}(t) = 3\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k}$$

at the point corresponding to $t = \frac{\pi}{4}$.

2. (15 pts) Consider the plane $2x + 3y + 6z + 6 = 0$.

   (a) (7 pts) Find parametric equations of the line that is perpendicular to the plane and passes through the point $(1, 1, 1)$.

   (b) (8 pts) Find the point on the plane that is closest to $(1, 1, 1)$.

3. (20 pts) Consider the function $f(x, y) = x^2 + xy + y^2$.

   (a) (10 pts) Find the equation of the tangent plane to the surface $z = f(x, y)$ when $(x, y) = (1, 2)$.

   (b) (5 pts) Use (a) to give an estimate of $f(0.9, 2.1)$.

   (c) (5 pts) In which direction is f increasing the fastest at the point $(x, y) = (1, 2)$?

4. (20 pts) Consider the function $f(x, y) = 4xy - x^4 - y^4$.

   (a) (10 pts) Find all the critical points of f.

   (b) (10 pts) Classify each critical point of f as a local maximum, local minimum, or saddle point.

5. (25 pts) Consider the triple integral $\iiint_E z^6 dV$, where

$$E = \{(x, y, z) \mid x^2 + y^2 + z^2 \le 4, x \ge 0, y \ge 0, z \ge 0\}.$$

(a) (15 pts) Write the triple integral in spherical coordinates.

(b) (10 pts) Use (a) to compute the triple integral.

6. (20 pts) A lamina occupies the region $D = \{0 \le x \le 2, -\sqrt{4 - x^2} \le y \le 0\}$. Find the mass of the lamina if the density at any point is proportional to the distance of the point to the origin.

7. (20 pts) Use Green's Theorem to evaluate the integral

$$\oint_{\partial D} \left(e^{x^2} - \frac{x^2y^2}{2} \right) dx + (xy^3 + \sin y) dy,$$

where ∂D is the boundary of the region $D = \{(x, y) \mid x^2 + y^2 \leq 4, y \geq 0\}$.

8. (20 pts) Let $\mathbf{F}(x, y, z) = (x^2 + 2xy, x^2 + 2yz, y^2)$.

(a) (12 pts) Is $\mathbf{F}$ is a conservative vector field? Why? If yes, find a function f such that $\mathbf{F} = \nabla f$.

(b) (8 pts) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the arc from $(1, 0, 0)$ to $(0, 1, 0)$ given by $\mathbf{r}(t) = (\cos\sqrt{t}, \sin\sqrt{t}, t^2 - \frac{\pi^2}{4}t)$, $0 \le t \le \frac{\pi^2}{4}$.

9. (30 pts) If $\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$, then calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, oriented counterclockwise as viewed from above.

   (a) (15 pts) Calculate the integral directly (as a line integral).

   (b) (15 pts) Calculate the integral using Stokes' Theorem.

10. (20 pts) Use the Divergence Theorem to evaluate the surface integral $\iint_{\partial E} \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F}(x, y, z) = xy^2\mathbf{i} + x^2y\mathbf{j} + y\mathbf{k}$$

and ∂E is the boundary of the solid $E = \{x^2 + y^2 \leq 1, -1 \leq z \leq 1\}$.