Math 126 Final Exam
May 11th, 2016

Directions. Fill out your name, signature and student ID number on the lines below **right now**, before starting the exam! Also, check the box next to the class for which you are registered.

You must **show all your work and justify your methods** to obtain full credit. Circle your final answers. Simplify your answers (unless the instructions say you do not have to). Do not use scratch paper; use the back of the previous page if additional room is needed. No calculators are allowed, but you may use the double sided HANDWRITTEN sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2} \times 11$ paper. Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

The table in the textbook of common Taylor series is provided on the last page of the test.

Name (please print): ______________________

Signature: ______________________

Student ID: ______________________

☐ D. Crombecque 9am	☐ D. Crombecque 11am	☐ N. Emerson 12pm
☐ C. Haskell 9am	☐ C. Haskell 1pm	☐ D. Searles 10am

Do not write on this page below this line!

1 (10 pts)	6 (10 pts)
2 (10 pts)	7 (15 pts)
3 (15 pts)	8 (15 pts)
4 (10 pts)	9 (15 pts)
5 (15 pts)	10 (15 pts)

130 points total

Problem 1. Evaluate the following limits.

(a) $\displaystyle\lim_{x \to 0} \frac{\arcsin(x^2)}{x^2}$

(b) $\displaystyle\lim_{n \to \infty} (-1)^n \left(1 + \frac{1}{2n^2}\right)^{n^2}$

Problem 2. Evaluate $\displaystyle\int_0^2 \frac{x^2}{(x^2+4)^2}\,dx.$

Problem 3. Compute the following antiderivatives.

(a) $\displaystyle\int (3x^2 + 2)\ln(x)\,dx$

Problem 3 cont.

(b) $\displaystyle\int \frac{3x^2+4x+2}{x(x+1)^2}\,dx$

Problem 4. Determine whether the following improper integral is convergent or divergent.

$$\int_0^{\frac{1}{2}} \frac{\cos^2 x}{\sqrt{x} + x^2}\, dx$$

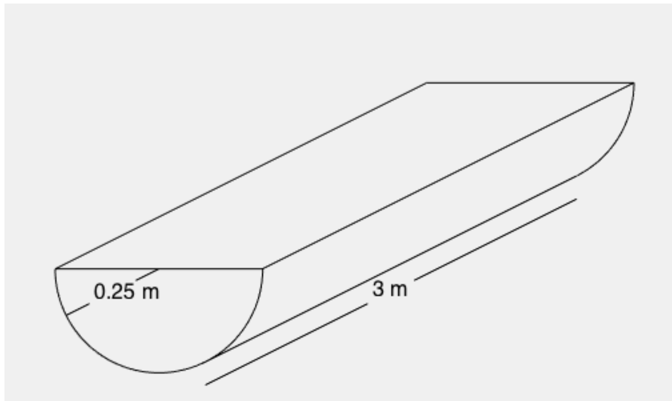
Problem 5. Let $\mathcal{R}$ be the region in the first quadrant bounded by

$$\begin{aligned} & y = e^{-x^2} \\ & x = 0 \\ \text{and} \qquad & y = e^{-1}. \end{aligned}$$

(a) Set up, but DO NOT EVALUATE, a definite integral whose value is the volume of the solid obtained by rotating $\mathcal{R}$ about THE y-AXIS.

(b) Set up, but DO NOT EVALUATE, a definite integral whose value is the surface area of the curved surface of the solid of revolution.

Problem 6. A water trough, shown below, is 3 m long and has a semicircular cross section with a radius of 0.25 m.



Suppose the trough is initially full to the brim. SET UP, but DO NOT EVALUATE, a definite integral whose value is the work required to empty the trough by pumping the water out over the top. The density of water is 1000 kg/m^3 and the acceleration due to gravity is $g = 9.8$ m/s^2.

Problem 7. Classify each series as absolutely convergent, conditionally convergent or divergent. Clearly state any tests you use and show that all the hypotheses of those tests are met.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln(n)}}$

Problem 7 cont.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n + e^n}$

Problem 8. Consider the following function:

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (x+3)^{2n}}{5^n \sqrt{n}}$$

(a) Find the interval of convergence of the series.

(b) Evaluate $f^{(32)}(-3)$. (You do not need to simplify your answer.)

Problem 9. Consider the function

$$f(x) = \sqrt{x}.$$

(a) Find $T_2(x)$, the Taylor Polynomial of degree 2 of f centered at $a = 4$.

(b) Use your Taylor polynomial to find an approximation for $\sqrt{3}$. You may leave your answer as a sum of fractions.

Problem 9 cont.

(c) Find the best bound you can for the error made in your approximation of $\sqrt{3}$.

Problem 10.

(a) Find the Maclaurin series expansion for $f(x) = x\cos(x^3)$. Write the power series using summation notation and also write out the first three non-zero terms.

(b) Write $\int_0^1 x\cos(x^3)\,dx$ as a series. (Either use summation notation or write out the first three non-zero terms.)

Problem 10 cont.

(c) Use your series in part (b) to find the value of $\int_0^1 x\cos(x^3)dx$ with an error of less than 10^{-4}. You may leave your answer as a sum of fractions.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \cdots \qquad R = 1$$