MATH 226 **FINAL EXAM** **December 14, 2011**

Last Name: ______________________ **First Name:**______________________

Student ID Number: ____________________ **Signature:**____________________

Circle your section:

Mancera (11am 1pm) Mikulevicius (10am 11am) Russell (9am 12pm) Sacker

INSTRUCTIONS

Answer all the questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators are not allowed. The exam is worth a total of $8 \times 25 = 200$ points.

Problem	Score
1	
2	
3	
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Total	

1. Given $f(x, y) = x\sqrt{3 - x^2 - y^2}$,

   (a) Find and sketch the domain of f.

   (b) Find the linearization of f at (1,1). Approximate $f(0.9, 1.1)$. Write an equation of the tangent plane to $z = x\sqrt{3 - x^2 - y^2}$ at $(1, 1, 1)$.

2. Consider the surface S given by the equation $x^2 + xy + 3y^2 - z = 0$.

   (a) Find a parametric equation of the line passing through $(1, 1, 5)$ and perpendicular to S.

   (b) Find an equation of the tangent plane to S at $(1, 1, 5)$.

   (c) Calculate the rate of change of $f(x, y, z) = x^2 + xy + 3y^2 - z$ at $(1, 1, 5)$ in the direction of the vector $\mathbf{v} = \langle 1, 2, 3 \rangle$.

3. Use Lagrange multipliers to find the absolute maximum and minimum values of the function $f(x, y) = x^2y$ subject to the constraint $2x^2 + y^2 = 3$.

4. (a) Evaluate $\int\int\int_E (x\sqrt{x^2+y^2+z^2} + zy)dV$, where

$$E = \left\{(x, y, z) | x^2 + y^2 + z^2 \leq 4, x \geq 0, z \leq 0\right\}.$$

(b) Evaluate by reversing the order

$$\int_0^4 \int_{\frac{\sqrt{y}}{2}}^1 \frac{5y}{8\sqrt{1+x^5}} dx dy.$$

5. Evaluate (a) directly and (b) using Greens Theorem the line integral $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathcal{C}$ is the counterclockwise oriented triangle with vertices $(0,0), (2,0)$, and $(2,2)$ and $\mathbf{F}(x,y) = \langle 5 - 2xy - y^2, 2xy - x^2 \rangle$.

6. Let $\mathbf{F}(x, y, z) = \langle 2x \cos y - 2z^3, 3 + 2ye^z - x^2 \sin y, y^2e^z - 6xz^2 \rangle$.

(a) Determine whether the vector field $\mathbf{F}(x, y, z)$ is conservative. If so, find its potential function.

(b) Evaluate the line integral $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathcal{C}$ is parameterized by

$$\mathbf{r}(t) = \left\langle t + 1, 4t - 4t^2, t^3 - t \right\rangle, 0 \leq t \leq 1.$$

7. The part of the paraboloid $z = 16 - 4x^2 - 4y^2$ that lies above the (x, y) plane intersects the (x, y) plane in a curve C. Use Stokes's Theorem to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where

$$\mathbf{F}(x, y, z) = \langle xz^2, x^2y, x^2z \rangle.$$

8. Use the Divergence Theorem to evaluate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = \langle y \sin z, 6x^2y, 2z^3 \rangle$ and S is the surface of the solid bounded by the cylinder $x^2 + z^2 = 2$ and the planes $y = 1$ and $y = 2$.

Some formulas you may need:

Differential: Given $z = f(x, y)$,

$$dz = f_x dx + f_y dy.$$

Given $w = f(x, y, z)$,

$$dw = f_x dx + f_y dy + f_z dz.$$

Curl: given a vector field $\mathbf{F} = \langle P, Q, R \rangle$, $\operatorname{curl} \mathbf{F} = \nabla \times \mathbf{F}$ with $\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$.

Jacobian: given the transformation $x = x(u, v), y = y(u, v)$, its Jacobian is

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}.$$

Spherical coordinates:

$$\begin{aligned} x &= \rho \sin\phi \cos\theta, \\ y &= \rho \sin\phi \sin\theta, \\ z &= \rho \cos\phi; \end{aligned}$$

absolute value of the Jacobian: $\left| \frac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)} \right| = \rho^2 \sin\phi$.

Evaluation of the surface integral: Given a surface $S : \mathbf{r}(u, v), (u, v) \in D$, and a function $f(x, y, z)$ on S,

$$\int\int_S f dS = \int\int_D f(\mathbf{r}(u, v)) |\mathbf{N}(u, v)| du dv,$$

where $\mathbf{N}(u, v) = \mathbf{r}_u(u, v) \times \mathbf{r}_v(u, v)$.

Evaluation of the flux integral: Given a surface $S : \mathbf{r}(u, v), (u, v) \in D$, with $\mathbf{n}(u, v) = \mathbf{N}(u, v)/|\mathbf{N}(u, v)|$ and a vector field $\mathbf{F}(x, y, z)$ on S,

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} = \int\int_S \mathbf{F} \cdot \mathbf{n} dS = \int\int_D \mathbf{F}(\mathbf{r}(u, v)) \cdot \mathbf{N}(u, v) du dv,$$

where $\mathbf{N}(u, v) = \mathbf{r}_u(u, v) \times \mathbf{r}_v(u, v)$.