Math 226 **Final Exam** **May 3, 2012**

Last Name: ____________________ **First Name:** ____________________

Signature: ____________________ **Student ID:** ____________________

Directions. Fill out your name, signature and student ID number on the lines above **right now** before starting the exam! Also, check the box next to the class for which you are registered.

- You must **show all your work and justify your methods** to obtain full credit. Write your final answers in the boxes provided.
- Simplify your answers to a reasonable degree. Any fraction should be written in lowest terms. You need not evaluate expressions such as $\ln 5$, $e^{0.7}$ or $\sqrt{226}$.
- Do not use scratch paper; use the back of the previous page if additional room is needed.
- No calculators are allowed. **Turn off your cell phone**.
- You may use the sheet of notes that you brought with you, this may be no more than one sheet of $8\frac{1}{2}'' \times 11''$ paper. You may have anything written on it (on both sides), but it must be written in your own handwriting.
- Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

☐ Chen (9:00)	☐ Sadhal (11:00)	☐ Williams (12:00)
☐ Chen (10:00)	☐ Emerson (12:00)	☐ Mancera (1:00)

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1 (25 pts)	5 (25 pts)
2 (25 pts)	6 (25 pts)
3 (25 pts)	7 (25 pts)
4 (25 pts)	8 (25 pts)

200 points total

Problem 1. Let $f(x, y) = x^3 + x(y^3 + 1) + e^{y-1}$. Let $(x_0, y_0) = (2, 1)$.

(a) Compute $f(2, 1)$.

(b) Compute $\frac{\partial f}{\partial x}(2, 1)$ and $\frac{\partial f}{\partial y}(2, 1)$.

(c) Use a linear approximation to estimate the change in f if x is increased by 0.2 and y is increased by 0.3.

(d) Suppose that x is increased by 0.2. Use a linear approximation to estimate how much y should be decreased so that f remains the same.

Problem 2. A spaceship is flying through deep space. The temperature at at point in space, in Kelvins, is given by

$$T(x, y, z) = e^{xy+z}.$$

Suppose that the spaceship follows the path $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$.

(a) Find their velocity of the spaceship at time $t = 2$.

(b) Determine how fast is the temperature changing at time $t = 2$.

(c) At time $t = 2$, find an expression for $\cos\theta$ where θ is the angle between the path of the space ship and the direction in which temperature is increasing most rapidly.

Problem 3. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = x + 2y$ subject to the constraints $x + y + z = 1$ and $y^2 + z^2 = 4$.

Problem 4. (a) Use Green's Theorem to evaluate $\oint_C x^2y dx - xy^2 dy$, where C is the circle $x^2+y^2=4$ with counterclockwise orientation.

(b) Let C be the curve of intersection of the plane $x+z=5$ and the cylinder $x^2+y^2=9$ that is oriented counterclockwise when viewed from above. Use Stokes' theorem to evaluate $\oint_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F} = xy\mathbf{i} + 2z\mathbf{j} + 3y\mathbf{k}$.

(c) Let W be a solid region whose boundary is a closed surface. Let $\mathbf{F} = (x^3 + y\sin z)\mathbf{i} + (y^3 + z\sin x)\mathbf{j} + (3z)\mathbf{k}$. Express the surface integral,

$$I = \iint_{\partial W} \mathbf{F}\cdot d\mathbf{S},$$

as a volume integral. *Do NOT evaluate the integral.*

Problem 5. Compute the volume below the surface

$$z = \frac{3x^2 + 4y^2}{\sqrt{x^2 + y^2}}$$

and above the disk in the xy-plane: $(x-1)^2 + y^2 = 1$. *Hint: Use polar coordinates in the xy-plane.*

Problem 6. Consider the tetrahedron with corners at $(0, 0, 0)$, $(a, a, 0)$, $(0, a, 0)$, and $(0, 0, a)$, with density:

$$\rho(x, y, z) = \rho_0 \left(\frac{z}{a}\right)^2,$$

where ρ_0 is a constant. Calculate the total mass m.

Problem 7. Use spherical coordinates to evaluate

$$\iiint_E x^2 dV$$

where E is the solid bounded by the hemispheres $y = \sqrt{9 - x^2 - z^2}$, $y = \sqrt{16 - x^2 - z^2}$, and the xz-plane.

Problem 8.

Let C be the spiral curve of points satisfying the polar-coordinate equation $r = \theta$ where $\pi \le \theta \le 4\pi$. In particular $x = r\cos\theta = \theta\cos\theta$ and $y = r\sin\theta = \theta\sin\theta$ for points on C.

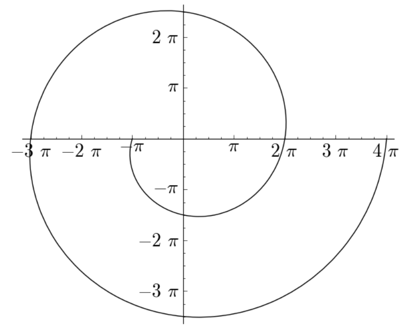

Figure 1: A plot of $\mathbf{r}(\theta) = \langle \theta\cos\theta, \theta\sin\theta \rangle$, for $\theta \in [\pi, 4\pi]$.

Let $f(x, y) = \sqrt{x^2 + y^2}$. Calculate the line integral $\displaystyle\int_C f(x, y)\, ds$.