Math 226 **Final Exam** **May 9, 2015**

Last Name: ________________________ **First Name:** ________________________

Signature: ________________________ **Student ID:** ________________________

Directions. Fill out your name, signature and student ID number on the lines above *right now* before starting the exam! Also, check the box next to the class for which you are registered.

- You must *show all your work and justify your methods* to obtain full credit. Clearly indicate your final answers.
- Simplify your answers to a reasonable degree. Any fraction should be written in lowest terms. You need not evaluate expressions such as $\ln 5$, $e^{0.7}$ or $\sqrt{226}$.
- You may use the sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2}'' \times 11''$ paper. You may have anything written on it (on both sides), but it must be written in your own handwriting.
- Do not use scratch paper; use the back of the previous page if additional room is needed.
- No calculators are allowed. *Turn off your cell phone.*
- Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

☐ Asok (9:00) ☐ Emerson (10:00) ☐ Zhuang (11:00) ☐ Sakai (1:00)
☐ Haskell (12:00) ☐ Emerson (1:00) ☐ Zhuang (12:00)

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1 (20 pts)	6 (20 pts)
2 (20 pts)	7 (25 pts)
3 (20 pts)	8 (25 pts)
4 (25 pts)	9 (25 pts)
5 (20 pts)	

200 points total

Problem 1 (20 points). Two particles are hurtling through space from time $t = 0$ until they collide. Their positions at time t are given by the vector-valued functions

$$\vec{r}_1(t) = \langle -3 + 2t,\ 2t,\ 2t \rangle \qquad \vec{r}_2(t) = \langle t,\ t(5-t),\ t(5-t) \rangle$$

a) Find all the points in space where the paths of the two particles intersect.

b) At what time do the particles collide?

c) Find the angle between the two paths when the particles collide.

Problem 2 (20 points). Consider the graph of the function f that is given by

$$f(x, y) = 5 + x\sin(2x - y).$$

a) Find an equation of the tangent plane at the point $(x, y) = (1, 2)$.

b) Use your answer to a) to estimate the value of $f(1.1, 1.8)$.

Problem 3 (20 points). Consider the function f that is given by

$$f(x, y) = y^3 - 2xy + x^2$$

a) Find all the critical points of f.

b) Classify each critical point you found above (if you can) as a local maximum, local minimum, or saddle point.

c) Does f have a *global* maximum on the plane? If it does, state its value and where it is attained. If not, explain why not.

d) Does f have a *global* minimum on the plane? If it does, state its value and where it is attained. If not, explain why not.

Problem 4 (25 points). A rectangular box without a lid is made of 48 in^2 of material. Using Lagrange multipliers, find the dimensions of the box with the maximum possible volume.

Problem 5 (20 points).

a) Consider the iterated integral:

$$\int_0^2 \int_{y^2}^4 y \ln(1+x^2)\, dx\, dy.$$

Reverse the order of integration. *Do not evaluate the integral.*

b) Consider the iterated integral:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} e^z\, dz\, dy\, dx.$$

Write the integral in spherical coordinates. *Do not evaluate the integral.*

Problem 6 (20 points). Let $\mathbf{F}(x, y) = \langle e^x \cos y,\ 2y - e^x \sin y \rangle$.

a) Find a function f such that $\mathbf{F} = \nabla f$.

b) Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve given by $\mathbf{r}(t) = \langle t + \sin \frac{\pi t}{2},\ t + \cos \frac{\pi t}{2} \rangle$, $0 \leq t \leq 1$.

Problem 7 (25 points). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where the vector field $\mathbf{F}$ is given by

$$\mathbf{F}(x, y) = \langle y^2 + \arctan x,\ x^2 - e^{\sin y} \rangle$$

and C is the boundary of the half-annular region $D = \{(x, y) : 1 \leq x^2 + y^2 \leq 4,\ y \geq 0\}$ oriented counter-clockwise.

Problem 8 (25 points). Let S be the surface defined by $z = x^2 + y^2$, $0 \leq z \leq 4$, oriented with the upward pointed normal vector. Let $\mathbf{G}$ be the vector field given by $\mathbf{G}(x, y, z) = -3xz^2\mathbf{i} + z^3\mathbf{k}$. Notice that $\mathbf{G}$ is the curl of the vector field $\mathbf{F}(x, y, z) = xz^3\mathbf{j}$. Use Stokes' theorem to evaluate

$$\iint_S \mathbf{G} \cdot d\mathbf{S}.$$

Problem 9 (25 points). A mountain in the ocean is bounded by the ocean floor $z = 0$ and the cone $z = 4 - 2\sqrt{x^2 + y^2}$. Suppose the heat flow in and around the mountain is described by the vector field

$$\mathbf{F}(x, y, z) = y^2(x + \sin y)\mathbf{i} + z\cos x\mathbf{j} + x^2 z\mathbf{k}.$$

Calculate the flux of heat flowing out of the mountain. *Hint: use the divergence theorem.*