Math 226 **Final Exam** **May 10, 2014**

Last Name: ______________________ **First Name:** ______________________

Signature: ______________________ **Student ID:** ______________________

Directions. Fill out your name, signature and student ID number on the lines above *right now* before starting the exam! Also, check the box next to the class for which you are registered.

- You must *show all your work and justify your methods* to obtain full credit. Clearly indicate your final answers.
- Simplify your answers to a reasonable degree. Any fraction should be written in lowest terms. You need not evaluate expressions such as $\ln 5$, $e^{0.7}$ or $\sqrt{226}$.
- You may use the sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2}'' \times 11''$ paper. You may have anything written on it (on both sides), but it must be written in your own handwriting.
- Do not use scratch paper; use the back of the previous page if additional room is needed.
- No calculators are allowed. *Turn off your cell phone.*
- Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

☐ Rose (9:00)	☐ Haskell (10:00)	☐ Emerson (11:00)
☐ Sadhal (1:00)	☐ Haskell (12:00)	☐ Emerson (12:00)

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1 (20 pts)	6 (25 pts)
2 (25 pts)	7 (25 pts)
3 (20 pts)	8 (20 pts)
4 (25 pts)	9 (20 pts)
5 (20 pts)	

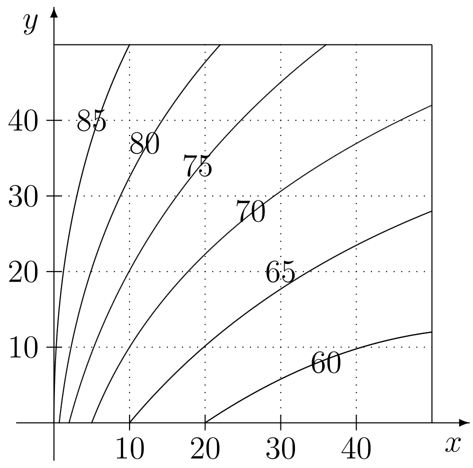
200 points total

Problem 1. Let S be the surface $(x + y)^2 + z^2 = 25$.

(a) Find the equation of the tangent plane to S at the point $(1, 2, 4)$.

(b) Let ℓ be the line $\langle 2t - 1, 3t - 1, 4\rangle$. Find the of angle of intersection of ℓ and the tangent plane to S at the point $(1, 2, 4)$.

Problem 2. Coordinates in a large room are chosen so that the origin is in a corner, the lines $x = 0$ and $y = 0$ are walls, and the coordinates x and y are measured in feet. A space heater is located at the origin. Let $T = f(x, y)$ denote the temperature (in degrees Fahrenheit) at the point (x, y). Shown below is a contour graph of f.



(a) Estimate the value of $f_x(20, 10)$. Remember to include units.

(b) Sketch the graph of $T = f(x, 10)$. Include as much detail as you can in your sketch.

(c) On the contour graph above, draw an arrow to indicate the direction in which $\nabla f(20, 10)$ points. Indicate how you obtained your answer here.

Problem 3. Consider the function $f(x, y) = e^y(x^2 + y^2)$.

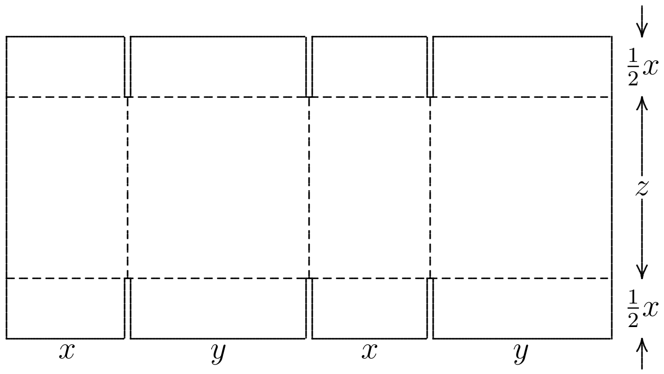
(a) Find all critical points of f.

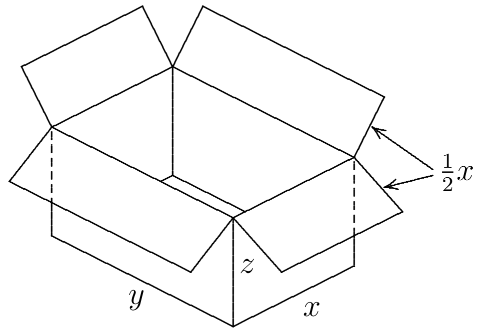
(b) Classify each critical point from part (a) as a local maximum, minimum, or saddle point.

(c) Does f have a *global* maximum? Briefly explain.

(d) Does f have a *global* minimum? Briefly explain.

Problem 4. A cardboard box is to be made from 18 square feet of material by cutting and folding as shown in the picture below. Find the dimensions x, y, and z of the box of maximum volume.





If the picture is confusing for you, it may help to notice that the flaps on the top and bottom of the box overlap, so the surface area of the box is actually less than the amount of cardboard used (18 square feet).

Problem 5. Consider the following double integral:

$$\iint_D \sin(x^2+y^2)\,dA = \int_1^2 \int_0^{\sqrt{4-x^2}} \sin(x^2+y^2)\;dydx \quad + \quad \int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} \sin(x^2+y^2)\;dydx$$

(a) Sketch the region of integration D.

(b) Evaluate $\iint_D \sin(x^2+y^2)\,dA$.

Problem 6. A solid object S occupies the region above the spherical surface $x^2 + y^2 + z^2 = 2z$, and below the conical surface $z = \sqrt{x^2 + y^2}$. The density of S is 1 at every point. Use spherical coordinates to calculate the following quantities.

(a) The mass of S.

(b) The center of mass of S.

Problem 7. Consider the force field $\mathbf{F}(x, y) = \langle y + e^x,\ x^2 + e^y \rangle$.

(a) Is $\mathbf{F}$ conservative? Briefly explain.

(b) Find the work done by $\mathbf{F}$ in moving a particle along the curve that is given by $\mathbf{r}(t) = \langle t^2,\ t \rangle$ for $0 \le t \le 1$.

(c) Use Green's theorem to find the work done by $\mathbf{F}$ in moving a particle counterclockwise along the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

Problem 8. Let S be the surface parameterized by $\mathbf{r}(u, v) = \langle u,\, uv,\, u^2 \rangle$ where $0 \leq u \leq 1$ and $0 \leq v \leq 1$. Find the surface area of S.

Problem 9. A goldfish bowl S is the part of the sphere, $x^2 + y^2 + (z + \sqrt{3})^2 = 4$, that lies below the xy-plane.

(a) Notice that the edge of the bowl (that is, the boundary of S) lies in the $z = 0$ plane. Show that the x and y coordinates of the points on the boundary satisfy $x^2 + y^2 = 1$.

(b) Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + e^{xz}\mathbf{k}$.