Math 226 **Final Exam** **May 15, 2013**

Last Name: ______________________ **First Name:** ______________________

Signature: ______________________ **Student ID:** ______________________

Directions. Fill out your name, signature and student ID number on the lines above *right now* before starting the exam! Also, check the box next to the class for which you are registered.

- You must *show all your work and justify your methods* to obtain full credit. Clearly indicate your final answers.
- Simplify your answers to a reasonable degree. Any fraction should be written in lowest terms. You need not evaluate expressions such as $\ln 5$, $e^{0.7}$ or $\sqrt{226}$.
- You may use the sheet of notes that you brought with you, this may be no more than one sheet of $8\frac{1}{2}'' \times 11''$ paper. You may have anything written on it (on both sides), but it must be written in your own handwriting.
- Do not use scratch paper; use the back of the previous page if additional room is needed.
- No calculators are allowed. *Turn off your cell phone.*
- Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

☐ Williams (9:00)	☐ Emerson (12:00)	☐ Tiruviluamala (10:00)
☐ Baxendale (11:00)	☐ Emerson (1:00)	☐ Tiruviluamala (12:00)

---

1 (20 pts)	6 (20 pts)
2 (20 pts)	7 (20 pts)
3 (20 pts)	8 (20 pts)
4 (20 pts)	9 (20 pts)
5 (20 pts)	10 (20 pts)

200 points total

Problem 1. Let C_1 be the curve parametrized by $\mathbf{r}_1(t) = \langle \sqrt{t}, t \rangle$, $t \geq 0$, and let C_2 be the curve parametrized by $\mathbf{r}_2(u) = \left\langle u, \dfrac{10}{u} + c \right\rangle$, $u > 0$, where c is a constant.

(a) Suppose $c = -1$. Find a point at which the two curves intersect.

(b) Find a value of $c \neq -1$ for which the curves intersect orthogonally.

Problem 2. One of the following vector fields is conservative.

(a) Determine which of these vector fields is conservative, and show that the other vector field is not the gradient of any function.

(i) $\mathbf{F}(x, y, z) = \langle 3x^2z^2 + 3z,\ 3y^2,\ -3x + 2x^3z \rangle$.

(ii) $\mathbf{G}(x, y, z) = \langle 3x^2y^2 + 3y,\ 3y^2 + 3x + 2x^3y,\ 0 \rangle$.

(b) For the above vector field that is conservative, compute its line integral along the curve $\mathbf{r}(t) = \langle 2\sin t, t, e^{3t} \rangle$ for $0 \le t \le \pi/2$.

Problem 3. Consider the function

$$f(x, y) = \sin^2 x - \cos y.$$

(a) Find all the critical points of $f(x, y)$ in the domain where $-1 \leq x \leq 2$ and $-1 \leq y \leq 4$.

(b) Classify as a local minimum, local maximum or saddle-point each critical point you found in part (a).

Problem 4. Let $f(x, y, z) = \dfrac{1}{x^2} + \dfrac{4}{y^2} + \dfrac{9}{z^2}$ for $x, y, z > 0$.

(a) Find the minimum value of $f(x, y, z)$ on the region $x^2 + y^2 + z^2 = 36$.

(b) Does $f(x, y, z)$ also have an absolute maximum on the above region? Explain why or why not.

Problem 5. Consider the integral

$$\int_0^8 \int_{x^{2/3}}^4 x\sqrt{1+y^2}\,dy\,dx$$

(a) Sketch the region of integration.

(b) Evaluate the integral by reversing the order of integration.

Problem 6. Let T be the solid tetrahedron with vertices $(0, 0, 0)$, $(2, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 3)$.

(a) Set up the integral

$$\iiint_T x \, dV.$$

Be sure to explicitly give the limits of integration.

(b) Find the value of the integral in (a).

Problem 7. Consider a point $P(3, 1, 2)$ on the surface S given implicitly by the equation

$$x^2 z - y^3 z^3 + 9z = 28.$$

(a) Find the equation of the tangent plane to the surface S at the point P.

(b) Suppose that Q is a point on the surface S near to P with x-coordinate 3.003 and y-coordinate 0.998. Use calculus to estimate the z-coordinate of Q.

Problem 8. Find the area of the surface S with parametric equations $x = u + v$, $y = uv$, $z = u - v$ for $u^2 + v^2 \leq 1$.

Problem 9. Let C be the piecewise linear path from $(0,0)$ to $(2,-1)$ to $(2,3)$ and back to $(0,0)$. Evaluate

$$\oint_C (xe^{x^2} + xy^2)\,dx + (x^3 + x^2 y - \arctan y)\,dy.$$

Problem 10. Let

$$\mathbf{F}(x, y, z) = -yz\,\mathbf{i} + x^2y\,\mathbf{j} + z\,\mathbf{k}.$$

Let S be the surface of the solid bounded by the paraboloid $x^2 + y^2 + 2z = 3$ and the plane $z = 1$. Calculate the flux of $\mathbf{F}$ outwards across S.