MATH 226 **FINAL EXAM** **December 13, 2014**

Last Name: ______________________ **First Name:** ______________________

Student ID Number: ____________________ **Signature:** ____________________

Circle your section:

Crombecque (10am 11am 1pm) Malikov Mikulevicius Rose (9am 12pm) Ryals (11am 12pm)

INSTRUCTIONS

1. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer.
2. Clearly indicate your answers. If you need more space, use the back of these pages and clearly indicate where the continuation may be found. Write as legibly as possible.
3. You may use one letter-sized sheet of handwritten notes. No other aides such as calculators, cell phones, laptops and textbooks are allowed.

Problem	Value	Score
1	20	
2	15	
3	15	
4	15	
5	20	
6	25	
7	20	
8	25	
9	20	
10	25	
Total	200	

1. (20 pts) Find parametric equations for the line through the point $(0, 1, 2)$ which is parallel to the plane $x+y+z=2$ and perpendicular to the line given by $x=1+t, y=1-t, z=2t$.

2. (15 pts) Consider the function $f(x, y) = x^2 - xy + y^3$.

(a) Find the equation of the tangent plane to the surface $z = f(x, y)$ when $(x, y) = (3, -1)$.

(b) Use what you found in (a) to approximate $f(2.96, -0.9)$.

3. (15 pts) Consider the function

$$f(x, y, z) = xe^{y^2 - z^2}.$$

(a) Find the direction of the maximum increase rate of f at the point $P(1, 1, -1)$. What is the value of the maximum increase rate?

(b) Find parametric equations of the normal line to the surface $xe^{y^2 - z^2} = 1$ at the point $(1, 2, 2)$.

4. (15 pts) Consider the function $f(x,y) = (y-1)(x^2-2) - y^2$.

(a) Find the critical points of $f(x,y)$.

(b) Classify the critical points as local min/max/saddle points.

5. (20 pts) Use Lagrange multipliers to find the point(s) on the sphere $x^2 + y^2 + z^2 = 1$ that are at the greatest distance from the point $(0, 1, 2)$? (Hint: It may be easier to maximize the square of the distance)

6. (25 pts) (a) Evaluate the integral

$$\int_0^3 \int_{y^2}^9 y \cos\left(x^2\right) dx dy.$$

(b) Find the intersection curve of the two paraboloids $z = x^2 + y^2$ and $z = 8 - 3x^2 - 3y^2$ and evaluate the volume between the two paraboloids.

7. (20 pts) Let $\mathbf{F}(x, y, z) = (2xy, x^2 + z^2, 2yz)$.

(a) Determine whether or not $\mathbf{F}$ is conservative . If so, find a function $f(x, y, z)$ such that $\mathbf{F} = \nabla f$.

(b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is parametrized by $\mathbf{r}(t) = (1 - t^2, t^3 + 1, 3t)$ with $0 \leq t \leq 1$.

8. (25 pts) Let C be the triangle with vertices $(0,0)$, $(1,0)$ and $(1,1)$, oriented counter-clockwise. Verify Green's theorem for $\mathbf{F} = \langle -1, xy \rangle$ by computing $\int_C \mathbf{F} \cdot d\mathbf{r}$ both directly and using the theorem.

9. (20 pts) Use the divergence theorem to evaluate the flux of $\mathbf{F} = \langle x^3, y^3, z^3 \rangle$ out of the sphere $x^2 + y^2 + z^2 = 1$.

10. (25 pts) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ by Stokes theorem, where $\mathbf{F} = \langle zy, -x, y \rangle$ and C is the oriented clockwise (as viewed from above) boundary curve of the part of the paraboloid $z = 1 - x^2 - y^2$ above the xy-plane.