Fall 2012 Math 226 Final

Please Print:

Your instructor's name:

Last name: First Name:

Signature: ID#

Problem	Score
1	
2	
3	
4	
5	
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7	
8	
9	
10	
Total	

INSTRUCTIONS:

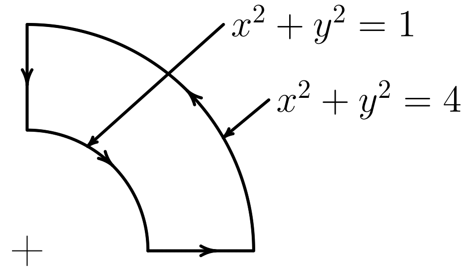
1. NO Calculators allowed. Cheating is not tolerated.
2. Clearly indicate your final answers by circling them.
3. Show all your work. Unsupported answers will not receive credit.
4. In general you do not need to "simplify" your answers, but you will need to evaluate simple numbers.
5. Point values are labeled and there are 100 total points possible.

Good luck!

1. [10 points] Use Green's theorem to evaluate $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$, where

$$\boldsymbol{F}(x, y) = < y^2, 3xy >,$$

and C is the boundary of the quarter-circular annulus as shown below with the path oriented counterclockwise.



2. [10 points] a) (3points) Is $\boldsymbol{F}(x, y, z) =< ye^{-x}, -e^{-x}, 2z >$ a conservative field.?(You need to justify your answer to get credit)

b) (7 points) If it is, find $f(x, y, z)$ so that $\boldsymbol{\nabla} f = \boldsymbol{F}(x, y, z)$.

3. [10 points] Use Stokes' theorem to evaluate

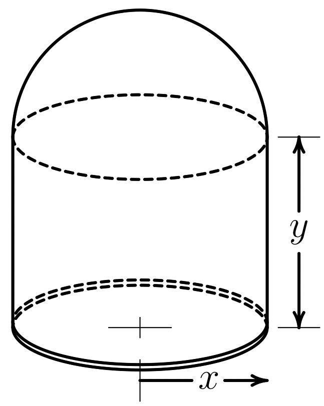
$$\int_C \boldsymbol{F} \cdot d\boldsymbol{r}, \quad \text{where} \quad \boldsymbol{F}(x, y, z) =< x^2y, \frac{1}{3}x^3, xy >,$$

and C is the curve of intersection of the elliptic paraboloid, $z = 2x^2+3y^2$ and the cylinder $x^2 + y^2 = 1$

4. [10 points] A sheet-metal tank consists of cylindrical section of radius x and height y, covered on top by a hemispherical dome of radius x. There is also the circular double-sheet base (i.e., twice the amount of sheet-metal for the base area) of radius x at the bottom of the cylinder.

(a) Calculate the surface area $A(x, y)$ of the sheet-metal used and $V(x, y)$ the volume enclosed by the tank.

(b) For a fixed amount of sheet-metal ($A = 8\pi$ is constant), obtain values of x and y for maximum volume.



5. [10 points] Find the flux of the vector field

$$\boldsymbol{F}(x, y, z) =< 2x^3+8xy^2, -y^3+2yz^2+e^y\cos(z), 4-x^2z+z^3-e^y\sin(z) >$$

top part of the hemisphere above the xy plane (the part of the sphere of radius 2 centered at the origin lying above the xy-plane) and outward orientation.

6. [10 points] Locate all relative extrema and saddle points of

$$f(x, y) = 3x^2 - 2xy + y^2 - 8y.$$

7. [10 points] Find the absolute maximum and minimum values of

$$f(x, y) = 3xy - 6x - 3y + 7$$

on the closed triangular region R with vertices $(0, 0)$, $(3, 0)$, and $(0, 5)$.

8. [10 points] Calculate the volume below the spherical surface $x^2 + y^2 + (z-1)^2 = 1$, and above the conical surface $z = \sqrt{x^2 + y^2}$.

9. [10 points]

a) Find an equation of the tangent plane of the parametric surface

$$x = 3u^2 + 2v, \quad y = v^2 - 3u, \quad z = u + v,$$

$0 \leq u \leq 2$ and $1 \leq v \leq 4$, at the point given by $u = 1$ and $v = 1$.

b) Write an integral that gives the area of the parametric surface (You do not need to evaluate the integral).

10. [10 points]

Let $\mathbf{F}(x, y, z) = 2z\mathbf{i} + 3x\mathbf{j} + 5y\mathbf{k}$. Compute the integral

$$\iint_S \text{curl } \mathbf{F} \cdot \mathrm{d}\mathbf{S}$$

where S is the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation.