Math 118: Business Calculus — Spring 2018

Final Exam — 09 May 2018

First Name: (as in student record)

Last Name: (as in student record)

USC ID: Signature:

Please circle your instructor and lecture time:

Haskell	Neshitov	Tabing	Tokorcheck
9am 1pm	10am	11am 12pm	2pm

- This exam has 10 problems, and will last 120 minutes.
- You may use any scientific **non-graphing** calculator.
- You may use one 8.5 x 11 handwritten formula sheet (front and back).
- Try to keep your solutions in the space provided for each question. You may continue solutions on other pages if you clearly indicate in that space where to find your solution.
- Show all of your work and justify every answer to receive full credit.

Do not write in the box below:

Q01	Q02	Q03	Q04	Q05	Partial 1
/20	/20	/20	/20	/20	/100
Q06	Q07	Q08	Q09	Q10	Partial 2
/20	/20	/20	/20	/20	/100

/200

Question 1 (20 points)**.** You sell handmade, made-in-the-USA T-shirts. The average cost function for your operation is given by

$$AC(q) = \frac{1200}{q} + 500 + 0.01(q - 100)^2,$$

where q is the **number of T-shirts** produced and $AC(q)$ is measured in **cents per shirt**.

(a) Find a formula for your total costs.

(b) What are your fixed costs?

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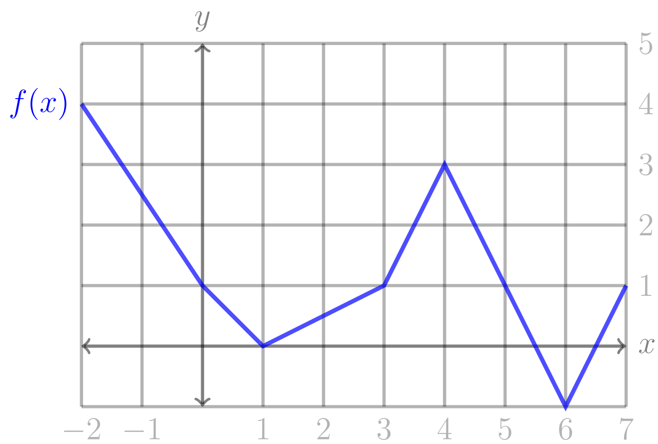
(c) Find a formula for your marginal cost.

(d) What is the minimum of marginal cost in the interval $100 \leq q \leq 400$?

Question 2 (20 points)**.** Suppose that $g(x) = 1 - 3x$, and $f(x)$ is represented by the graph:



(a) Find $\dfrac{d}{dx}\left[f(x)g(x)\right]$ at the point $x = 5$.

(b) Find $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]$ at the point $x = 2$.

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Question 3 (20 points). A hockey team plays in an arena with a seating capacity of $15,000$. At \$12 per ticket, they have an average attendance of $11,000$ fans. A recent survey shows that for every dollar they decrease their ticket price, their attendance will increase by $1,000$ fans. Where should they set their price for maximum revenue?

Question 4 (20 points)**.** You make beer. Let $C(q)$ denote the cost in dollars of producing q pints, $MC(q)$ denote the marginal cost, and $AC(q)$ the average cost.

(a) Suppose $\displaystyle\int_{40}^{70} MC(q)\,dq = 20$. What does this tell us? Choose the best option..

(i) The marginal cost, when you have produced 40 pints of beer, is \$20 per pint.

(ii) Suppose you have already made 40 pints of beer. The next 30 pints will cost, on average, \$20 per pint.

(iii) Producing 20 pints of beer will cost $70 - 40 = 30$ dollars.

(iv) Suppose you have already made 40 pints of beer. It will cost \$20 to produce the next 30 pints.

(b) Suppose $\displaystyle\frac{1}{35}\int_{100}^{135} MC(q)\,dq = \frac{1}{2}$. What does this tell us? Choose the best option.

(i) The marginal cost, when you have produced 100 pints of beer, is \$0.50 per pint.

(ii) Suppose you have already made 100 pints of beer. The next 35 pints will cost, on average, \$0.50 per pint.

(iii) Producing 1/2 a pint of beer will cost 35 dollars.

(iv) Suppose you have already made 100 pints of beer. It will cost \$0.50 to produce the next 35 pints.

(c) Consider the value of q where $MC(q) = AC(q)$. Which of the following are true about this value of q?

(i) This is the quantity where the profit is maximized.

(ii) This is the quantity where you break even.

(iii) This is a critical point of the average cost function.

(iv) This is a critical point of the marginal cost function.

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Question 5 (20 points)**.** Given that $f(x, y) = \ln\left(x\sqrt{x + y^2}\right)$, compute the derivatives:

(a) $\dfrac{\partial f}{\partial x}$

(b) f_y

(c) $\dfrac{\partial^2 f}{\partial x \partial y}$

Question 6 (20 points). Compute the following integrals:

(a) $\int 2^{x-1}\, dx$

(b) $\int_2^3 2te^{t^2}\, dt$

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(c) $\int (\ln y)^2 \; dy$

Question 7 (20 points)**.** Albert is currently in Kindergarten, and his parents decide to set up a bank account for his college education that will start in 12 years. Assume the annual interest rate is 3%, compounded continuously, and they will deposit money continuously throughout the 12-year period.

(a) If they deposit the money at a constant rate of \$9,000 per year, how much money will be in the account at the end of the 12-year period?

(b) If they expect that Albert will need \$200,000 for college education, at what rate should they deposit money?

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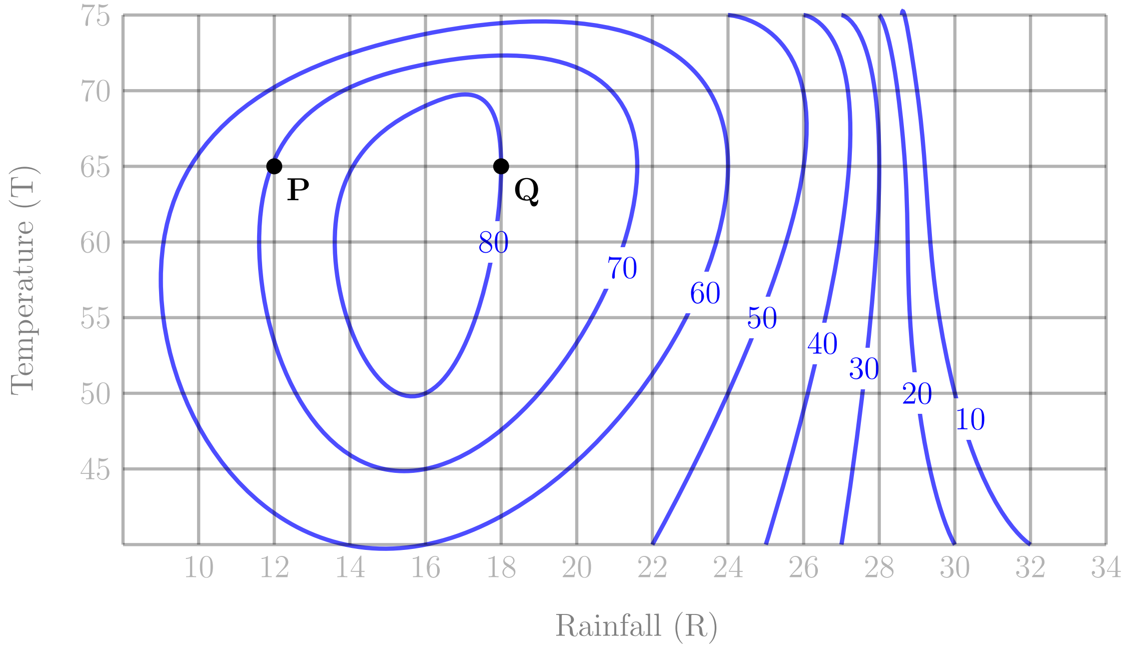
Question 8 (20 points). Find the **critical points** of the following function:

$$f(x, y) = 2x^2 + y^2 + 8xy - 6y + 20$$

Use the second derivative test to determine if your critical points represent local maximums, local minimums, or saddle points.

Question 9 (20 points)**.** The following contour diagram shows the corn production C (in millions of kg) in a given year, based on that year's average rainfall R (in inches) and average temperature T (in degrees Fahrenheit):



(a) Estimate the values of $\dfrac{\partial C}{\partial R}$ and $\dfrac{\partial C}{\partial T}$ at the point $(12, 65)$ marked **P**, including units.

(b) Use the derivatives in Part (a) to estimate the corn production C at $(11.5, 65.8)$.

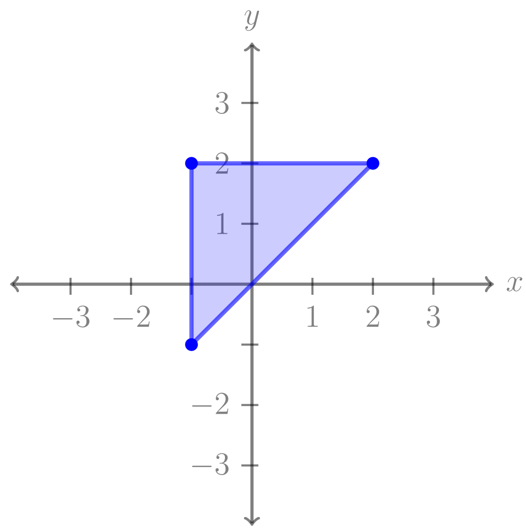
(c) At the point $(18, 65)$ marked **Q**, is $\dfrac{\partial^2 C}{\partial R^2}$ positive or negative? Explain how you know.

Question 10 (20 points).

Consider the function

$$f(x,y) = x^2 + y,$$

and the triangle T in the x,y-plane with its vertices at $(-1,-1)$, $(-1,2)$, and $(2,2)$.



Set up the integral $\iint_T f(x,y)\, dA$ in two ways, with the appropriate bounds:

(a) ... as an iterated integral in the order $dx\, dy$,

(b) ... as an iterated integral in the order $dy\, dx$,

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(c) Compute $\iint_T f(x,y)\,dA$, using the ordering that you prefer.

(d) Find the **average value** of $f(x,y)$ over the triangle T.

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