Math 126 Final Exam
May 9th, 2015

Directions. Fill out your name, signature and student ID number on the lines below **right now**, before starting the exam! Also, check the box next to the class for which you are registered.

You must **show all your work and justify your methods** to obtain full credit. Circle your final answers. Simplify your answers (unless the instructions say you do not have to). Do not use scratch paper; use the back of the previous page if additional room is needed. No calculators are allowed, but you may use the double sided HANDWRITTEN sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2} \times 11$ paper. Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

Name (please print): ______________________________

Signature: ______________________________

Student ID: ______________________________

☐ D.Crombecque 11am	☐ D Crombecque 12pm	☐ Y.Lin 10am
☐ N. Tiruviluamala 9am	☐ N. Tiruviluamala 11pm	☐ C. Wang 1pm
☐ G. Yildirim 9am	☐ G. Yildirim 10am	

Do not write on this page below this line!

1 (10 pts)	6 (20 pts)
2 (20 pts)	7 (20 pts)
3 (15 pts)	8 (20 pts)
4 (10 pts)	9 (15 pts)
5 (10 pts)	10 (20 pts)

160 points total

Problem 1 Evaluate the following limits.

(a) $\lim_{x \to 0} \dfrac{\tan(x)}{x + \sin(2x)}$

(b) $\lim_{x \to \infty} x^2 \ln(1 + \dfrac{1}{x})$

Problem 2 Compute the following antiderivatives.

(a) $\displaystyle\int \ln(1 + x^2)dx$

(b) $\displaystyle\int \frac{x^2+3x+1}{x^2+2x+1}dx$

Problem 3 Determine whether each of the following improper integrals is convergent or divergent.

(a) $\displaystyle\int_1^\infty \frac{x^2+1}{\sqrt{4x^6-x}}dx$

(b) $\displaystyle\int_0^1 \frac{1}{\cos^2(x)x\sqrt{x}}dx$

(c) $\int_0^4 x^2 \ln(x) dx$

Problem 4 Let T be the bounded region between the graphs of
$y = x^2 + 1$ and $y = 9 - x^2$
Set up, but DO NOT EVALUATE, an integral representing the volume of the solids obtained by rotating T about:

(a) THE X-AXIS

(b) THE LINE $x = 10$.

Problem 5 A tank is filled with sea water of density ρ in kg/m^3. Its shape is obtained by rotating the curve $y = \sqrt{x}$, with $0 \leq x \leq 4$ AROUND the Y-AXIS. DRAW A SKETCH AND SET UP, but DO NOT EVALUATE, an integral representing the work that is required to pump all the sea water out of the tank (from the top of the tank). All the length mentioned are in unit of meter (m). Note that the gravitational constant is g (m/s^2).

Problem 6

(a) Is $\sum_{0}^{\infty} \frac{(-1)^n e^n}{n!}$ convergent or divergent? Justify.

(b) Find the limit if it exists of the sequence given by $u_n = \frac{(-1)^n e^n}{n!}$. Justify.

(c) Find the limit if it exists of the sequence given by $u_n = (\frac{n+3}{n+1})^n$. Justify.

(d) Is $\sum_{0}^{\infty}(\frac{n+3}{n+1})^n$ convergent or divergent? Justify.

Problem 7 Determine whether the following series are ABSOLUTELY CONVERGENT, CONDITIONALLY CONVERGENT or DIVERGENT using any method. State the method(s) that you are using.

(a) $\displaystyle\sum_{1}^{\infty} \frac{(-1)^n \ln(n)}{n}$

(b) $\displaystyle\sum_{2}^{\infty} \frac{n\cos(n+5)}{n^3-1}$

Problem 8 Consider the following series:

$$\sum_{2}^{\infty} \frac{(-1)^n (x-4)^n}{n^2 - 1}$$

(a) Find the interval of convergence of the series. (Namely, find all the values of x for which the series is convergent.)

(b) Evaluate the series from part (a) when $x = 3$.

Problem 9 Consider the function

$$f(x) = x^{\frac{1}{3}}$$

(a) Find $T_2(x)$, the Taylor Polynomial of degree 2 of f centered at $a = 27$

(b) Use part (a) to find an approximation of $26^{\frac{1}{3}}$ (you can leave your answer as a sum of fractions)

(c) Use Taylor Formula to provide an estimate of the error made in the approximation of $26^{\frac{1}{3}}$ in part (b)

Problem 10 Recall that for $|x| < 1$,

$$\ln(1+x) = \sum_{1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

Namely, $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

(a) Find a power series expansion for $f(x) = x \ln(1 + \frac{1}{9}x^2)$.
(Find a general formula and then write out the first three terms of the power series explicitly.)

(b) Evaluate $f^{(39)}(0)$ (you do not need to simplify your answer).

(c) Write $\int_0^1 x \ln(1 + \frac{1}{9}x^2)dx$ as a series.

(d) Approximate $\int_0^1 x \ln(1 + \frac{1}{9}x^2)dx$ with an error of at most 0.001. (You can leave your answer as a sum of fractions.)