MATH 126 FINAL EXAM SPRING 2013

Last Name (Print):________________________ First Name (Print):_____________________

USC ID: __________________ Signature: __________________________________

Circle your lecture section:

Kalligiannaki at 10AM	Lin at 10AM	Mancera at 11AM
Rusin at 11AM	Mancera at 12 PM	Lanski at 1PM

INSTRUCTIONS

Turn off all electronic devices. Calculators, notes, books, or consultation with others is not allowed. Only the proctor may answer questions that arise. **SHOW YOUR WORK** and make your answers to each problem clear. The backs of sheets may be used for scratch paper, but if any part of a solution is on the back of a sheet, then you must indicate that to the grader.

Problem 1 (30 points) ________

Problem 2 (30 points) ________

Problem 3 (30 points) ________

Problem 4 (20 points) ________

Problem 5 (15 points) ________

Problem 6 (20 points) ________

Problem 7 (20 points) ________

Problem 8 (20 points) ________

Problem 9 (15 points) ________

Total ________

1) (30 points) For each part below, carefully justify mathematically whether the limit exists, including $\pm\infty$, or does not exist. If the limit exists find it.

a) $\lim\limits_{x\to 0} \dfrac{x-\tan x}{x-\sin x}$

b) $\lim\limits_{x\to 0^+} (\cos x)(\ln x)$

c) $\lim\limits_{x\to\infty} \left(\dfrac{x}{x+3}\right)^x$

2) (30 points) a) Find $\int \frac{x^2+x^3}{\sqrt{1-x^2}}dx$

b) $\int x^2(\ln x)^2 dx$

c) Why does the integral $\int_0^{\infty} \frac{dx}{1+x^{2013}}$ converge or why does it diverge?

3) (30 points) Let A be the region under the graph of $f(x) = \dfrac{x^2 + x + 1}{x^2(x^2 + 1)}$, above the x-axis, and to the right of $x = 1$. Determine if A has finite area. If so, find it, and if not, explain why.

4) (20 points) A conical tank 20 feet high with a circular top 10 feet in diameter is filled with water (62.5 lb/ft^3). Find an integral that represents:

a) the work done in pumping the water out the top of the tank;

b) the hydrostatic force on the inner wall (curved surface) of the tank.

5) (15 points) i) If $a_n = \dfrac{\sqrt{n}\sin(n!)}{n+1}$, carefully determine if $\lim\limits_{n\to\infty} a_n$ exists, and if so, find it.

ii) Let $c > 0$, set $b_1 = c$, $b_2 = \sqrt{c} + 1$ and in general let $b_{n+1} = \sqrt{b_n} + 1$.

a) Show that the sequence (b_n) is increasing or decreasing, depending on c.

b) Show that (b_n) is bounded.

c) Justify whether (b_n) converges or diverges.

d) If (b_n) converges find its limit.

6) (20 points) In each case below, justify mathematically why the series converges or diverges. If the series converges, find its sum if possible, using results from class.

a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n}{10n+1}$

b) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2+n+1}{n^3+n^2}$

c) $\displaystyle\sum_{n=1}^{\infty}\frac{2^{n-1}+3^n}{6^n}$

7) (20 points) Carefully determine the interval of convergence of $\displaystyle\sum_{n=2}^{\infty} \frac{(x+3)^{3n}}{2^n \ln(n)}$.

8) (20 points) Using Taylor's Formula, estimate $\int_0^{1/10} (e^x + e^{-x})\,dx$ with an error of at most 10^{-4}.

Your answer should be a fraction.

9) (15 points) a) Sketch a graph in polar coordinates of $r = 2\sin(\theta) + 1$.

b) Find an expression using integrals that represents the area of the region enclosed by the graph of $r = 2\sin(\theta) + 1$.