MATH 126 - Fall '14

Final Exam

Name: **Student Number:**

Please read all of the following rules carefully before proceeding.

- Check that this Exam contains 11 pages.
- Unless otherwise instructed, please clearly indicate all work involved in the solution of each problem. You will receive partial credit for partial progress toward a solution.
- You may use one 8 x 11 in. sheet of paper with notes (both sides); you may not refer to any other books or notes during the course of the exam.
- You may **not** use a calculator on the exam.

Problem	Possible	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

Please encircle the name of your instructor:

A.Asok **N. Emerson** **L. Goldstein** **W. Hu** **Y. Lin**

Problem 1 (10 pts.)

Evaluate the integral

$$\int x^2 \sin^{-1} x \, dx.$$

Problem 2 (10 pts.)

Evaluate the following integral:

$$\int \frac{3x^2 + x + 1}{x(x^2 + 1)}\, dx.$$

Problem 3 (10 pts.)

Evaluate the following integrals.

a) $\displaystyle\int_0^\infty xe^{-2x}\,dx.$

b) $\displaystyle\int \frac{dx}{(3-x^2)^{3/2}}.$

Problem 4 (10 pts.)

Let $\mathcal{R}$ be the bounded region between the graphs of $x = -y^2 + 1$, $x = 0$ and $y = 0$. Set up, but DO NOT EVALUATE, integrals representing the volumes for the solids obtained by rotating $\mathcal{R}$ about:

a) the line $x = 0$;

b) the line $y = -1$.

Problem 5 (10 pts.)

Consider the function

$$f(x) = \int_0^x \sqrt{e^t - 1}\, dt.$$

a) Find the length of the curve $y = f(x)$ for $1 \le x \le 2$.

b) Set up the integral to find the area of the surface obtained by rotating the curve $y = f(x), 1 \le x \le 2$ about the x-axis. DO NOT EVALUATE THE INTEGRAL.

Problem 6 (10 pts.)

(Differential Equations) Initially, a tank contains 100 gallons of pure water. Starting at time $t = 0$, a brine solution of concentration 0.2 lbs salt/gallon is added to the tank at a rate of 3 gallons/minute. As the brine solution is added, the tank is mixed and drained at a rate of 3 gallons/minute.

a) Write a differential equation for the amount of salt the tank contains at time t.

b) Solve the differential equation in Part (a).

c) How long does it take for the *concentration* of salt in the tank to reach 0.1 lbs/gal?

Problem 7 (10 pts.)

Determine whether each series is absolutely convergent, conditionally convergent or divergent; state precisely which convergence test(s) you use to determine your answer.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{3n} - 4}{11^n}$.

b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt[3]{n+2}}$.

c) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{n^3 + n^2 + 2}{3n^3 - n^2 + 1}\right)^{2n}$.

Problem 8 (10 pts.)

Consider the function $f(x) = \sinh^{-1} x$; recall that $f'(x) = \frac{1}{\sqrt{1+x^2}}$.
Below, when you are asked to write down a series, please give explicitly the first 3 non-zero terms as well as the general term.

a) Write down a Taylor series expansion for $f(x)$ about the point $x = 0$.

b) Determine the radius of convergence of this Taylor series.

c) Determine the value of $f^{(31)}(0)$.

Problem 9 (10 pts.)

Throughout this problem, set $f(x) = e^{-x^2}$.
Below, when you are asked to write down a series, please give explicitly the first 3 non-zero terms as well as the general term.

a) Write down the Maclaurin series for $f(x)$.

b) Use the above series to find a series representation of $\int_0^1 e^{-x^2}\, dx$.

c) Determine how many terms from the series in Part (b) are necessary if we want to compute the integral with an error of at most $1/1000$.

Problem 10 (10 pts.)

Consider one leaf of the four leaf rose, $r = \cos(2\theta)$, $-\pi/4 \leq \theta \leq \pi/4$.

a) Determine the values of θ for which this curve has a horizontal tangent (Hint: first write down an equation whose solutions govern points with horizontal tangents and then use double angle and Pythagorean identities to solve this equation).

b) Determine the values of θ for which this curve has a vertical tangent (Refer to the hint given in Part (a)).

c) Find the area of this (one) leaf.