Math 126 Final Exam
May 7th, 2014

Directions. Fill out your name, signature and student ID number on the lines below **right now**, before starting the exam! Also, check the box next to the class for which you are registered.

You must **show all your work and justify your methods** to obtain full credit. Write your final answers in the boxes provided. Simplify your answers. Any fraction should be written in lowest terms. You need not evaluate expressions such as $\ln 5$, $e^{0.7}$, and $\sqrt{3}$. Do not use scratch paper; use the back of the previous page if additional room is needed. No calculators are allowed, but you may use the sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2} \times 11$ paper. You may have anything written on it (on both sides), but it must be written *in your own handwriting*. Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

Name (please print): ______________________

Signature: ______________________

Student ID: ______________________

☐ D.Crombecque 10am ☐ D Crombecque 11am ☐ D. Crombecque 1pm
☐ T.Sakai 10am ☐ N. Tiruviluamala 11am ☐ N. Tiruviluamala 12pm

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1 (15 pts)	6 (10 pts)
2 (15 pts)	7 (10 pts)
3 (15 pts)	8 (15 pts)
4 (10 pts)	9 (20 pts)
5 (10 pts)	10 (20 pts)

140 points total

Problem 1 (15 points) Evaluate the following limits.

(a) $\displaystyle\lim_{x\to\infty} \frac{\ln(1+x^2)}{x}$

(b) $\displaystyle\lim_{x\to\frac{\pi}{2}^-} (\sin x)^{\tan x}$

(c) $\lim_{x \to 0} \frac{1 - \cos(x^2)}{x^4 + x^6}$

Problem 2 (15 points) Compute the following antiderivatives.

(a) $\int x^{\frac{3}{2}} \ln(x) dx$

(b) $\int \frac{1}{x^2 - x - 2} dx$

(c) $\int \frac{1}{x^2\sqrt{4+x^2}}dx$

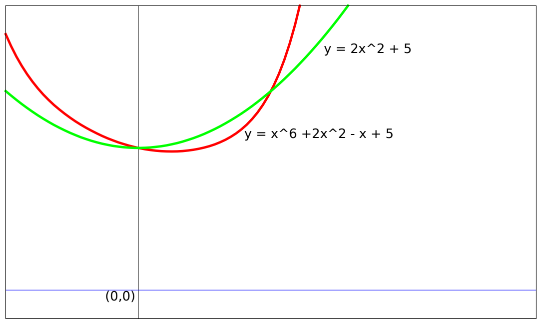
Problem 3 (15 points) Determine whether each of the following improper integrals is convergent or divergent.

(a) $\displaystyle\int_1^\infty \frac{e^x}{x}\,dx$

(b) $\displaystyle\int_0^1 \frac{1}{(x-1)^{\frac{1}{3}}}\,dx$

(c) $\displaystyle\int_1^\infty \frac{\pi - \sin(x)}{x^2 + \ln(x)}\,dx$

Problem 4 (10 points) Let T be the bounded region between the graphs of $y = x^6 + 2x^2 - x + 5$ and $y = 2x^2 + 5$.

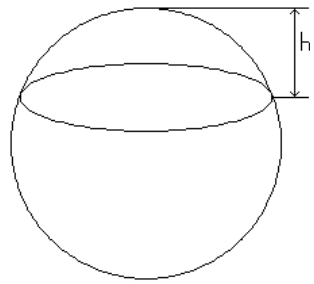


Set up, but DO NOT EVALUATE, an integral representing the volume of the solids obtained by rotating T about:

(a) THE LINE $x = -1$

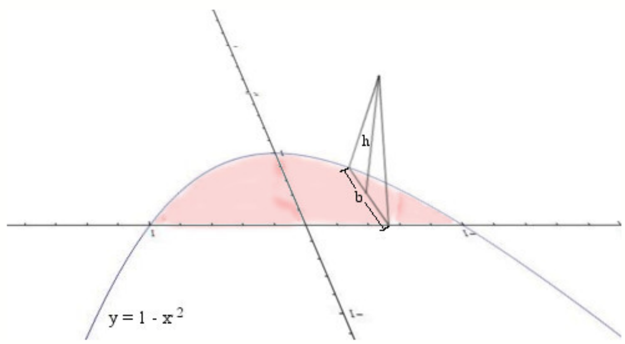
(b) THE LINE $y = 10$.

Problem 5 (10 points) A spherical tank with a radius of 3 meters contains a liquid of density $\rho = 2000$ kilograms per cubic meter. The tank is filled all the way up to 1 meter BELOW the TOP of the tank ($h = 1$ meter).



Set up, but DO NOT EVALUATE, an integral representing the the work (in Joules) required to pump ALL of the liquid out of the tank.
Assume $g = 9.8$ m/s^2.

Problem 6 (10 points) The base of a solid S is the region enclosed by the parabola $y = 1 - x^2$ and the x-axis. The cross sections of S which are perpendicular to the x-axis are isosceles triangles with base equal to the height ($b = h$).
Find the volume of S.



Problem 7 (10 points) Evaluate the area of the surface generated by revolving the curve

$$x = \frac{1}{3}y^3, \qquad\qquad 0 \leq y \leq 1$$

ABOUT THE Y-AXIS.

Problem 8 (15 points) Determine the convergence or divergence of the following series using any method. State the method(s) that you are using.

(a) $\sum_{1}^{\infty} \frac{1}{3^n + n^4}$

(b) $\sum_{0}^{\infty} \frac{(-1)^n (2n)!}{3^n}$

(c) $\sum_{2}^{\infty} \frac{\sqrt{n}}{n^3 - 5}$

(d) $\sum_{1}^{\infty} (n^2 \sin(\frac{1}{n}))^n$

Problem 9 (20 points) Consider the following series:

$$\sum_{0}^{\infty} \frac{(-1)^n (3x-4)^n}{3^{2n}}$$

(a) Find the interval of convergence of the series. (Namely, find all the values of x for which the series is convergent.)

(b) Evaluate the series when $x = 1$.

(c) When the series is convergent, find its value explicitly in terms of x. Simplify your answer.

Problem 10 (20 points) Recall that for all x,

$$\cos(x) = \sum_{0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

Namely, $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

(a) Find a power series expansion for $f(x) = x\cos(\frac{1}{2}x^3)$.
(Find a general formula and then write out the first three terms of the power series explicitly.)

(b) Evaluate $f^{(13)}(0)$.

(c) Write $\int_0^1 x\cos(\frac{1}{2}x^3)dx$ as a series.

(d) Approximate $\int_0^1 x\cos(\frac{1}{2}x^3)dx$ with an error of at most 0.0001. (You can leave your answer as a sum of fractions.)