Math 125 Final December 6, 2017

NAME:____

USC ID Number:____

Circle your section in the list below:

T. Do (9am)	G. Dreyer (9am)	R. Mancera (10am)	D. Searles (10am)
R. Mancera (11am)	D. Searles (11am)	G. Dreyer (12pm)	A. Mazel-Gee (12pm)
G. Reyes Souto (1pm)	C. Wang (1pm)	G. Reyes Souto (2pm)	

Instructions:

- No note sheets or electronics are permitted.
- Carefully justify all your answers.
- For each problem, draw a box around your final answer.
- There are 200 points available on the exam.

1. (20 points) Find the value of the following limits. Carefully justify your answers.

   (a) $\lim_{x \to 0} \dfrac{\sin^2(x)}{x \tan(3x)}$

   (b) $\lim_{x \to \infty} \dfrac{e^{\cos(3x)}}{\sqrt{x}}$

(c) $\lim_{x \to \infty} x + 1 - \sqrt{x^2 + x + 1}$

2. (16 points) Find the value of a for which the function

$$f(x) = \begin{cases} x^2 - a & \text{if } x \leq 1 \\ \frac{x^2-1}{x^2+2x-3} & \text{if } x > 1 \end{cases}$$

is continuous at $x = 1$.

3. (16 points) Show that the function

$$f(x) = \begin{cases} x^3 \sin \frac{1}{x^3} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is differentiable at $x = 0$.

4. (16 points)

(a) Suppose g is a differentiable function with $g(1) = 2$ and $g'(1) = -1$.
If $f(x) = 2g(x)\ln(g(x) - x)$, find $f'(1)$. Express your answer as a single number.

(b) Find the derivative $f'(x)$ of $f(x) = x^{\sqrt{x}}$.

5. (16 points) Water is being poured into a cylindrical tank at a rate of 5 ft^3/min. If the depth of the water is increasing at a rate of 7 ft/min, what is the radius of the tank?

6. (16 points) Find the linear approximation to the function $f(x) = \sqrt[3]{x+2}$ at 62, and use this to approximate $f(63)$.

7. (16 points) Find all points in the interval $[-2, 2]$ on which the function

$$f(x) = \frac{x^{\frac{2}{3}}}{2 + x^2}$$

has a local maximum or minimum, and find the absolute maximum and minimum values of f on this interval.

8. (16 points) Find the area of the largest rectangle with one of its sides on the x-axis that can be inscribed in the closed region bounded by the x-axis and the graph of $y = 2 - x^2$.

9. (16 points) Let $f(x) = \ln(2 + e^{x-3})$.

(a) Indicate the domain of f. Show that f is one-to-one over its domain and determine the range of f.

(b) Find an expression for f^{-1}, and find the domain of f^{-1}.

(c) Evaluate the derivative of f^{-1} at $x = 5$.

10. (20 points) Find the following integrals.

(a) $\displaystyle\int_0^2 |1-x|dx$

(b) $\displaystyle\int x^5\sqrt{x^2+3}dx$

(c) $\displaystyle\int_{-\ln 7}^{\ln 7} \frac{x^7}{e^{-7x} + e^{7x}} dx$

11. (16 points) Find the interval where the function

$$g(x) = \int_0^x \frac{dt}{t^2 - t + 1}$$

is concave up.

12. (16 points) The weight of a certain extraterrestrial creature increases at a rate proportional to its weight. A researcher found such a creature at 9am on Monday morning. The researcher weighed the creature at 9am on Tuesday morning, and found it weighed 50g. The researcher weighed the creature again at 9pm on Thursday evening, and found it weighed 150g.

    How much did the creature weigh when the researcher found it? Express your answer as single fraction, without any exponentials or logarithms.