MATH 118: BUSINESS CALCULUS — FALL 2017
FINAL EXAM

06 December 2017

First Name: (as in student record)

Last Name: (as in student record)

USC ID: Signature:

Please circle your instructor and lecture time:

Zhang 9am	Tabing 9am 11am	Dreyer 10am	Wang 10am	Tokorcheck 12pm 1pm	Hall 12pm 2pm	Haskell 1pm

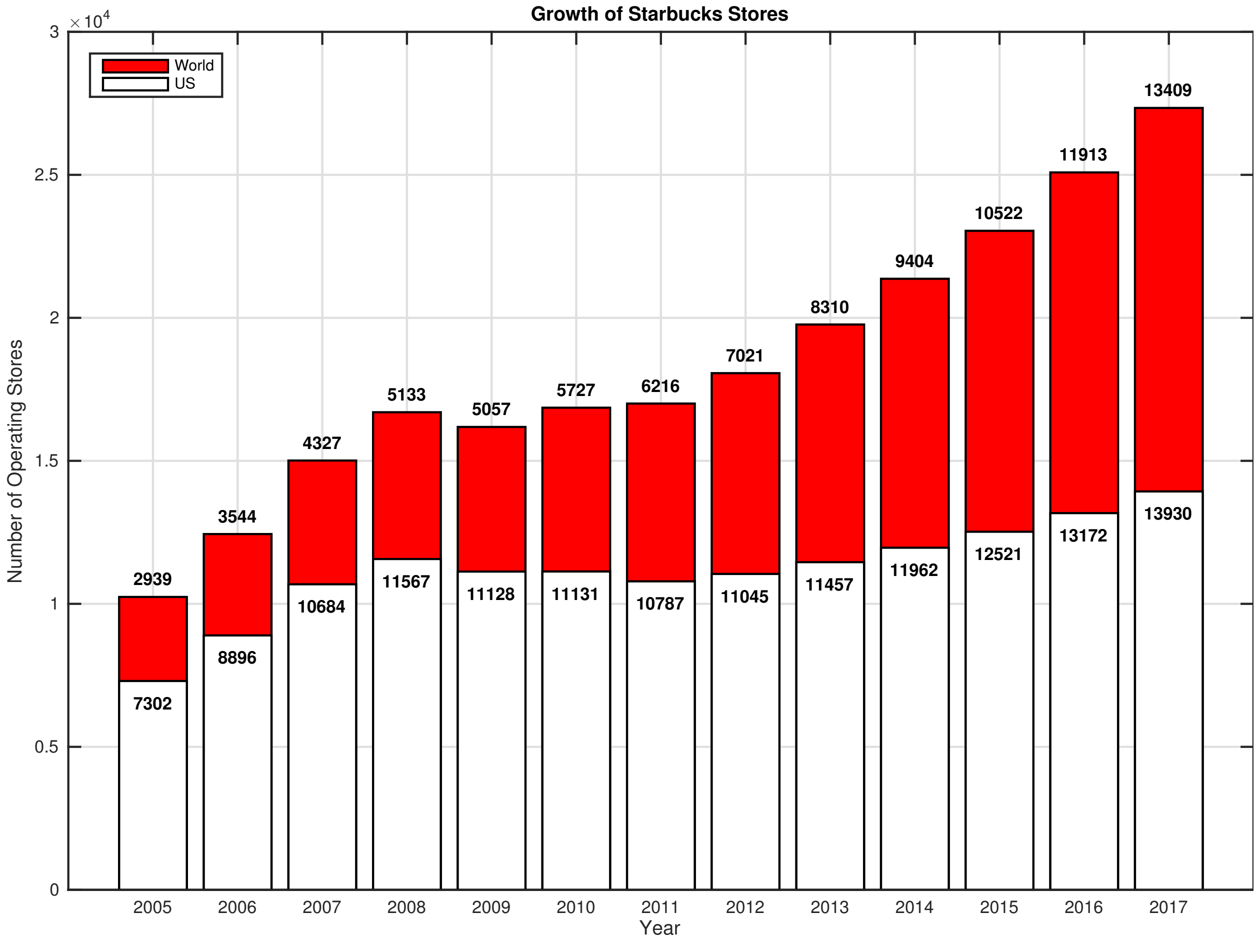
- This exam has 11 problems, and will last 120 minutes.
- You may use any scientific **non-graphing** calculator.
- You may use one 8.5 x 11 handwritten formula sheet (front and back).
- Try to keep your solutions in the space provided for each question. You may continue solutions on other pages if you clearly indicate in that space where to find your solution.
- Show all of your work and justify every answer to receive full credit.

Do not write in the box below:

Q01	Q02	Q03	Q04	Q05	Q06	Partial 1
/12	/12	/10	/10	/10	/12	/66
Q07	Q08	Q09	Q10	Q11		Partial 2
/12	/10	/12	/10	/10		/54

/120

Question 1 (12 points)**.** The growth of the Starbucks stores over the years is shown in the figure below. The number of stores in the US and outside of the US are represented by the heights of the white and shaded bars, respectively.



(Source: https://www.statista.com/statistics/218366/number-of-international-and-us-starbucks-stores/)

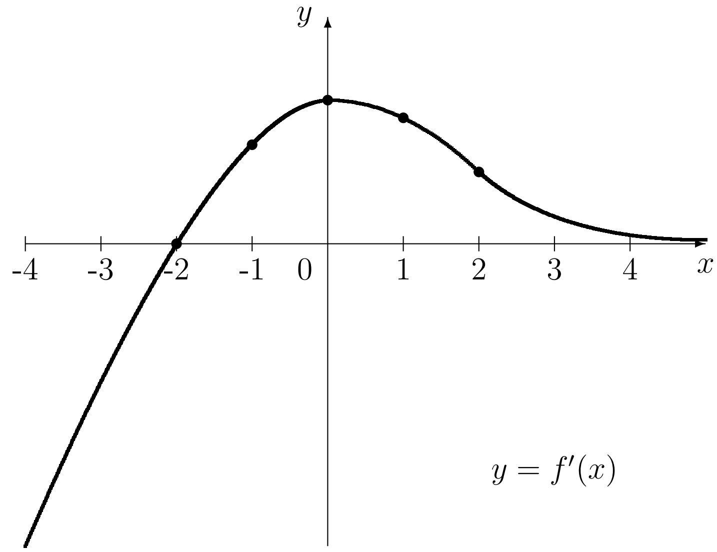
(a) Find the average rate of growth m of the chain **worldwide** (including US) between 2010 and 2017.

(b) Assuming the above rate of growth m is maintained in the future, what would be the number of Starbucks stores worldwide by 2020?

(c) If the growth from 2010 is assumed to be **exponential**, use the data from 2010 and 2017 to find the exponential rate of growth k, and a formula for the number of chains worldwide t years after 2010.

(d) If the growth from 2010 is assumed to be **exponential**, what would be the number of Starbucks stores worldwide by 2020?

Question 2 (12 points)**.** Shown below is the graph of the **derivative** f' of a function f.



(a) For each quantity below, determine if it is positive, negative, equal to 0, or whether the information is insufficient to determine this. Give a brief explanation in each case.

(i) $f(-1)$

(ii) $f'(-1)$

(iii) $f''(-1)$

(b) Consider the interval $(-1, 0)$. Determine if each function below is increasing or decreasing on this interval. Give a brief explanation in each case.

(i) f

(ii) f'

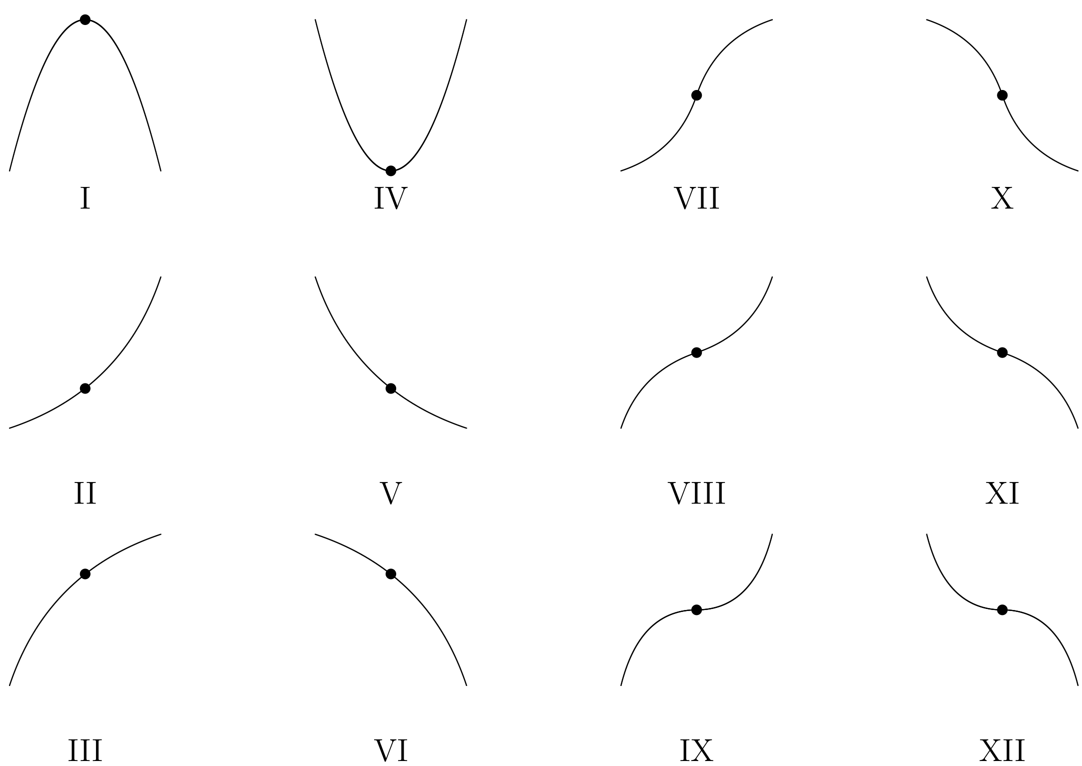
(iii) f''

(c) For each input value below, choose one of the sketches to show what the graph of the function f looks like in a neighborhood of that point. A sketch may be used once, more than once, or not at all.

(i) $x = -2$ ☐

(ii) $x = -1$ ☐

(iii) $x = 0$ ☐



Question 3 (10 points)**.** It is estimated that the cost of constructing an office building that is n floors high is

$$C = 3n^2 + 500n + 1810,$$

where C is measured in thousands of dollars. How many floors should a building have in order to minimize the **average cost** per floor?

(**Hint**: If the critical point is not an integer, check neighboring integer points.)

Question 4 (10 points)**.** Consider this integral of $f(x) = 2x$ with unknown endpoints:

$$F(a, b) = \int_a^b 2x \, dx$$

(a) For which values of a and b will $F(a, b) = 0$? Circle all answers that are correct, and leave blank any that are incorrect.

(i) $a = 2, \quad b = 3$

(ii) $a = 0, \quad b = 0$

(iii) $a = -50, \quad b = 50$

(iv) $a = 0, \quad b = 1$

(v) $a = 3, \quad b = 3$

(b) For which values of a and b will $F(a, b) > 0$? Circle all answers that are correct, and leave blank any that are incorrect.

(i) $a = 2, \quad b = 3$

(ii) $a = 0, \quad b = 0$

(iii) $a = -1, \quad b = 2$

(iv) $a = 0, \quad b = 1$

(v) $a = -2, \quad b = 1$

(vi) $a = 3, \quad b = 3$

(c) If we assume that a and b are both positive ($0 < a < b$), then which combination of horizontal width (w) and vertical height (h) describes a rectangle whose area is equal to the area represented by $F(a, b)$? Circle all answers that are correct, and leave blank any that are incorrect.

(i) $w = b - a, \quad h = b^2 - a^2$

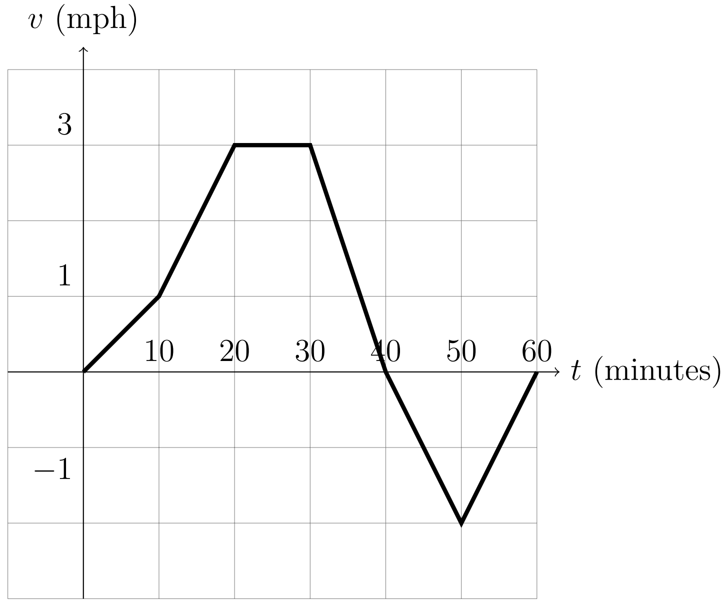
(ii) $w = b - a, \quad h =$ the average value of $f(x) = 2x$ on $[a, b]$

(iii) $w = a, \quad h = f(b)$

(iv) $w = a, \quad h =$ the maximum value of $f(x) = 2x$ on $[a, b]$

Question 5 (10 points)**.** A woman lives on Venice Blvd, 1.2 miles from the ocean. (Venice Blvd runs east-west from downtown LA all the way to the ocean.)

She takes a walk, heading west from her house toward the ocean. The graph below shows her velocity $v(t)$ during her walk, t minutes from when she leaves her house.



(a) From 30 to 40 minutes into her walk, is she walking towards the ocean, or towards her home? **Explain**.

(b) From 40 to 50 minutes into her walk, is she walking towards the ocean, or towards her home? **Explain**.

(c) Between 0 and 20 minutes into the woman's walk, is she speeding up or slowing down? **Explain**.

(d) How many miles is the woman from her home, one hour into her walk?

Question 6 (12 points)**.** Compute the following integrals:

(a) $\displaystyle\int (7x-1)\, e^{2x}\, dx$

(b) $\displaystyle\int_0^1 x^2(x^3+1)^4\, dx$

(c) $\int x^3 e^{-\frac{x^2}{2}}\ dx$

Question 7 (12 points)**.** For each function below, find the indicated partial derivative:

(a) $f(x, y) = \sqrt{1 + xy}$ $f_x =$

(b) $P(u, v) = u \cdot 10^{uv}$ $\dfrac{\partial P}{\partial v} =$

(c) $T(x, y) = \dfrac{x + y}{x - y}$ $T_{xy} =$

Question 8 (10 points)**.** When a company sells its product for p dollars and spends A dollars per month on advertising, its monthly revenue is $R(p, A)$ dollars. Suppose the company presently sells its product for \$200 and spends \$10,000 per month on advertising.

(a) Also suppose that $\left.\dfrac{\partial R}{\partial A}\right|_{(200,10000)} = 1.75$. What does this tell us about the company?

Circle only the best answer.

(i) If the company increases both the selling price of its product and the amount it spends on advertising, then its revenue will increase by \$1.75.

(ii) The company's revenue is presently approximately \$200.

(iii) If the company increases the amount it spends on advertising by \$200, its revenue will increase by approximately \$1.75.

(iv) If the company increases the amount it spends on advertising by \$1, its revenue will increase by approximately \$1.75.

(v) If the company increases its revenue by \$1, then the amount it spends on advertising will increase by approximately \$1.75.

(b) Further suppose that

$$R(200, 10000) = 125,000 \quad \text{and} \quad \left.\frac{\partial R}{\partial p}\right|_{(200,10000)} = -210$$

Estimate the company's monthly revenue if it increases the selling price of the product by \$1.70 and decreases the amount spent on advertising by \$150.

Question 9 (12 points)**.** A company produces two versions of its product – one for domestic markets and one for export. In the domestic market, the demand curve is

$$p_1 = 45 - q_1$$

while in the export market the demand curve is

$$p_2 = 55 - q_2.$$

(a) Write an expression for the company's **revenue** R in terms of q_1 and q_2.

(b) The cost of producing q_1 units of the domestic version and q_2 of the export version is

$$C(q_1, q_2) = 1000 + 5q_1 + 5q_2 + q_1q_2.$$

Find the **critical points** (q_1, q_2) for the company's **profit** P, also considered as a function of q_1 and q_2.

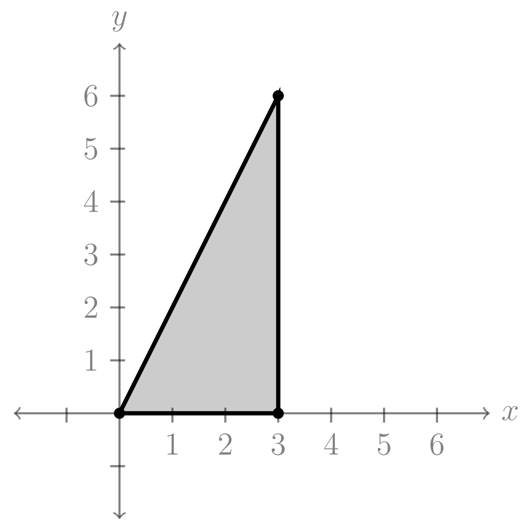
(c) Use the second derivative test to determine if your critical points represent local maximums, local minimums, or saddle points.

Question 10 (10 points)**.** Let f be the function $f(x, y) = 5x^2y^3$.

Find the **average value** of f on the rectangle defined by $0 \leq x \leq 3$ and $-4 \leq y \leq 4$.

Question 11 (10 points)**.** Let T be the triangle having vertices at $(0,0)$, $(3,0)$, and $(3,6)$.

Evaluate the integral of $f(x,y) = x^2y$ over the region bounded by the triangle T.



BLANK PAGE