Math 118: Business Calculus — Fall 2021

Final Exam — 08 Dec 2021

DO NOT OPEN THIS EXAM UNTIL YOU ARE INSTRUCTED TO DO SO

First Name: (as in student record)

Last Name: (as in student record)

USC ID: Signature:

- This exam has 10 questions and will last **120 minutes**.
- You may use a non-graphing standard **scientific calculator**.
- You may use one 8.5 x 11 handwritten **formula sheet** (front and back).

- Include your **signature on each page** in the space provided at the top. When you scan your solutions at the end of the exam, this signature must be visible on every page.
- Try to keep your solutions on the same page as the question you're answering. The (blank) last page is for extra work - if you use it, please let the grader know on the original question page.
- Show all of your work and justify every answer to receive full credit.

Please circle your instructor and lecture time:

Tokorcheck 9am 10am	Dreyer 10am	Hall 11am	Cremaschi 12pm	Wickham 12pm 1pm	Gerhardt 1pm 2pm

---

Question 1 (8 points)**.** The number of medical physicians that are active in the US is increasing, as indicated in the table to the right.

This seems to follow an exponential growth model $N(t) = N_0 e^{kt}$, where $t = 0$ corresponds to 2006.

Year	**Active Physicians**
2006	$680,500$
2015	$758,600$
2020	$805,800$
2025	$859,300$

(a) Use the data for 2006 and 2015 to find the explicit expression for $N(t)$.

(b) Use your function $N(t)$ to predict the number of active physicians in 2030.

(c) According to $N(t)$, how long does it take for the number to increase by 30%?

Question 2 (12 points)**.**

The demand curve of a product is $p = \dfrac{3}{e^q} + 10$, with p being the price per unit in thousands of dollars.

A company evaluates the production costs, also in thousands of dollars, to be $C(q) = \dfrac{9}{e^q} + 10q$.

(a) Find a formula for the **profit** as a function of q.

(b) Can profits be maximized? Carefully justify your findings with a derivative test.

(c) Find a formula for the **average cost** as a function of q.

(d) Can the average production costs be minimized? Carefully justify your findings with a derivative test.

Question 3 (10 points)**.** The values for a smooth function $z = f(x, y)$ are provided in the table below. The y-values are listed vertically down the side, and the x-values are listed listed horizontally across the top:

y/x	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	3	0	-1	0	3	7	7
2	4	1	0	1	4	10	11
3	3	0	-1	0	3	7	14
4	0	-3	-4	-3	0	4	16
5	-5	-8	-9	-8	-5	5	17
6	-7	-10	-11	-14	-7	7	10
7	-8	-11	-12	-21	-8	10	5

(a) Estimate $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at the point $(2, 4)$.

(b) Using your answers from Part (a), estimate the value of $f(2.3, 3.8)$.

(c) Which of the following could be critical points of $z = f(x, y)$? Circle each correct answer.

$(4, 3)$ $(2, 4)$ $(3, 2)$

(d) At which of the following points could the property $f_{yy} < 0$ hold? Circle each correct answer.

$(6, 5)$ $(4, 3)$ $(7, 2)$

Question 4 (10 points)**.**

A company's production output $f(N, V)$ is given in tons and is a function of the number of workers, N, and the value of the equipment, V, in units of **thousands of dollars.** Suppose the production function is

$$f(N, V) = 10N^{0.8}V^{0.2}.$$

(a) Compute $f(50, 25)$ rounded to two decimal places and **include units.**

(b) Compute the partial derivative $\dfrac{\partial f}{\partial N}$.

(c) Compute $f_V(50, 25)$ rounded to two decimal places and include units.

(d) "If we increase the equipment value V from _____ thousand dollars to _____ thousand dollars

while maintaining _____ workers, we expect an additional _____ tons of production."

Question 5 (12 points)**.** Compute the indicated derivatives, leaving your work **unsimplified**.

(a) Compute $\dfrac{dy}{dx}$ for the function $y = x^3 \ln(e^x + e^{-x})$.

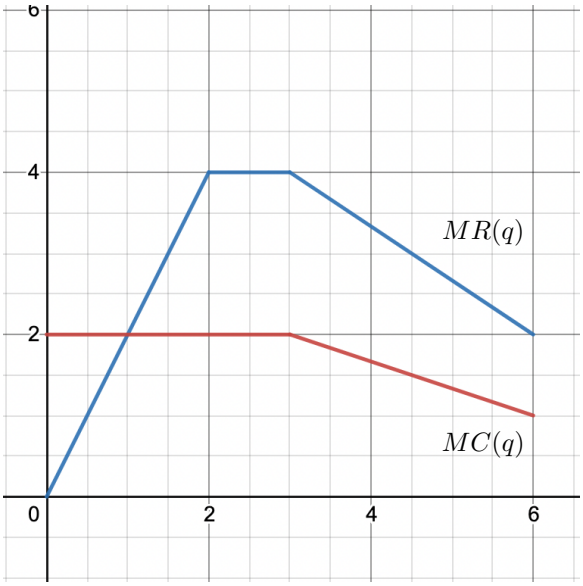
(b) Compute $\dfrac{\partial z}{\partial x}$ for the function $z = \dfrac{x+y}{(x+1)^2}$.

(c) Compute $f_x(x, y)$ for the function $f(x, y) = x^y + y^x$.

Question 6 (10 points)**.** Find all critical points of the function and use the second derivative test to classify them as local maxima, local minima, or saddle points:

$$f(x, y) = \frac{1}{3}x^3 + y^2 + 2xy - 6x - 3y + 4$$

Question 7 (8 points)**.** The marginal revenue $MR(q)$ and marginal cost $MC(q)$ for producing q units of California ceramics are each graphed below. Both are listed in **hundreds of dollars per ceramic**.



If the fixed costs are \$100, find the profit from producing 6 ceramics.

Question 8 (8 points)**.**

Starting on December 8, 2021, cash flows continuously into an initially empty bank account at the rate of $S(t) = 5t^2$ dollars per year for 10 years. The account earns 3% annual interest compounded continuously.

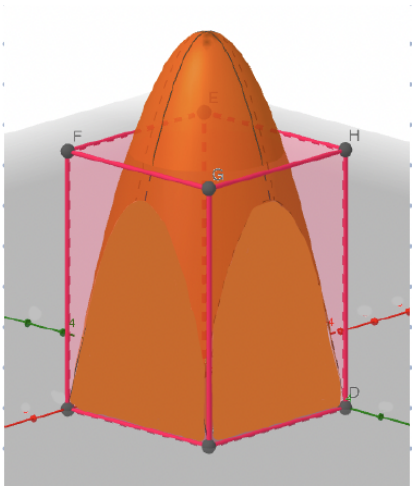
(a) Compute the present value of this cash flow on December 8, 2021, to the nearest \$10.

(b) What is the future value at the end of the 10 years?

Question 9 (10 points)**.** As a holiday gift you receive a candle in a box, as pictured below. The box itself has a 2" $\times$ 2" square base where

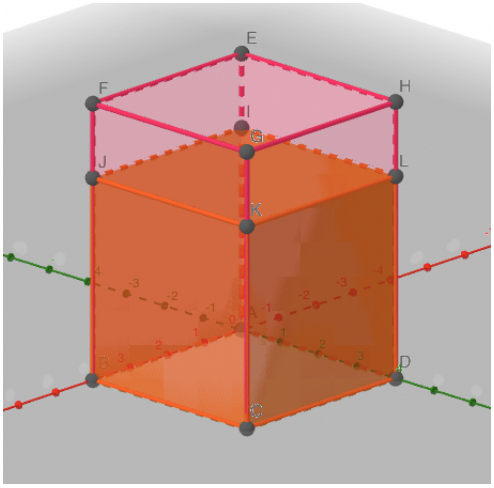
$$1 \leq x \leq 3 \qquad \text{and} \qquad 1 \leq y \leq 3.$$

The surface of the candle is described by the equation $f(x, y) = -x^2 + 4x - e^{x-2}(y-2)^2 + 6$.



(a) Find the volume of the candle. To receive full credit, be sure to show all work in your calculations.

(b) You leave the candle in your apartment on a hot day and the wax melts into the following prism:



After the wax has melted, what is the height of the wax in the box?

Question 10 (12 points)**.** Compute the indicated integrals.

(a) $\displaystyle\int \frac{x^3}{(1+x^4)^{\frac{1}{3}}}\,dx$

(b) $\displaystyle\int x^2\,\ln(x)\,dx$

(c) $\displaystyle\int \frac{\left(e^{-x}+2e^{3x}\right)^2}{e^{2x}}\,dx$

SIGNATURE:

INDICATE ON THE ORIGINAL QUESTION PAGE IF THERE IS WORK TO BE GRADED HERE

SIGNATURE:

INDICATE ON THE ORIGINAL QUESTION PAGE IF THERE IS WORK TO BE GRADED HERE