Math 118, Fall 2022

Final

Name: ____________________

USC ID: ____________________

Instructions:

- Write your name and USC ID in the spaces indicated above. Once you're insructed to begin the exam, you should also write your name on each page.
- You have 2 hours to complete your work. You can use a handwritten sheet of notes (8.5x11, front and back) and a non-graphing calculator as you solve these problems.
- Make sure to clearly show each step of your solution for full credit - correct answers without any work will not receive credit. Some questions may remind you to show your work explicitly, but you should assume each question requires you to justify your steps by making your thought process clear in your solution.
- The very last page is blank and is intended to be used if you need extra space to solve any of the problems. If you do use this last page to write a solution to one or more of the problems, indicate on those problems' pages that some of your work appears on the last page.
- When units are provided in a problem, your numerical answers should be given with the appropriate units.
- All work you submit should represent your own thoughts and ideas. If the graders suspect otherwise, you can expect your instructor to file a report with USC Student Judicial Affairs and Community Standards.
- Indicate your section by circling the instructor and time below.

Tabing, 9am   J. Zhang, 10am   Qiu, 10am   Z. Zhang, 11am   Tabing, 12pm

Warner, 12pm   Warner, 1pm   Jayanti, 1pm   Quijada, 2pm

1. (20 points) The table shows the weight of a fetus (in ounces) over days 77 through 104 of a pregnancy.

Days in pregnancy, t (days)	77	84	93	104
Weight, w (oz)	1.9	2.4	2.8	3.0

(a) Which of the following are correct interpretations of the quantity $W'(84)$. Circle all that apply.

A. $W'(84)$ is the average weight gain of the fetus per day over the first 84 days of the pregnancy.

B. $W'(84)$ is the approximate weight gain of the fetus on the 84th day of the pregnancy.

C. $W'(84)$ is the rate of change of W with respect to t when t is 84.

D. $W'(84)$ is the rate of change of t with respect to W when t is 84.

E. $W'(84)$ is the weight of the fetus on the 84th day of the pregnancy.

F. $W'(84)$ is the slope of the secant line at $t = 84$.

(b) Find the average rates of change of $W(t)$ on the intervals $[77, 84]$ and $[84, 93]$. Report your answers with the appropriate units.

(c) Using your answer to part (b), does it appear that $W''(84)$ is positive or negative? Explain.

(d) If $W'(84) = 0.05$, estimate $W(87)$. Is this an overestimate or underestimate? Explain.

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2. (20 points) Consider the functions

$$f(x) = e^{x^2-2x} \qquad \text{and} \qquad g(x) = \frac{\ln x}{1+\sqrt{x}}$$

For each of the following, you do not need to simplify your answer.

(a) Compute $f'(x)$.

(b) Compute $f''(x)$.

(c) Find the rate of change of $g(x)$ at $x = 1$.

3. (20 points) The demand curve of a product is $p = 60 - 2q$, with p being the price per unit in thousands of dollars.

A company evaluates the production costs, also in thousands of dollars, to be $C(q) = q^3 - 20q^2 + 120q$. Suppose the company only has the capacity to produce at most $q = 20$ units.

(a) Find a formula for the **profit** as a function of q.

(b) Find the quantity at which profits are maximized. You must justify that you have found the global maximum.

(c) Find a formula for the **average cost** as a function of q.

(d) Find the quantity at which average costs are minimized. You must justify that you have found a local minimum.

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4. (20 points) Peter is a purchasing manager at ABC grocery store. The amount of cabbages he restocks every day depends on customer demand for cabbage. Peter models the demand curve of cabbages as

$$q = 1000 - 10p^3$$

where p is the price, and q is the quantity of cabbages sold each day. Currently, one cabbage is priced at \$2.

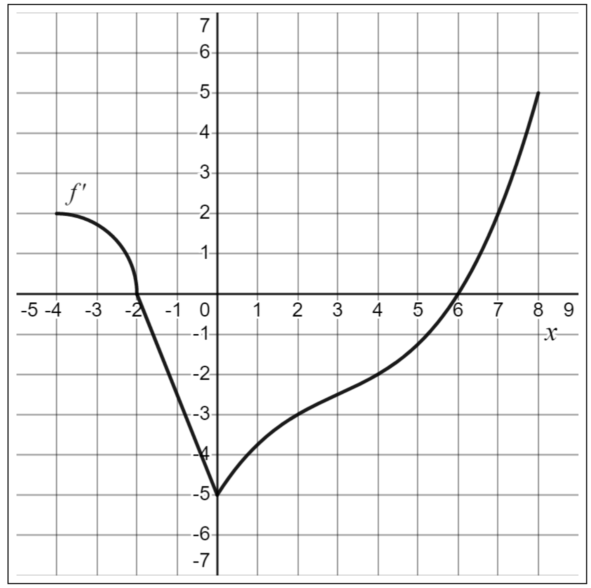
(a) Find the elasticity of demand at the current price. Is the demand for cabbages elastic or inelastic?

(b) Use the elasticity you found in part (a) to estimate the percent change in demand given a 10% increase in price.

(c) If the price of one cabbage increases slightly from \$2, do you expect the revenue to increase or decrease? Explain.

5. (20 points) Consider a function $f(x)$ defined on the interval $[-4, 8]$ such that $f(0) = 4$ and whose derivative $f'(x)$ is given by the graph below. The part of the graph on the interval $[-4, -2]$ is a quarter circle, and the part of the graph on the interval $[-2, 0]$ is a line.

(a) Use a right-Riemann sum with $\Delta x = 2$ to estimate the value of $f(8)$. State whether you think this is an overestimate or an underestimate, and briefly explain why.



(b) Identify all critical points of f. At each critical point, determine whether f has a local maximum, local minimum, or neither.

(c) Find the exact value of $f(-4)$.

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6. (20 points) Evaluate the following definite and indefinite integrals. You do not need to simplify your answers.

(a) $\displaystyle\int \left(\frac{2}{3t} - t^{\frac{1}{2}} + e^{1.3t} \right) dt$

(b) $\displaystyle\int_1^2 \frac{1}{x^2 \left(\frac{1}{x} + 1 \right)} dx$

7. (20 points) Sam's dream car costs \$100,000.

(a) Sam starts to save money for his dream car. He deposits money into an investment account, which earns 10% interest compounded continuously, at a rate of \$10,000 per year. Calculate the balance in the account at the end of year 2.

(b) Instead of saving for the car, Sam decides to take out a \$100,000 loan with an interest rate of 2%, compounded continuously. Suppose Sam pays the loan at a rate of $100e^t$ dollars per year. How long will it take for Sam to pay off the loan? In other words, over what period of time is the present value of Sam's payment stream equal to the \$100,000 loan?

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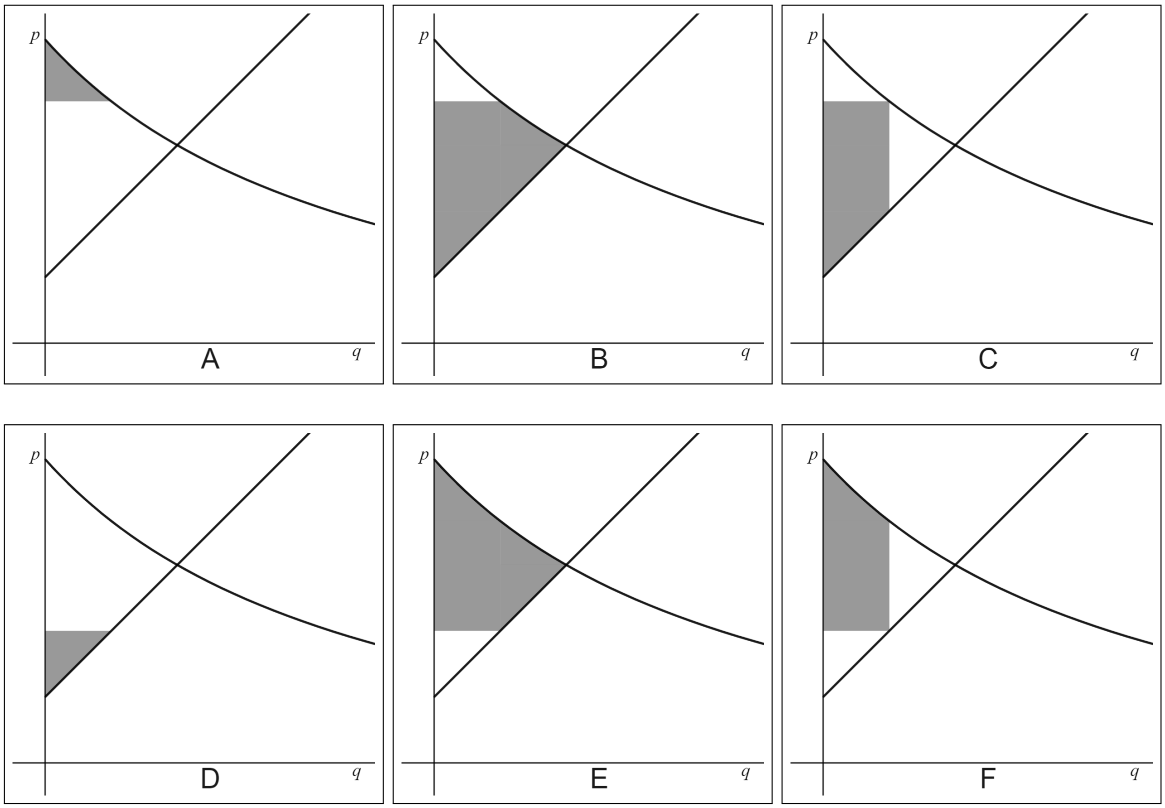
8. (20 points) The supply and demand curves for a product are

$$p_s(q) = q + 1 \qquad \text{and} \qquad p_d(q) = \frac{28}{q+5} - 1$$

(a) What is the consumer surplus at market equilibrium?

(b) Suppose the government fixes the price at $p = 2$. What is the consumer surplus when $p = 2$?

Consider the graphs of $p_s(q)$ and $p_d(q)$ shown below, labeled A through F.



(c) Which shaded region represents the consumer surplus at $p = 2$?

(d) Which shaded region represents the producer surplus at $p = 2$?

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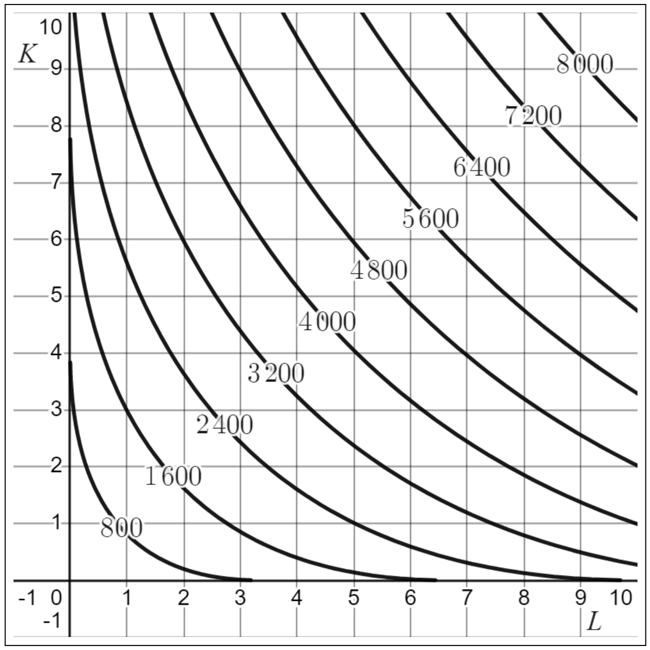
9. (20 points) Consider the following contour diagram for a company's production function $Q(L, K)$. Here Q represents the company's output, L is the labor they employ, and K is the capital they own.



(a) Use the contour diagram to estimate $Q_L(9, 4)$. The work you show should clearly demonstrate how you found your estimate.

(b) Which of the following are correct interpretations of $Q_L(9, 4)$. Circle all that apply.

A. $Q_L(9, 4)$ is the approximate difference in output between a company that employs 10 units of labor and 4 units of capital and a company that employs 9 units of labor and 4 units of capital.

B. $Q_L(9, 4)$ is the approximate difference in output between a company that employs 9 units of labor and 5 units of capital and a company that employs 9 units of labor and 4 units of capital.

C. $Q_L(9, 4)$ is the rate of change of Q with respect to L at $L = 9$ assuming K is held constant at 4.

D. $Q_L(9, 4)$ is the rate of change of L with respect to Q at $L = 9$ assuming K is held constant at 4.

E. $Q_L(9, 4)$ is the derivative of the single-variable function $Q(9, K)$ at $K = 4$.

(c) Suppose now we know that the formula for the production function

$$Q(L,K) = 2\left(11\sqrt{L} + 10\sqrt{K}\right)^2$$

Use this formula to find the exact value of $Q_K(9,4)$.

(d) Use the partial derivatives you found in parts (a) and (c) to approximate $Q(9.5,3)$. Credit will only be awarded to an approximation found using partial derivatives.

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10. (20 points) Suppose you are selling wine. The revenue generated not only depends on quantity sold but on the age of the wine as well. Let q and a denote the number of wine bottles sold and the age of the wine (in years). Here, we will use the following specific form for the revenue:

$$R(q, a) = 100\ q^{\frac{1}{2}}\ a^{\frac{3}{2}}$$

Since older wines are usually more expensive, you find that the number of bottles sold decreases with the age as given by the constraint:

$$4q + a = 80$$

(a) Find the values of q and a that maximize revenue subject to the given constraint and find the value of this maximum revenue. You do not need to justify that the value you found is a maximum.

(b) Calculate the value of the Lagrange multiplier, and then use it to approximate the new maximum revenue if the constraint is changed to $4q + a = 82$.

(You can use this page if you need extra space to solve any of the problems. If you write work on this page for a certain problem, indicate on that problem's page that some of your solution appears here.)