

MATH 118: BUSINESS CALCULUS
FINAL EXAM

FALL 2018
05 December 2018

First Name: _____ (as in student record)

Last Name: _____ (as in student record)

USC ID: _____ Signature: _____

Please circle your instructor and lecture time:

Tokorcheck 9am	Pawlowski 9am 11am	Heilman 10am 12pm	Gao 10am 12pm	Rooney 1pm 2pm	Tabing 1pm
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- This exam has 10 problems, and will last 120 minutes.
- You may use any scientific **non-graphing** calculator.
- You may use one 8.5x11in handwritten formula sheet (front and back).
- Try to keep your solutions in the space provided for each question. You may continue solutions on other pages if you clearly indicate in that space where to find your solution.
- Show all of your work and justify every answer to receive full credit.

Do not write in the box below:

Q01	Q02	Q03	Q04	Q05	Partial 1
/15	/18	/15	/18	/20	/86
Q06	Q07	Q08	Q09	Q10	Partial 2
/18	/20	/18	/18	/20	/94

Question 1 (15 points). The table below summarizes some data about a function $f(x)$. It is also given information that:

- $f''(x) > 0$ on the interval $(5, 7)$
- $f''(x) < 0$ on the interval $(7, 9)$
- $f''(x) > 0$ on the interval $(9, 10)$

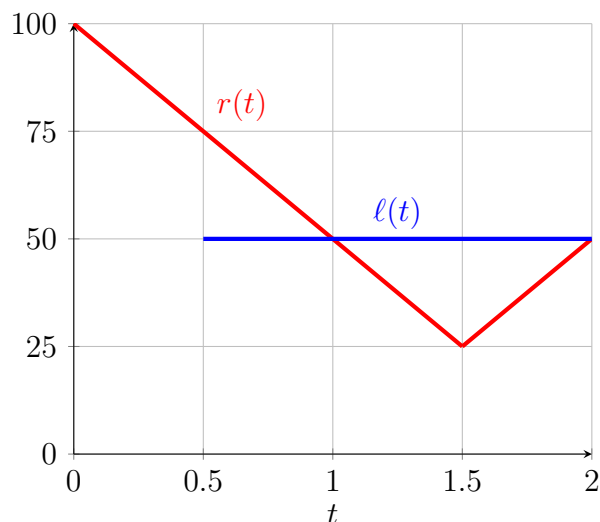
At each x -value from 1 to 10, determine whether or not $f(x)$ has a **local maximum**, **local minimum**, or **inflection point** there, and circle the appropriate response in each column. If it is not possible to make that determination using the data at hand, circle “Inconclusive”.

You do not need to show any work for this question.

x	$f'(x)$	$f''(x)$	local maximum	local minimum	inflection point
1	0	-3	Yes / No / Inconclusive	Yes / No / Inconclusive	Yes / No / Inconclusive
2	0	0	Yes / No / Inconclusive	Yes / No / Inconclusive	Yes / No / Inconclusive
3	0	1	Yes / No / Inconclusive	Yes / No / Inconclusive	Yes / No / Inconclusive
4	1	0	Yes / No / Inconclusive	Yes / No / Inconclusive	Yes / No / Inconclusive
5	-1	2	Yes / No / Inconclusive	Yes / No / Inconclusive	Yes / No / Inconclusive
6	0	2	Yes / No / Inconclusive	Yes / No / Inconclusive	Yes / No / Inconclusive
7	4	0	Yes / No / Inconclusive	Yes / No / Inconclusive	Yes / No / Inconclusive
8	1	-3	Yes / No / Inconclusive	Yes / No / Inconclusive	Yes / No / Inconclusive
9	0	0	Yes / No / Inconclusive	Yes / No / Inconclusive	Yes / No / Inconclusive
10	3	0	Yes / No / Inconclusive	Yes / No / Inconclusive	Yes / No / Inconclusive

Question 2 (18 points). A function $r(t)$ records the **rate** at which rain is falling into a bucket at time t , in cubic inches per hour. At $t = 0.5$ hours, the bucket springs a leak, and at each time t water is **escaping** at a rate of $\ell(t)$ cubic inches per hour.

Shown below is a graph of both $r(t)$ and $\ell(t)$.



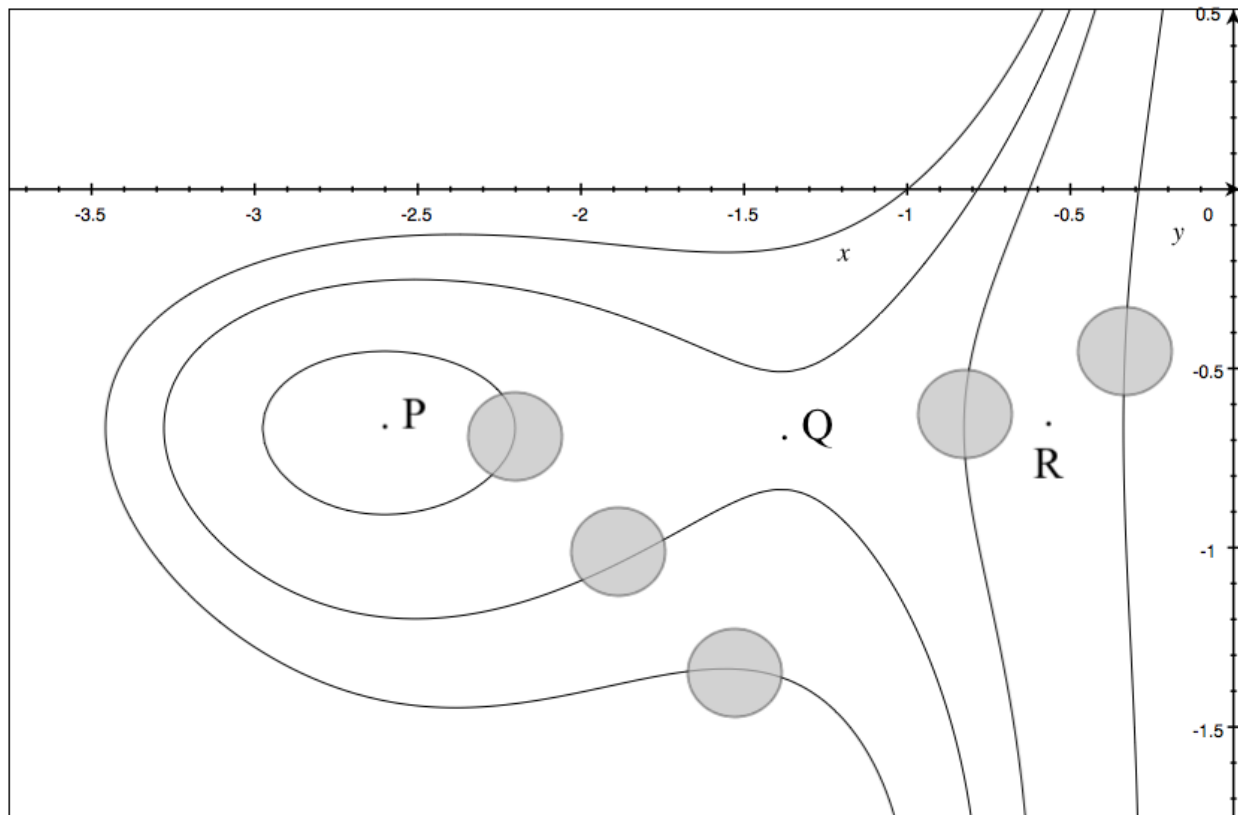
- (a) If the bucket is **empty** at time $t = 0$, how much water is in the bucket at time $t = 2$? Justify your answer, either by obtaining it from an integral, or indicating how it comes from the graphs.
- (b) When does the bucket contain the **largest** amount of water?
- (c) Construct an integral that gives the **average** rate at which rain enters the bucket between $t = 0$ and $t = 1.5$. You do not need to evaluate the integral.

Question 3 (15 points). The contour diagram below shows the contours of a function $z = f(x, y)$ for the z -values $z = 0, 1, 2, 5$.

There is a **local maximum** at P , a **saddle point** at Q , and these are the only critical points in this domain.

If we also know that $f_x(R) > 0$, label the contours by putting the correct z -value in the grey circles that overlap the contour. Each z -value may be used more than once.

You do not need to show any work for this question.



Question 4 (18 points). A carpenter sells desks for \$30 each and, at this price, sells 100 desks per month. The carpenter estimates that for each \$5 increase in price, she sells 10 fewer desks per month. If the desks are manufactured at a cost of \$2 per desk, at what price should they be sold to generate the greatest possible profit?

Carefully show why your answer yields the greatest possible profit and show all of your work.

Question 5 (20 points). The following table is included with every new California Driver's License. It shows the Blood Alcohol Content $B(d, w)$ for males and females as a function of the number of drinks they consume, d , and their body weight in pounds, w .

It is illegal in California to drive with a BAC of 0.08% or higher. You rode to a party with a female friend who just had **one 12oz beer** and weighs **140 lbs**.

Number of Drinks		Body Weight in Pounds							Driving Condition	
		100	120	140	160	180	200	220		240
0	M	.00	.00	.00	.00	.00	.00	.00	.00	Only Safe Driving Limit
	F	.00	.00	.00	.00	.00	.00	.00	.00	
1	M	.06	.05	.04	.04	.03	.03	.03	.02	Driving Skills Impaired
	F	.07	.06	.05	.04	.04	.03	.03	.03	
2	M	.12	.10	.09	.07	.07	.06	.05	.05	
	F	.13	.11	.09	.08	.07	.07	.06	.06	
3	M	.18	.15	.13	.11	.10	.09	.08	.07	
	F	.20	.17	.14	.12	.11	.10	.09	.08	
4	M	.24	.20	.17	.15	.13	.12	.11	.10	Legally Intoxicated
	F	.26	.22	.19	.17	.15	.13	.12	.11	
5	M	.30	.25	.21	.19	.17	.15	.14	.12	
	F	.33	.28	.24	.21	.18	.17	.15	.14	

Subtract .01% for each 40 minutes that lapse between drinks.
 1 drink = 1.5 oz. 80 proof liquor, 12 oz. 5% beer, or 5 oz. 12% wine.
Fewer than 5 persons out of 100 will exceed these values.

(a) For females, use the data to estimate the value of $B_w(1, 140)$.
 As an estimate, different answers can receive full credit as long as method is correct.

(b) For females, use the data to estimate the value of $B_d(1, 140)$.
 As an estimate, different answers can receive full credit as long as method is correct.

(c) Use your estimates from Parts (a) and (b) to estimate your friend's BAC if she had another 8oz of beer (1.75 drinks total), and weighed 147 pounds. Can she legally drive?

(d) If a **male** friend had 2.3 drinks and weighed 173 pounds, could they legally drive?

Question 6 (18 points). Compute the derivatives indicated in each part without using a calculator. Show every step, and highlight the derivative rules that you use.

- Find $\frac{df}{dt}$ when $f(t) = (2^t + 5t)^9$.

- Find R_q when $R(p, q) = \frac{q}{p^2 + q^2}$.

- Find $\frac{\partial^2 f}{\partial x \partial y}$ when $f(x, y) = x \ln(xy)$.

Question 7 (20 points). Compute the following definite or indefinite integrals without using a calculator. Show every step, and highlight the integration techniques that you use.

- $\int_1^2 \frac{3x^5 - 2x^4 + 7x^3 - 5x^2 + 1}{x} dx$

- $\int \frac{(\ln q + 1)^2}{q} dq$

- $\int_0^2 (t^2 - 3t + 4) e^{-3t} dt$

Question 8 (18 points). Find all critical points of the following function. Classify each critical point using the second derivative test:

$$f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$$

Question 9 (18 points). Find the maximum/minimum values of the function $f(x, y) = 3xy$, when the variables x and y must also satisfy the constraint $2x^2 + y^2 = 1$. For each point you find that is a candidate for a max/min, identify it as a max, min or neither.

Question 10 (20 points). Let $f(x, y) = 3x + 5y$, and let R be the region in the x, y -plane bounded by the curves

$$y = 4x^2,$$

$$y = 12x.$$

Set up the integral $\iint_R f(x, y) dA$ in two ways, with the appropriate bounds:

(a) ... as an iterated integral in the order $dx dy$,

(b) ... as an iterated integral in the order $dy dx$,

(c) Compute $\iint_R f(x, y) dA$, using the ordering that you prefer.

(d) Construct a formula for the **average value** of $f(x, y)$ over the region R . You do not need to evaluate this formula.

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