

# A Note on Conditionals and Compositionality\*

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Higginbotham (1986) noted that the interpretation of conditional *if* and alternative *unless* depended upon the nature of quantifiers appearing in the main clause. In (1), for example, the conditional can be interpreted as a connective of a familiar sort; whereas in (2) that would lead to the wrong meaning.

- (1) Everyone will succeed if he works hard.
- (2) No one will succeed if he goofs off.

The data thus constitute a *prima facie* counterexample to a narrow version of compositionality; for it follows that the conditional must be sensitive to the presence of a higher syntactic feature. In this note, I show that a popular dismissive response to the point just made fails entirely, and fails even for the examples that would seem best suited for it. I then explore a possible response that invokes the conditional as in possible-worlds semantics, showing that, whereas the Stalnaker conditional, with its characteristic principle of Conditional Excluded Middle, would restore compositionality, it is nevertheless doubtful that this principle holds in general. Compositionality might be restored, however, to the extent that the presupposition of Conditional Excluded Middle can be built into the semantics.

In Higginbotham (1986) I considered two examples of possible non-compositionality in English. One, namely the thesis that complement clauses denote only relatively to what they are embedded in, had been suggested also by others in various forms, and has been debated since. The other, which is the substance of this note, concerned indicative conditionals, as in minimal pairs like (1)-(2):

- (1) Everyone will succeed if he works hard.
- (2) No one will succeed if he goofs off.

I observed that the word *if*, while interpretable as a conditional connective in (1), could not be interpreted the same way in (2); for, (2) would then mean that there is no one whose goofing off is, or would be, a sufficient condition for his success. It appeared, therefore, that the interpretation of *if* was sensitive to the semantic nature of the quantifier within whose scope it fell. But that conclusion conflicts with compositionality, at least as narrowly construed.

A counter-thesis, which has acquired something of folkloric status, is that in both (1) and (2) the conditional *if* is meaningless: the clause that it marks constitutes merely the restriction on the quantifier, as made explicit in the paraphrases (3)-(4):

- (3) Everyone who works hard will succeed.
- (4) No one who goofs off will succeed.

I will argue that this *riposte* fails in general (even if it appears to succeed for these cases), and that the counterexample still stands. But there will be a further moral to the story. Compositionality can be restored under certain assumptions about the meaning, or the presuppositions, of conditionals. However, at present I am not aware of any way of grounding these presuppositions that is not stipulative.

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The problem of (1)-(2), and the counter-thesis sketched above, were, so far as I know, first adumbrated in a few swift remarks in Lewis (1975: 14-15). Lewis did not, at least as I read his work, intend the counter-thesis to be universally applicable; that is, applicable to absolutely all conditionals. Rather, it was to apply to cases of what he called "unselective binding," illustrated, for instance, by Frege's example (5) and by others in Lewis's important article.

(5) If a number is less than 1 and greater than 0, then its square is less than 1 and greater than 0.

In Higginbotham (1986), I observed that the generalization, that *if* and *unless* are interpreted differently depending upon the nature of the higher quantifier, extended to a contrast between all monotone increasing quantifiers such as *every*, on the one hand, and all monotone decreasing quantifiers such as *no* on the other.<sup>1</sup> I was there short on examples, however, which I now supply. Suppose that we are speaking of the 30 students now enrolled in Philosophy 300, and consider (6) where *they* is construed as bound to *most students*. First of all, (6) must be sharply distinguished from (7), the result of absorbing the *if*-clause into the restriction:

- (6) Most students will get A's if they work hard.
- (7) Most students who work hard will get A's.

For: (6) is true iff in counting up the students *x* of whom it is true to say that *x* will get an A if *x* works hard, the total amounts to most of them; whereas (7) is true or false depending upon whether, of those students who in fact work hard, most get A's. So (6) and (7) are logically independent. That is enough to show that the absorption method suggested by Lewis for his cases will not work in general; and as we will see below, it fails also even for the universal and negative existential quantifiers. Consider now (8) again with *they* construed as bound to the subject.

(8) Few students will get A's if they work hard.

This example certainly does not mean that few students are things *x* such that *x* will get an A if *x* works hard.<sup>2</sup> We will return to the question what exactly it does signify, but for immediate purposes it is sufficient to note that it is not equivalent to the straightforward (9) and, even if it were, the problem for compositionality would remain: for the question how, if at all, the subordinating conjunction *if* is to be interpreted could not be locally determined.

(9) Few students who work hard will get A's.

Returning now to the issues posed by the original examples, I want to propose a general account of the conditional that sorts out the phenomena. First, let us reduce the range of data. As remarked above, and noted in my earlier work, the issue of how to interpret the connective arise for *unless* as well as for *if*. Thus we may contrast (10) and (11):

- (10) Every student will get an A unless he goofs off.
- (11) No student will get an A unless he works hard.

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<sup>1</sup> A restricted quantifier *Q* is monotone increasing if *Q A are B* and *All B are C* implies *Q A are C*, and monotone decreasing if *Q A are B* and *All C are B* implies *Q A are C*. See further below for the model-theoretic version of these notions.

<sup>2</sup> Although I will not expand upon the point here, it should be noted that (i) and (ii) need not be equivalent in this setting. In examples of the form (ii), the conditional always carries its proper force; i.e., the force it would have in *x is A if x is B*; so there is no problem about compositionality. To put the matter in terms congenial to Lewis's discussion, the quantifier in (ii) is a whole clause away from the *if*-clause, so absorption of it into the quantifier restriction is blocked, presumably for syntactic reasons. Compositionality is an issue only for examples of the form (i).

(i) *Q* things are *A* if they are *B*

(ii) *Q* things are things such that they are *A* if they are *B*

(10) has it that for every student  $x$ ,  $x$  will get an A provided that  $x$  doesn't goof off, but (11) cannot be taken as meaning that for no student  $x$ ,  $x$  will get an A provided that  $x$  doesn't work hard.

Now, one is taught in Logic 101 to "translate" *unless* by disjunction ' $\vee$ '; and this is OK, but only as it were by accident. The reason it is OK is that, whereas  $p$  *unless*  $q$  pretty clearly amounts to  $p$  *if*  $\neg q$ , the schemata ' $\neg q \rightarrow p$ ' and ' $p \vee q$ ' are truth-functionally equivalent. The equivalence of course fails in general where the *if*-clause does not express the material conditional. Even so, if we take *unless*-clauses as decomposed or else taken up in some semantic fashion (depending upon how one treats the conditional) as what we might call *if-not*-clauses, then the problem posed by (10)-(11) immediately reduces to the previous case, (10) being equivalent to (12), and (11) to (13). Hence I confine the examples in what follows to the conditional *if*.

(12) Every student will get an A if he doesn't goof off.

(13) No student will get an A if he doesn't work hard.

On reflection, it is clear, I believe, that some but not all universal indicative conditionals of the type that we have been discussing, where the subject universal binds a place in the *if*-clause, can be treated by the method suggested in Lewis. For an example that clearly pulls them apart: suppose that the university is going to offer generous pensions to some 20% of its 422 professors, hoping to induce early retirement; but has not yet decided, or even drawn up criteria for deciding, which 20% this will be. Concluding as I do, that generous pensions will infallibly induce early retirement, I believe (14). (14), of course, implies (15), the result of absorbing the *if*-clause into the restriction.

(14) Every professor will retire early if offered a generous pension.

(15) Every professor offered a generous pension will retire early.

But the converse is false: there might be many professors (but even one will do) who we can be sure will not retire early, quite independently of any pension they may be offered. Similar examples may be constructed for the negative existential, showing that absorption in this case fails too. It may be that I have taken a poll of the professors, determining the truth of (16):

(16) No professor will retire early if not offered a generous pension.

That will imply (17):

(17) No professor not offered a generous pension will retire early.

But again the converse is false: if Professor X is going to retire early, period, then he is a counterexample to (16). But if he is amongst those offered a generous pension, then he is no counterexample to (17), whose truth or falsehood depends only upon whether any of those in the 80% not offered a generous pension retire early.

Examples can be multiplied; but I want now to take more theoretical steps. We take up the indicative conditional as suggested by Stalnaker (1968):  $q$  *if*  $p$  is true in  $w$  iff  $q$  is true in the closest  $p$ -world  $w' = f(p, w)$  to  $w$ , or else there are no worlds in which  $p$  is true; and moreover if  $p$  is true in  $w$ , then  $f(p, w) = w$ . Writing the Stalnaker conditional as ' $\Rightarrow$ ', we have the validity of (CEM), or Conditional Excluded Middle a point that will play a major role in what follows.<sup>3</sup>

$$(CEM) (\phi \Rightarrow \psi) \vee (\phi \Rightarrow \neg\psi)$$

The Stalnaker conditional  $\phi \Rightarrow \psi$  implies the material conditional  $\phi \rightarrow \psi$ ; for if the latter is false in  $w$  we must have  $\phi \& \neg\psi$  in  $w$ , contradicting  $\phi \Rightarrow \psi$ . The wedge between the material conditional and the Stalnaker conditional is in fact very small: we can have  $\phi \rightarrow \psi$  true in  $w$  whilst  $\phi \Rightarrow \psi$  is false in  $w$

<sup>3</sup> Kai von Fintel and Sabine Iatridou (2002) have independently observed the significance of (CEM) for compositionality.

only where, in  $w$ ,  $\neg\phi$ , and in  $f(\phi,w)=w'\neq w$ ,  $\phi\&\neg\psi$ . Inversely, the Stalnaker conditional and the material conditional are equivalent in the following three cases: (a)  $f(\phi,w)$  is undefined (or, alternatively, is a sink state); (b)  $\phi$  is itself true in  $w$  (in which case  $f(\phi,w)=w$ , by definition); and (c)  $f(\phi,w)=w'\neq w$  is defined,  $\psi$  is true in  $w$ , and (we know in advance that) the truth value of  $\psi$  in  $w'$  is whatever it is in  $w$ . Each case will parley into a case where an *if*-clause can be taken as merely restricting a universal quantifier, as follows. Any sentence *Everything is  $\psi$  if it is  $\phi$* , taken as (18) will be equivalent to (19) whenever, for each object  $a$ , either (a')  $\phi(a)$  is false in every world; or (b')  $\phi(a)$  is true in every world; or (c') neither (a') nor (b'), but, if  $w'=f(\phi(a),w)$  and  $w'\neq w$ , then  $\psi(a)$  holds in  $w$  and  $w'$ . Intuitive examples of the equivalence of (18) and (19) include Frege's (5), and any similar mathematical case.

(18)  $(\forall x) (\phi(x) \Rightarrow \psi(x))$

(19)  $[\forall x: \phi(x)] \psi(x)$

Consider now an example such as (20):

(20) Every book on that shelf is boring if it has a red cover.

One is ready to regard (20) as equivalent to (21):

(21) Every book on that shelf with a red cover is boring.

And it would appear that such is the case because we know in advance that giving a book a red cover does not alter its contents, so does not affect whether it is boring. Let  $b$  be a book on that shelf with a blue cover. In the closest possible world, whatever it is, in which  $b$  has a red cover, it is boring or not, just as it is boring or not as things are. But then it seems that (20) should be false if there are non-boring books on the shelf whose covers are not red, although they might have been! Let  $b$  be such a book. Then  *$b$  has a red cover  $\Rightarrow b$  is boring* is false, whereas  *$b$  has a red cover  $\rightarrow b$  is boring* is true; so  $b$  is a counterexample to (20), but not to (21); whereas what we want is that they should be equivalent.

If the intuition about (20)-(21) is correct, then the conditions (a')-(c') above therefore do not exhaust the cases in which we are prepared to take (18) and (19) as equivalent. A further step is wanted.

I introduce, as a technical device, the notion that  $\phi$  is *counterfactually irrelevant* to  $\psi$  in  $w$  if either  $\phi$  is necessarily false, or  $(\phi \Rightarrow \psi) \leftrightarrow \psi$  holds in  $w$ ; and that  $\phi$  is counterfactually irrelevant to  $\psi$  if irrelevant in  $w$  for every  $w$ . Extending this notion to open sentences, I will say that  $\phi(x)$  is counterfactually irrelevant to  $\psi(x)$  if  $\phi(a)$  is counterfactually irrelevant to  $\psi(a)$  for every  $a$ . I propose the generalization (I):

(I) If  $\phi(x)$  is counterfactually irrelevant to  $\psi(x)$ , then (18) and (19) are equivalent, for every  $w$ .

The converse of (I) is false, as it may just happen that (18) and (19) are equivalent because  $\phi(a)$  and  $\psi(a)$  both fail as things are, whereas both hold in  $w'=f(\phi(a),w)$ . Evidently, mathematical universal conditionals, and in fact all cases falling under (a')-(c') above, will show counterfactual irrelevance. But so will examples like (20) (assuming it quite impossible that the color of a book's cover should have any influence on whether its contents are boring). On the other hand, it is the counterfactual relevance of  $\phi(x)$  to  $\psi(x)$  that points out the difference between (14) and (15), repeated here:

(14) Every professor will retire early if offered a generous pension.

(15) Every professor offered a generous pension will retire early.

To assess the truth value of (14), we must know whether professor Y, who is not offered a generous pension, and does not in fact retire early, would have retired early had she been offered one. Nothing like that is at stake for (15).

I think that we tend to reject (or perhaps simply to find baffling) indicative conditionals with false antecedents, where it is manifest that the antecedent is counterfactually irrelevant to the consequent. If so (although this exceeds what is given simply through the Stalnaker conditional), then the counterexamples to (20), like those for (21), are just the books with red covers that are not boring, so that (I) is vindicated.

The notion of counterfactual relevance perhaps belongs to the pragmatics, not the semantics, of conditionals. As noted by Kai von Fintel and Sabine Iatridou (2002) (though not in the present terms) some cases of counterfactual irrelevance of consequent to antecedent lead to anomaly. So, for instance, *Every coin is silver if it is in Jones's collection* is weird, even if it is known that Jones, as a matter of principle, only collects silver coins; likewise *Every coin is silver if it is in my pocket*. Of course, universal conditionals *Every coin in Jones's collection/in my pocket is silver* are fine. At the same time, the notion of counterfactual irrelevance respects necessary equivalence; i.e., if  $\phi_1$  is counterfactually irrelevant to  $\psi_1$  in  $w$ ,  $(\phi_1 \leftrightarrow \phi_2)$ , and  $(\psi_1 \leftrightarrow \psi_2)$ , then  $\phi_2$  is counterfactually irrelevant to  $\psi_2$  in  $w$ .

I should add a word about the assessment of counterfactual irrelevance in particular cases. Suppose  $X$  is a two-headed coin, and consider the conditional, *If  $X$  were tossed, it would not land tails*. The conditional is true, even if (we know that)  $X$  will never be tossed. But if  $X$  is never going to be tossed, then it is never going to land tails; and it would appear that, by our definition,  *$X$  is tossed* is counterfactually irrelevant to  *$X$  does not land tails*. To restore counterfactual relevance, we should understand the conditional as: *If  $X$  were tossed, then it would land as a result of that toss, and it would not land tails*. There being a toss of  $X$  is then counterfactually relevant to the consequent. (Similar remarks go for the case of (27) and (28), discussed below.)

Having, if I am right, vindicated the independence of the conditional meaning for cases like (14), we are brought back to the problem posed by the inequivalent (16) and (17). (16) in particular cannot, it would appear, be understood as in (22):

(22) (No  $x$ ) ( $x$  is not offered a generous pension  $\Rightarrow x$  will retire early).

But, as not being offered a generous pension is counterfactually relevant to retiring early (for we need to know whether professor  $X$ , who did in fact retire early, would have done so had he not been offered a generous pension), (16) is not equivalent to (17) either.

Thus we are brought, I believe, to an obvious hypothesis. We may decompose *for no  $x$ ,  $A$  as for all  $x$ , not- $A$* , and note, by the general principle (CEM) that characterizes the Stalnaker conditional, that  $\neg(\phi \Rightarrow \psi)$  is equivalent to  $\phi \Rightarrow \neg\psi$ . By this double transformation, (16) is then equivalent to (23):

(23) Every professor will not retire early if not offered a generous pension.

But now in *this* expression, the link between the *if*-clause and the main clause is rightly expressed by ' $\Rightarrow$ '. But that means that (16) can be understood as in (22) after all!

We have, then, the following dilemma: either (i) the intuitive inequivalence of (16) and (17) is an illusion, or (ii) (CEM) is mistaken, in that it makes them equivalent. We generalize to other cases before proceeding to address it.

The trick that we just pulled with *no* can be pulled with any monotone decreasing quantifier. The lexicography of quantifiers, as Frege taught us, is that they map concepts onto truth values (or, in natural languages, as Frege also observed, ordered pairs of concepts into truth values; I will confine the discussion here to the unrestricted case, the extension to restricted quantifiers like *every student* being immediate). Recasting this lexicography in model-theoretic terms, a quantifier  $Q$  on a non-empty domain  $D$  is monotone increasing if for any subsets  $X$  and  $Y$  of  $D$ ,  $Q(Y) = \text{True}$  if  $Q(X) = \text{True}$ , and  $X \subseteq Y$ ; and it is monotone decreasing if  $Q(Y) = \text{True}$  if  $Q(X) = \text{True}$ , and  $Y \subseteq X$ . So for each monotone decreasing  $Q$  there is a unique monotone increasing  $Q'$  such that for all  $X$ :

$$Q'(D-X) = Q(X)$$

Consider in this light the puzzling example (8), repeated here:

(8) Few students will get A's if they work hard.

We may decompose *for few x, A* as *for most x, not-A*, and apply (CEM). According to this transformation, (8) should amount to (24):

(24) Most students will not get A's if they work hard.

The dilemma of (16)-(17) thus presents itself with respect to (8)-(24). I am unable to convince myself whether (24) is indeed equivalent to (8), or just anomalous: see also below.<sup>4</sup>

What we have seen, if the views advanced here are correct, is that in many cases the compositionality of conditionals can be restored, not indeed by making the clauses introduced by *if* or *unless* part of the quantifier restriction, but rather by what I have casually called a kind of decomposition and transformation of the sentences in question. For the monotone decreasing quantifiers such as *no*, suppose that the syntactic structure that is the input to semantics for (25) is as in (26) (I use QR, but this is inessential):

(25) No student will get an A if he goofs off.

(26) [[No student]<sub>i</sub>, [t<sub>i</sub> will get an A [if he, goofs off]]]

Despite initial appearances, the correct compositional result is obtained, but only by exploiting the law (CEM) that characterizes the Stalnaker conditional, a controversial assumption.

We may be able to push the analysis farther. Suppose we adopt the interpretation of the indicative conditional inspired by Lewis (1973) (who, however, had different views about its application), and accept the Limit Assumption, so take  $p \Rightarrow q$  as true in  $w$  iff  $q$  holds at every closest world in which  $p$  is true. Then Conditional Excluded Middle may fail. Define counterfactual irrelevance as above. Then (I) may again be suggested. However, we now admit cases in which *Nothing is B if it is A* need not amount to *Everything is not B if it is A*.

The examples that we have given to this point are not examples that, in the most intuitive sense, support Conditional Excluded Middle. Thus we are not ready to say, in typical settings, *Either Professor X will retire early if offered a generous pension, or Professor X will not retire early if offered a generous pension*, or *Either student Z will not succeed if she goofs off, or student Z will succeed if she goofs off*. But there are cases where, again on the most intuitive level, Conditional Excluded Middle applies. Suppose a bowl on the table containing a large quantity of peanuts, enough to supply everyone at the reception. Each particular person is either allergic to peanuts, or else not. So I can volunteer, for each person  $x$ : either, if  $x$  eats those peanuts,  $x$  will have an allergic reaction to them, or, if  $x$  eats those peanuts,  $x$  will not have an allergic reaction to them. Now, I know that allergy to peanuts is rare, and so am confident in saying (27):

(27) Few people will have an allergic reaction if they partake of those peanuts.

Obviously, this is to be distinguished from (28):

(28) Few people who partake of those peanuts will have an allergic reaction.

Note that, even amongst those people with no allergy who do not partake of the peanuts, and so *a fortiori* do not have an allergic reaction to them, the consequent, more fully unpacked as *they have an allergic reaction as a result of partaking of those peanuts*, is something to which the truth of the antecedent is counterfactually relevant. It is not a conjunction, but rather the single sentence, *There is a partaking of the peanuts by  $x$  which is not followed by an allergic reaction* that is false as things are, but true in the counterfactual situation.) But now the question is whether (27) amounts to (29) or, more tententiously, (30):

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<sup>4</sup> I have used "decomposition" above for expository purposes only: the proof that (8), assuming (CEM), is equivalent to (24) can be given directly.

- (29) Most people will not have an allergic reaction if they partake of those peanuts.  
 (30) There are few people (among those at the reception) such that their partaking of those peanuts is a sufficient condition for their having an allergic reaction to them.

If the answer to this question is affirmative, then we may propose that conditionals *Q* things are *B* if *A*, where *Q* is monotone decreasing, are anomalous if Conditional Excluded Middle fails (for some values of the variable), otherwise equivalent to *Q'* things are not-*B* if *A*, where *Q'* is the monotone increasing quantifier corresponding to *Q*.

What of conditionals with quantifiers that are neither monotone increasing nor monotone decreasing, *exactly three*, for instance, or *between 4 and 17*, or *some odd number of*? It appears to me that the conditional is satisfactorily interpreted as the conditional connective in contexts where (CEM) holds, but is anomalous otherwise. Thus, for instance, it is unclear what *Exactly three students will pass unless they goof off* is supposed to mean, whereas *Exactly three guests will have allergic reactions unless they avoid those peanuts* is pretty clear.

To summarize:

1. The problem of the compositionality of quantified conditionals is genuine: it cannot, save in a few accidental cases, be dismissed by absorbing the antecedent *if*-clause into the quantifier restriction.
2. If we may assume (I), thus putting aside the cases of counterfactual irrelevance, then, on the Stalnaker semantics with (CEM), quantified conditionals with monotone increasing and monotone decreasing quantifiers submit to the same treatment.
3. Waiving (CEM) as a general principle, but retaining the rest of the Stalnaker semantics, it seems that the cases where (CEM) may intuitively be assumed are all compositional, even for quantifiers that are neither monotone increasing nor monotone decreasing.

We may therefore suggest the generalization (II):

- (II) (Assertions of) quantified conditionals whose quantifiers are not monotone increasing presuppose (CEM).

Compositionality (for this restricted class of cases, at least: we have not in this note considered multiple quantification) is then restored. However, (II), and for that matter (I) above, have a stipulative character that invites further inquiry.

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