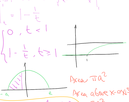
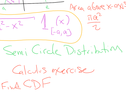
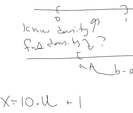
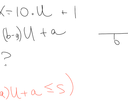
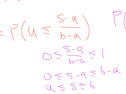
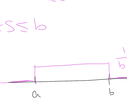


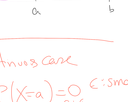


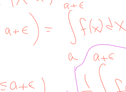


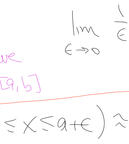


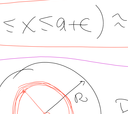


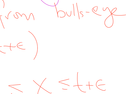


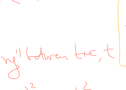


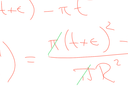


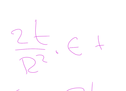


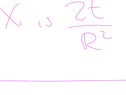


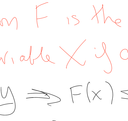


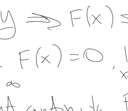


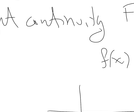


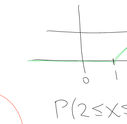


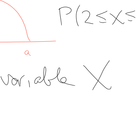












$F_1(t) = \begin{cases} 0, & t < 1 \\ 1 - \frac{1}{t}, & t \geq 1 \end{cases}$

Example:
Consider a

$f(x) = \frac{2}{\pi a^2}\sqrt{a^2 - x^2}\ \mathbb{1}_{[-a,a]}(x)$

Area πa^2
Area above $x = a^2$
$\frac{\pi a^2}{2}$

$x^2 + y^2 = a^2$
$y^2 = a^2 - x^2$
$y = \sqrt{a^2 - x^2}$

Semi Circle Distribution

Calculus exercise
Find CDF

$\int \sqrt{a^2 - x^2}\, dx$

$F(t) = \int_{-\infty}^{t} f(x)\, dx$

CDF $\quad$ density
$F'(t) = f(t)$

know density?
find down to?

$U \sim U[0,1]$

$X = 10 \cdot U + 1$

$X = (b-a)U + a$

density of X?

$P(X \leq s) = P((b-a)U + a \leq s)$

$= P((b-a)U \leq s - a) = P\left(U \leq \frac{s-a}{b-a}\right)$

$= \frac{s-a}{b-a}$

$P(U \leq t) = t \quad 0 \leq t \leq 1$

$0 \leq \frac{s-a}{b-a} \leq 1$

$0 \leq s - a \leq b - a$

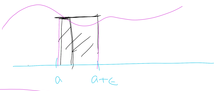
$a \leq s \leq b$

$F_X(s) = \frac{s-a}{b-a}, \quad a \leq s \leq b$

$f_X(s) = F_X'(s) = \frac{1}{b-a}$

$\approx f(a)\epsilon$

Area $= P(a \leq X \leq a + \epsilon)$

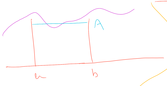


Discrete
$P_X(a) = P(X = a)$

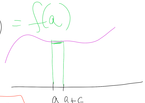
Continuous case
$P(X = a) = 0$ $\quad \epsilon$: small

$P(a \leq X \leq a + \epsilon) = \int_a^{a+\epsilon} f(x)\, dx$

$\frac{P(a \leq X \leq a+\epsilon)}{\epsilon} \approx \boxed{\frac{1}{\epsilon}\int_a^{a+\epsilon} f(x)\, dx \approx f(a)}$



$\lim_{\epsilon \to 0} \frac{1}{\epsilon}\int_a^{a+\epsilon} f(x) = f(a)$



$A = \frac{1}{b-a}\int_a^b f(x)\, dx$

Average value of f over $[a,b]$

Area box $\times$ b
$A \times (b-a) = \int_a^b f(x)\, dx$

$\boxed{P(a \leq X \leq a + \epsilon) \approx \epsilon f(a)}$

$P(X = a) = p_X(a)$

Dartboard
Area πR^2

Dart hits uniformly on the board
X distance from bull-eye

$P(t \leq X \leq t + \epsilon)$

Area where $t \leq X \leq t + \epsilon$

$P(X \leq t) = \frac{\text{Area where } X \leq t}{\text{Total area}}$

$= \frac{\pi t^2}{\pi R^2} = t^2/R^2$

Hit in "ring" between $t + \epsilon$, t

Area $\pi(t+\epsilon)^2 - \pi t^2$

$P(\text{hit "ring"}) = \frac{\pi(t+\epsilon)^2 - \pi t^2}{\pi R^2} = \frac{\cancel{t^2} + 2\epsilon t + \epsilon^2 - \cancel{t^2}}{R^2}$

$= \frac{2t}{R^2} \cdot \epsilon + \frac{\epsilon^2}{R^2} \approx \frac{2t}{R^2} \cdot \epsilon$

Density of X is $\frac{2t}{R^2}$

negligible

The function F is the CDF of a random variable X if and only if

a) $x < y \Rightarrow F(x) \leq F(y)$

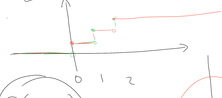
b) $\lim_{x \to -\infty} F(x) = 0, \quad \lim_{x \to \infty} F(x) = 1$

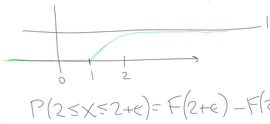
Domain $[0,1]$
non-negative

c) right continuity $\quad F(t) = \lim_{s \downarrow t} F(s)$

Eg $X \sim B(2, p)$

$f(x) = \frac{1}{x^2}\mathbb{1}_{[1,\infty)}(x)$





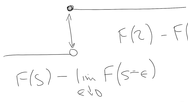
$P(X = 2)$

$P(2 \leq X \leq 2 + \epsilon) = F(2 + \epsilon) - F(2)$

$\lim_{\epsilon \to 0} F(2 + \epsilon) = F(2)$

Given F for random variable X

Find $P(X = a)$



$F(2) - F(2 - \epsilon) =$ height of jump at 2

$F(s) - \lim_{\epsilon \downarrow 0} F(s - \epsilon)$

$F(s) - F(s-) = P(X = s)$