Birthday Problem

How many people do we need to make sure that there is a 50% chance that two share a birthday?

Assume 365 days, no leap years, and all days are equally likely.

$P(\text{at least 2 share a birthday}) = 1 - P(\text{no two share a birthday})$

$P(\text{no two share a birthday}) = \frac{\text{number of ways no two share a birthday}}{\text{total number of ways}}$

$P(\text{no two share a birthday}) = \frac{\text{number of ways no two share a birthday}}{365^n}$

$\frac{\text{number of ways no two share a birthday}}{365^n} = \frac{365 \cdot 364 \cdots (365-n+1)}{365^n}$

$= \frac{365!}{(365-n)!\,365^n}$

$P(\text{at least 2 share a birthday}) = 1 - \frac{365!}{(365-n)!\,365^n} = 0.50$

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$\frac{365 \cdot 364 \cdots (365-n+1)}{365^n} = 0.50$

$\frac{365}{365} \cdot \frac{364}{365} \cdots \frac{(365-n+1)}{365} = 0.50$

$1 \cdot \left(1 - \frac{1}{365}\right) \cdots \left(1 - \frac{n-1}{365}\right) = 0.50$

$\prod_{i=1}^{n-1}\left(1 - \frac{i}{365}\right) = 0.50$

$\prod_{i=1}^{n-1}\left(1 - \frac{i}{365}\right) = \frac{n}{365}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$

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$P(A)