Make sure to understand [illegible]

Ind. Mult. (3,3) [illegible]

$\binom{3}{1}\binom{3}{1}\binom{3}{1}$

[illegible]

[illegible] $\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}$

$\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}$

Case 1111 Mult. for breakdowns

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$P(H)=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{16}$ (Efficient Mult.)

$P(T)=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{16}$

$P(HH)+P(TT)=\frac{1}{16}+\frac{1}{16}$

$P(HH)+P(TT)+P(HT)+P(TH)=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=2$

HHH $P(\text{max run})=\frac{2}{3}$

THH HHT Bonus Game

Ind. Mult. of Problems

n independent runs gets a hit on probability p of success

What is the probability that no runs get this success hit?

(What is the behavior as $n\to\infty$)

A_i = {independent runs this success hit}

$\bigcap_{i=1}^{n} A_i$ = {ind. hits from previous get this success hit}

$P(\bigcap_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \dots$

$+ (-1)^{n-1} P(A_1 \cap A_2 \cap \dots \cap A_n)$

$n=2$ $P(A_1 \cap A_2) = P(A_1) - P(A_1 \cap A_2)$

$P(A_1) = \frac{1}{n}$ $P(A_1 \cap A_2) = \frac{1}{n(n-1)}$ $P(A_1 \cap A_2 \cap A_3) = \frac{1}{n(n-1)(n-2)}$

$P(\bigcap_{i=1}^{n} A_i) = \sum_{i=1}^{n} \frac{1}{n} - \sum_{i<j} \frac{1}{n(n-1)} + \sum_{i<j<k} \frac{1}{n(n-1)(n-2)} - (-1)^{n-1} \frac{1}{n!}$

$= \binom{n}{1}\frac{1}{n} - \binom{n}{2}\frac{1}{n(n-1)} + \binom{n}{3}\frac{1}{n(n-1)(n-2)} - \dots + (-1)^{n-1}\frac{1}{n!}$

$\binom{n}{1}\frac{1}{n} = \frac{n}{n!}$ $\binom{n}{2}\frac{1}{n(n-1)} = \frac{1}{2!}$

$\binom{n}{3}\frac{1}{n(n-1)(n-2)} = \frac{1}{3!}$

$\binom{n}{k}\frac{1}{n(n-1)\dots(n-k+1)} = \frac{1}{k!}$

$= 1 - \frac{1}{2!} + \frac{1}{3!} - \dots$

$e^{x} = 1 + x + \frac{x^2}{2!} + \dots$

$e^{-x} = 1 - x + \frac{x^2}{2!} - \dots$

$e^{-1} = 1 - 1 + \frac{1}{2!} - \dots = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots$

$1 - e^{-1} = 1 - \frac{1}{2!} + \frac{1}{3!} - \dots$

$\lim_{n\to\infty} P(\text{no runs gets this success hit}) = e^{-1} \approx \frac{1}{3}$

$\lim_{n\to\infty} P(\text{no runs gets this success hit}) = 1 - e^{-1}$

$\lim_{n\to\infty} e^{x}$ $f^{(n)}(x) = e^{x}$ Taylor expansion $\left(\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n\right)$

$f(x) = e^{x}$

$f'(x) = e^{x}$

$f''(x) = e^{x}$

$f^{(n)}(0) = 1 \neq 0$

$\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\sum_{n=0}^{\infty} a_n$ $n=0$ to ∞

$\sum_{n=0}^{\infty} a_n - \sum_{n=0}^{\infty} a_n$

$f(1)$ $f(x)$

Random Variables

$(\Omega, \mathcal{F}, P)$ $P: \mathcal{F} \to [0,1]$

Ω sample space $\Omega = \{1,2,3,4,5,6\}^2$

$X: \Omega \to \mathbb{R}$ $\omega \in \Omega$ $\omega = (\omega_1, \omega_2)$ $\omega_i \in \Omega$

$X(\omega) = \omega_1 + \omega_2$ sum of two dice $X(3,4) = 7$

$\Omega = [0, \infty)$ T time until failure

$P(T \geq 1)$ random variables X, Y, T, W

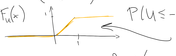
Distribution function: $F_X(x)$ $x \in \mathbb{R}$

$F_X(x) = P(X \leq x)$

Uniform value, on $[0,1]$, say U

$P(U \leq \frac{1}{2}) = \frac{1}{2}$

$P(U \leq x) = x$, $x \in [0,1]$



$P(U \leq -1)$ $P(U \leq 2)$

$P(U \in [\frac{1}{3}, \frac{2}{3}]) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

Distribution of X. $\{P(X \in B) : B \in \mathcal{F}\}$

$P(X \in [2,3]$ or $X \in [5,7])$ $P(X \in (a,b]) = F_X(b) - F_X(a)$

probability mass function (only for discrete variables)

$\Omega = \{x_1, x_2, \dots\}$ finite or countable

p.m.f. $p_X(x) = P(X = x)$

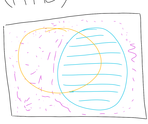
Conditional Probability & Independence

$A \in \mathcal{F}$ some event, $P(A)$

B another event, has occurred.

Is the event A influenced by the occurence of B?

$P(A|B)$ probability of A given B, $P(B) > 0$



A

B

not relevant

$P(A|B) = 0$

$A = B$

$P(A|B) = 1$

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

$\Omega = \{1,2,3,4,5,6\}^2$, uniform Probability

X: result of first roll, Y: result of 2nd roll

$A = \{X \text{ is an even number}\}$, $B = \{X + Y = 4\}$

$P(A) = \frac{1}{2}$ unconditional probability

$P(X+Y=4) = P(\underbrace{\{(1,3), (2,2), (3,1)\}}_{B}) = \frac{3}{36} = \frac{1}{12}$

$A \cap B = \{(2,2)\}$ $B = \{(1,3), (2,2), (3,1)\}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/36}{3/36} = \frac{1}{3} < \frac{1}{2} = P(A)$

known want

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ or $\boxed{P(A \cap B) = P(A|B)P(B)}$

5 red

3 green

urn ↑ Conditional on 1st draw

Draw out 2 chips one at a time, w/ that replacement

Probability get Green, then red

$P(G \cap R_2) = P(R_2|G_1)P(G_1) = \frac{5}{7} \cdot \frac{3}{8}$

$AB = A \cap B$

$P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_n|A_1 \cap \dots \cap A_{n-1}) P(A_{n-1}|A_1 \cap \dots \cap A_{n-2}) \dots P(A_2|A_1) P(A_1)$

check $\frac{P(A_1 \cap \dots \cap A_n)}{P(A_1 \cap \dots \cap A_{n-1})} \cdot \frac{P(A_1 \cap \dots \cap A_{n-1})}{P(A_1 \cap \dots \cap A_{n-2})} \cdot \frac{P(A_1 \cap A_2)}{P(A_1)} \cdot \frac{P(A_1)}{1}$

E E T E A T

fact

$P((E_1 E_2 T_3 E_4 A T_6))$ same! $P((A_1 E_2 T_3 T_4 E_5 E_6))$

getting ordered sequence n elements

unordered

multiple

$P(\{A, E, T, T, E, E\}) = \text{factor} \times P((A, E, T, T, E, E))$

$\frac{n}{\quad}$

$\binom{n}{k_1, k_2, \dots, k_r} = \frac{n!}{k!}$