

14.5 Let (X, \mathcal{M}, μ) be a measure space. Assume $\mu(X) = 1$, and that the measure μ is *non-atomic*, that is, for every $E \in \mathcal{M}$ with $\mu(E) > 0$ there exists $F \subset E$ such that $0 < \mu(F) < \mu(E)$. Prove that

$$\{\mu(E) : E \in \mathcal{M}\} = [0, 1].$$