In p.16, line 4, the second term has the wrong sign and the correct one should be:

$$\Big[\mathbb{E}^{\mathbb{P}}_{\tau_{2i}}[\hat{L}_{\tau_{2i+1}}] - \hat{U}_{\tau_{2i}}\Big]\mathbf{1}_{\{\tau_{2i}<T\}} - \mathbb{E}^{\mathbb{P}}_{\tau_{2i}}\Big[[\hat{L}_{\tau_{2i+1}} - \xi]\mathbf{1}_{\{\tau_{2i}<T=\tau_{2i+1}\}}\Big].$$

Note that, on the set $\{\{\tau_{2i} < T = \tau_{2i+1}\}$, by (3.6) either $\hat{L}_T = L_T \le \xi$, or by the first equation of (3.29),

$$\hat{L}_T - \xi = L_{T-} - Y_T \le Y_{T-} - Y_T = -\Delta K^-_T \le 0.$$

Then in both cases we have $[\hat{L}_{\tau_{2i+1}} - \xi]\mathbf{1}_{\{\tau_{2i}<T=\tau_{2i+1}\}} \le 0$, and consequently the inequality in line 5 cannot hold true.

To fix the gap, we modify (3.6) as:

$$\hat{L}_t := \big[L_t\mathbf{1}_{\{t<T\}} + \xi\mathbf{1}_{\{t=T\}}\big] \vee L_{t-}, \quad \hat{U}_t := \big[U_t\mathbf{1}_{\{t<T\}} + \xi\mathbf{1}_{\{t=T\}}\big] \wedge U_{t-}, \tag{1}$$

Then

$$\mathbb{E}^{\mathbb{P}}_{\tau_{2i}}\Big[[\hat{L}_{\tau_{2i+1}} - \xi]\mathbf{1}_{\{\tau_{2i}<T=\tau_{2i+1}\}}\Big] = \mathbb{E}^{\mathbb{P}}_{\tau_{2i}}\Big[[\hat{L}_T - \xi]\mathbf{1}_{\{\tau_{2i}<T=\tau_{2i+1}\}}\Big] \ge 0,$$

and hence we recover the inequality in line 5.

Equivalently we improve Assumption 3.1 (iii) to $L_T = \xi = U_T$. Note that $Y_T = \xi$ is already given, so this improvement does not change anything in the BSDE (3.1).

We should remark though that the modified $(\hat{L}, \hat{U})$ in (1) or the improved Assumption $L_T = \xi = U_T$ could increase the norm $\|(L,U)\|_{\mathbb{P}}$ in (3.13) slightly. This, however, does not change the spirit of our main results.