- p. 14, Theorem 1.3.4 (ii): the two "X_n" there should be "X^n".
- p. 15, Theorem 1.3.10 (ii), the last condition: $X_n \in \overline{\mathcal{H}}$.
- p. 67, (3.2.7): the last term should be "$\int_0^T |\Delta\sigma_t(X_t^1)|^2 dt$".
- p. 68, line -2: $|\beta_t^n \Delta X_t^n|^2$.
- p. 71, line -3: the last term should be "$+\int_0^T |\Delta\sigma_t^n(X_t)|^2 dt$".
- p. 75, line -12: the term "$\langle X\rangle_t$" should be "$\langle M\rangle_t$".
- p. 78, Problem 3.7.8 (iii): $\eta \in \mathbb{L}^2(\mathcal{F}_0, \mathbb{R}^{d_1})$.
- p. 81, (4.1.4): $\int_0^t \Gamma_s[\alpha_s ds + \cdots$
- p. 82, (4.2.5): the last term should be "$-2\int_t^T Y_s Z_s dB_s$".
- p. 83, line 5: the last term should be "$C\int_t^T [|Y_s|^2 + |Y_s Z_s|]ds$".
- p. 85, line (-5): in the term $2\int_t^T [\Delta Y_s^n[\cdots$, the first "[" shouldn't be there.
- p. 86, line 4: in the first term in the right side, "ΔY_s^n" should be "ΔY_t^n".
- p. 90, line 2-4: all the "I_p^2" there should be "I_p^p".
- p. 96, Problem 4.7.1, (4.7.1):

$$Y_t^i = \xi + \int_t^T \big[\sum_{j=1}^2 [\alpha_s^{ij} Y_s^j + \beta_s^{ij} Z_s^j] + \gamma_s^i\big] ds - \int_t^T Z_s^i dB_s, i = 1, 2.$$

- p. 104, (5.1.5): the signs in the right side should be changed:

$$Y_s^{t,x} = g(X_T^{t,x}) + \int_s^T f(r, X_r^{t,x}, Y_r^{t,x}, Z_r^{t,x}) dr - \int_s^T Z_r^{t,x} dB_r;$$
$$\mathcal{Y}_s^{t,\eta} = g(\mathcal{X}_T^{t,\eta}) + \int_s^T f(r, \mathcal{X}_r^{t,\eta}, \mathcal{Y}_r^{t,\eta}, \mathcal{Z}_r^{t,\eta}) dr - \int_s^T \mathcal{Z}_r^{t,\eta} dB_r.$$

- p. 125, (5.5.13): the second $\overline{Y}$ inside f should be $\overline{Z}$:

$$\bar{Y}_s = \varphi(\tau, X_\tau) + \int_s^\tau f(r, X_r, \bar{Y}_r, \bar{Z}_r) dr - \int_s^\tau \bar{Z}_r dB_r.$$

- p. 125, line -10: the $\overline{X}$ inside σ should be X:

$$d\widehat{Y}_s = \big[\partial_t\varphi + \frac{1}{2}\partial_{xx}\varphi\sigma^2 + \partial_x\varphi b\big](s, X_s)ds + \partial_x\varphi\sigma(s, X_s)dB_s$$

- p. 125, line −3: there should be a negative sign in the right side:

$$d\Gamma_s = -\Gamma_s[\alpha_s ds + \beta_s dB_s], \quad \Gamma_t = 1.$$

- p. 125, line −1: a sign in the right side should be changed:

$$d(\Gamma_s \Delta Y_s) \geq \frac{c}{2}\Gamma_s ds + \Gamma_s[\Delta Z_s - \beta_s \Delta Y_s]dB_s.$$

- p. 150, line −4 and −3: First, there is a typo in the definition of K:

$$K_t := Y_0 - Y_t - \int_0^t [f_s^0 + h_s]ds + \int_0^t Z_s dB_s.$$

  More seriously, the weak convergence of $K^n \to K$ is not sufficient to conclude that K is increasing in t, a.s. A rigorous argument is as follows.

  First, by Mazur's lemma, there exist convex combination

$$\tilde{h}^n := \sum_{i \geq n} \alpha_i^n h^i, \quad \tilde{Z}^n := \sum_{i \geq n} \alpha_i^n Z^n$$

  such that $(\tilde{h}^n, \tilde{Z}^n) \to (h, Z)$ strongly in $\mathbb{L}^2$. Denote

$$\tilde{F}_t^n := \int_0^t [f_s^0 + \tilde{h}_s^n]ds, \;\; \tilde{M}_t^n := \int_0^t \tilde{Z}_s^n dB_s, \;\; F_t := \int_0^t [f_s^0 + h_s]ds, \;\; M_t := \int_0^t Z_s dB_s$$

  Then, possibly along a subsequence, $(\tilde{F}^n - F)_T^* + (\tilde{M}^n - M)_T^* \to 0$, a.s. Denote

$$\tilde{Y}^n := \sum_{i \geq n} \alpha_i^n Y^i, \quad \tilde{K}^n := \sum_{i \geq n} \alpha_i^n K^i. \tag{1}$$

  Then $\tilde{K}^n$ is increasing in t, $\tilde{Y}^n \to Y_t$, $0 \leq t \leq T$, a.s. Note that

$$\tilde{K}_t^n = \tilde{Y}_0^n - \tilde{Y}_t^n - \tilde{F}_t^n + \tilde{M}_t^n. \tag{2}$$

  Then $\tilde{K}_t^n \to K_t$, $0 \leq t \leq T$, a.s. and thus K is also increasing in t, a.s.

- p. 207, line -2: the $\mathbb{P}$ should be $\mathbb{P}_0$.

- p. 229, Step 3 in the proof of Theorem 9.3.2: the current arguments use Problem 9.6.2, which unfortunately is wrong. A new argument is as follows.

  Fix τ and a version of $\{\mathbb{P}^{\tau,\omega} : \omega \in \Omega\}$, $\{\mathbb{P}^{t,\omega} : \omega \in \Omega\}_{t \in [0,T]}$. Define

$$E_t := \{\tau = t\} \in \mathcal{F}_t, \quad \tilde{\mathbb{P}}^{t,\omega} := \begin{cases} \mathbb{P}^{\tau,\omega}, & \omega \in E_t; \\ \mathbb{P}^{t,\omega}, & \omega \notin E_t. \end{cases} \tag{3}$$

Then clearly $\tilde{\mathbb{P}}^{t,\omega}$ satisfies (9.3.8) for all $\omega \in \Omega$. It remains to show that, for any fixed t, $\tilde{\mathbb{P}}^{t,\omega}$ is also an r.c.p.d. of $\mathbb{P}$ and

$$\tilde{\mathbb{P}}^{t,\omega} = \mathbb{P}^{t,\omega}, \quad \mathbb{P}\text{-a.e. } \omega. \tag{4}$$

First, for $\omega \in E_t$, $\mathbb{P}^{\tau,\omega}$ is a probability measure on $\mathcal{F}_T^{\tau(\omega)} = \mathcal{F}_T^t$. Then by (4) we see that $\tilde{\mathbb{P}}^{t,\omega}$ is a probability measure on $\mathcal{F}_T^t$ for all $\omega \in \Omega$, verifying Definition 9.3.1 (i).

Next, for any $\xi \in \mathbb{L}^1(\mathcal{F}_T, \mathbb{P})$, note that

$$\begin{aligned} \mathbb{E}^{\tilde{\mathbb{P}}^{t,\omega}}[\xi^{t,\omega}] &= \mathbb{E}^{\mathbb{P}^{\tau,\omega}}[\xi^{t,\omega}]\mathbf{1}_{E_t}(\omega) + \mathbb{E}^{\mathbb{P}^{t,\omega}}[\xi^{t,\omega}]\mathbf{1}_{E_t^c}(\omega) \\ &= \mathbb{E}^{\mathbb{P}^{\tau,\omega}}[\xi^{\tau,\omega}]\mathbf{1}_{\{\tau=t\}}(\omega) + \mathbb{E}^{\mathbb{P}^{t,\omega}}[\xi^{t,\omega}]\mathbf{1}_{E_t^c}(\omega) \end{aligned}$$

Note that $\omega \mapsto \mathbb{E}^{\mathbb{P}^{\tau,\omega}}[\xi^{\tau,\omega}]$ is $\mathcal{F}_\tau$-measurable, then by the definition of $\mathcal{F}_\tau$ we see that $\omega \mapsto \mathbb{E}^{\mathbb{P}^{\tau,\omega}}[\xi^{\tau,\omega}]\mathbf{1}_{\{\tau=t\}}(\omega)$ is $\mathcal{F}_t$-measurable. Now it is clear that $\omega \mapsto \mathbb{E}^{\tilde{\mathbb{P}}^{t,\omega}}[\xi^{t,\omega}]$ is also $\mathcal{F}_t$-measurable, verifying Definition 9.3.1 (ii).

Moreover, for any $\xi \in \mathbb{L}^1(\mathcal{F}_T, \mathbb{P})$ and $\eta \in \mathbb{L}^\infty(\mathcal{F}_t, \mathbb{P})$, we have

$$\begin{aligned} &\mathbb{E}^{\mathbb{P}}[\eta \mathbb{E}^{\tilde{\mathbb{P}}^{t,\cdot}}[\xi^{t,\cdot}] = \mathbb{E}^{\mathbb{P}}\Big[\eta \mathbb{E}^{\mathbb{P}^{\tau,\cdot}}[\xi^{\tau,\cdot}]\mathbf{1}_{E_t} + \eta \mathbb{E}^{\mathbb{P}^{t,\cdot}}[\xi^{t,\cdot}]\mathbf{1}_{E_t^c}\Big] \\ &= \mathbb{E}^{\mathbb{P}}\Big[\eta \mathbb{E}^{\mathbb{P}}[\xi|\mathcal{F}_\tau]\mathbf{1}_{E_t} + \eta \mathbb{E}^{\mathbb{P}}[\xi|\mathcal{F}_t]\mathbf{1}_{E_t^c}\Big] = \mathbb{E}^{\mathbb{P}}\Big[\mathbb{E}^{\mathbb{P}}[\eta\xi\mathbf{1}_{E_t}|\mathcal{F}_\tau] + \mathbb{E}^{\mathbb{P}}[\eta\xi\mathbf{1}_{E_t^c}|\mathcal{F}_t]\Big] \\ &= \mathbb{E}^{\mathbb{P}}\Big[\eta\xi\mathbf{1}_{E_t} + \eta\xi\mathbf{1}_{E_t^c}\Big] = \mathbb{E}^{\mathbb{P}}[\eta\xi] = \mathbb{E}^{\mathbb{P}}\Big[\eta\mathbb{E}^{\mathbb{P}}[\xi|\mathcal{F}_t]\Big]. \end{aligned}$$

This implies (9.3.7) for $\tilde{\mathbb{P}}^{t,\omega}$.

Finally, if $\mathbb{P}(E_t) = 0$, then (4) holds true. We now consider the case that $\mathbb{P}(E_t) > 0$. Recall (9.3.9) and denote

$$\eta_n^\tau := \mathbb{E}^{\mathbb{P}}[\xi_n|\mathcal{F}_\tau], \quad \eta_n^t := \mathbb{E}^{\mathbb{P}}[\xi_n|\mathcal{F}_t].$$

It is clear that $\mathbb{P}\Big(\{\eta_n^\tau \neq \eta_n^t\} \cap E_t\Big) = 0$ for all n, and thus

$$\mathbb{P}(E_t') = 0, \quad \text{where} \quad E_t' := \cup_n \{\eta_n^\tau \neq \eta_n^t\} \cap E_t.$$

By (9.3.12), we have

$$\mathbb{E}_\tau^\omega[\xi] = \mathbb{E}_t^\omega[\xi], \quad \forall \omega \in \Omega_1 \cap (E_t \backslash E_t'),\ \xi \in C_b^0(\Omega).$$

Now following the arguments in Steps 1 and 2 we see that $\mathbb{P}^{\tau,\omega} = \mathbb{P}^{t,\omega}$ for all $\omega \in \Omega_2 \cap (E_t \backslash E_t')$. Combining with (4) we obtain $\tilde{\mathbb{P}}^{t,\omega} = \mathbb{P}^{t,\omega}$ for all $\omega \in \Omega_2 \backslash E_t'$, which implies (4) immediately.

- p. 241, Problem 9.6.2 is wrong. Indeed, let $(\tilde{\Omega}, \tilde{\mathcal{F}}, \tilde{\mathbb{P}})$ be an arbitrary probability space and $\tilde{\xi}$ a random variable bounded by 1. Denote $\tilde{X}_t := t\tilde{\xi}$, $\mathbb{P} := \tilde{\mathbb{P}} \circ (\tilde{X})^{-1}$, and $\xi := (-1) \vee X_1 \wedge 1$. Then it is clear that $\mathbb{P} \in \mathcal{P}_\infty$ and $\xi \in C_b^0(\Omega)$. For any $t > 0$, we have $\mathbb{E}^{\mathbb{P}}[\xi|\mathcal{F}_t] = \mathbb{E}^{\tilde{\mathbb{P}}}[\tilde{\xi}|\mathcal{F}_t^{\tilde{X}}] = \tilde{\xi}$, but $\mathbb{E}^{\mathbb{P}}[\xi|\mathcal{F}_0] = \mathbb{E}^{\tilde{\mathbb{P}}}[\tilde{\xi}|\mathcal{F}_0^{\tilde{X}}] = \mathbb{E}^{\tilde{\mathbb{P}}}[\tilde{\xi}]$. Then $\lim_{t\downarrow 0} \mathbb{E}^{\mathbb{P}}[\xi|\mathcal{F}_t] \neq \mathbb{E}^{\mathbb{P}}[\xi|\mathcal{F}_0]$ when $\tilde{\xi}$ is not a constant.