Math 225 Final Exam, May 3, 2017

Each of the ten problems is worth 10 points. To get credit, work must be shown. No calculator or book allowed, but you may bring one page of notes (both sides of one piece of paper allowed) in your own handwriting.

1. Show that A is nondefective and find e^{At}.

$$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

2. Find an orthogonal matrix S such that $S^T AS = diag(\lambda_1, \lambda_2)$ where

$$A = \begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix}$$

3. Find the Jordan canonical form J for the matrix A, and determine an invertible matrix S such that $S^{-1}AS = J$.

$$A = \begin{bmatrix} 4 & -4 & 5 \\ -1 & 4 & 2 \\ -1 & 2 & 4 \end{bmatrix}$$

Hint: you may assume that the characteristic polynomial of A is

$$-(\lambda - 2)(\lambda - 5)^2.$$

4. Solve the differential equation

$$\frac{dy}{dx} = 2xy.$$

5. Solve the differential equation

$$y' + 2xy = 2x^3.$$

6. Use the concept of the inverse of a matrix to find the solution to the given linear system.

$$\begin{aligned} x_1 + 3x_2 &= 1 \\ 2x_1 + 5x_2 &= 3 \end{aligned}$$

7. Let

$$A = \begin{bmatrix} 1 & -2x & 2x^2 \\ 2x & 1-2x^2 & -2x \\ 2x^2 & 2x & 1 \end{bmatrix}$$

(a) Show that $det(A) = (1+2x^2)^3$
(b) Use the adjoint method to find A^{-1}.

8. Let A be a real matrix. Prove that if the rowspace of A is equal to the nullspace of A, then A must have an even number of columns.

9. Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by the given set of vectors:

$$\{(1, 1, -1, 0), (-1, 0, 1, 1), (2, -1, 2, 1)\}$$

10. Consider the linear transformation $T : \mathbb{R}^3 \mapsto \mathbb{R}$ defined by

$$T(v) = < u, v >$$

where u is a fixed nonzero vector in $\mathbb{R}^3$.

(a) Find $Ker(T)$ and $dim[Ker(T)]$ and interpret this geometrically.

(b) Find $Rng(T)$ and $dim[Rng(T)]$.