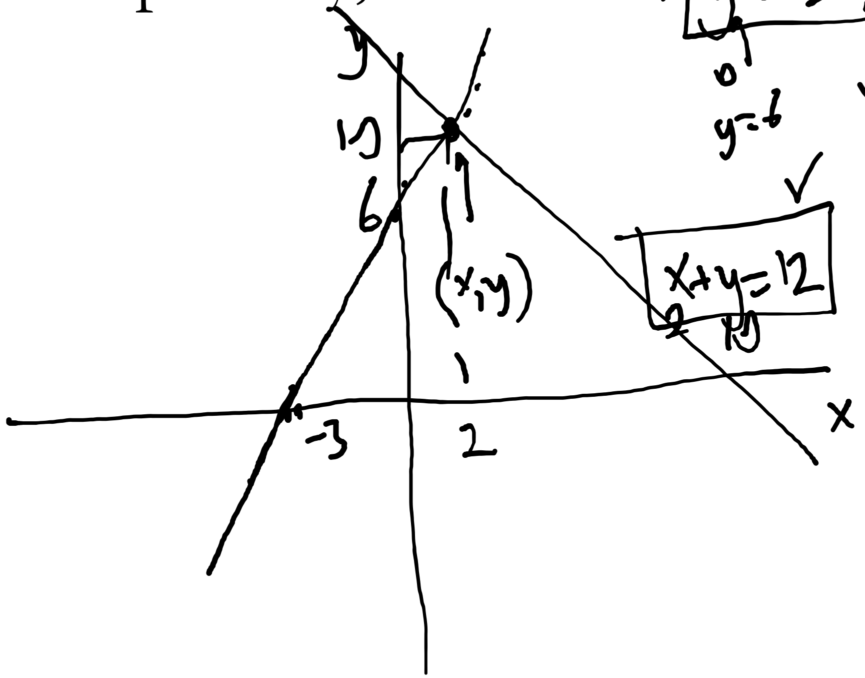
(6) Systems of Linear Equations

- We might need to determine the <u>values</u> of 2 unknowns: **x** and **y.**
- As <u>one</u> linear equation represents a line in the Cartesian plane, we need **two linear equations** generically, to pin down a *unique* **x** and a *unique* **y**.
- Graphically;



$\boxed{y = 2x + 6}$

$y = 6$

$0 = 2x + 6$

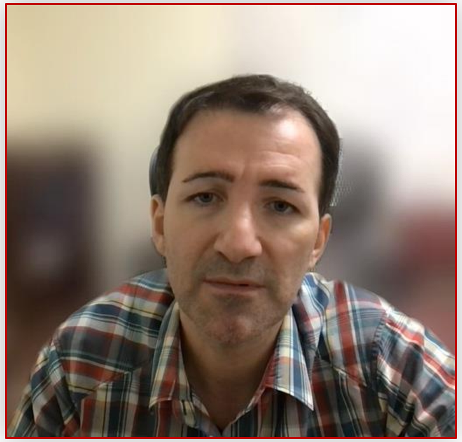
$-3 = x$

$\boxed{x + y = 12}$

(x, y)

$x = 2$

$y = 10$



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(6) Equating to the Same Variable

- One method is to write each of the two linear equations in the format $\mathbf{y = ax + b}$ and equate the *right hand sides* of the two equations.
- Consider the *Demand* and *Supply* equations

$$\mathbf{Q = -4p + 2100} \text{ and } \mathbf{Q = 2p + 300}$$

- **What is the equilibrium Quantity and price ?**

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$$\mathbf{2100 - 300 = 2p + 4p \rightarrow 1800 = 6p}$$

$$\boxed{\mathbf{p = 300}} \rightarrow \boxed{\mathbf{Q = 2 \cdot 300 + 300 = 900}}$$

$(p, Q) = (300, 900)$

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(6) Substitution

- Alternatively, we can ***substitute*** what we know about one variable from one equation into the other equation. For example, *Demand* and *Supply* equations

$$\mathbf{Q = 1700 - 400p} \text{ and } \mathbf{p = 0.02Q + 2}$$

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$$Q = 1700 - 400p \text{ and } \boxed{p = 0.02\overset{100}{Q} + 2}$$

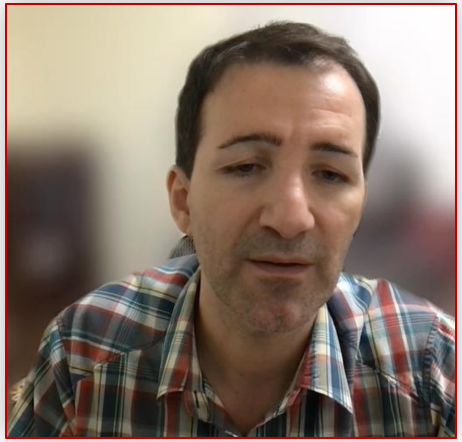
$$\boxed{Q = 1700 - 400(0.02Q + 2)}$$

- Now we have a single variable linear equation to solve:

$$Q = 1700 - 8Q - 800 \quad \rightarrow \quad 9Q = 900$$

$$\rightarrow \boxed{Q = 100} \quad \rightarrow \quad \boxed{p = 0.02 \cdot 100 + 2 = 4}$$

$$(Q, p) = (100, 4)$$



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(6) Row Operations

- In general, we can multiply each of the two equations by some numbers and some them up to <u>get rid of one of the variables</u>, reducing the system again down to a <u>single</u> linear equation with a <u>single</u> unknown. For example:

$$\mathbf{2x + 3y = 4} \quad \text{and} \quad \mathbf{3x - 5y = -13}$$

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(6) Row Operations

- In general, we can multiply each of the two equations by some numbers and some them up to get rid of one of the variables, reducing the system again down to a single linear equation with a single unknown. For example:

$$2x + 3\overset{2}{y} = 4 \quad \text{and} \quad 3x - 5y = -13$$

$$3 \times (2x + 3y = 4)$$

$$+ \quad -2 \times (3x - 5y = -13)$$

$$\cancel{6x} + 9y - \cancel{6x} + 10y = 3 \times 4 + (-2) \times (-13) = 38$$

$$19y = 38 \rightarrow \boxed{y = 2} \rightarrow 2x + 3(2) = 4 \rightarrow \boxed{x = -1}$$

$(-1, 2)$

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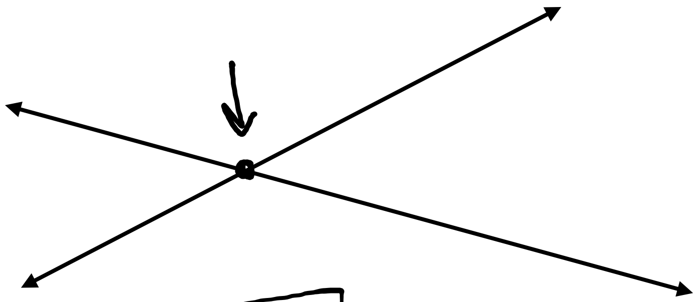
(6) Unique solution? Many solutions? No solution?

$\boxed{y = ax + b}$ and $\boxed{y = a'x + b'}$

Unique Solution | **Many (infinite) Solutions** | **No Solution**



$\boxed{a \neq a'}$

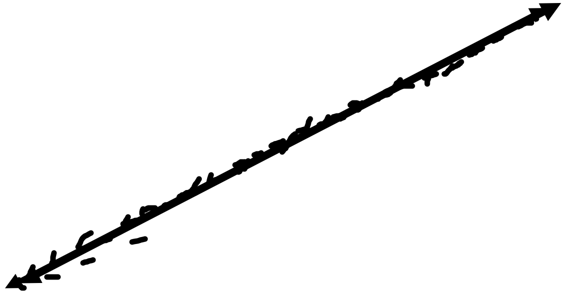
$ax + b = y = a'x + b'$

$ax - a'x = b' - b$

$(a - a')x = b' - b$

$x = \dfrac{b' - b}{a - a'}$

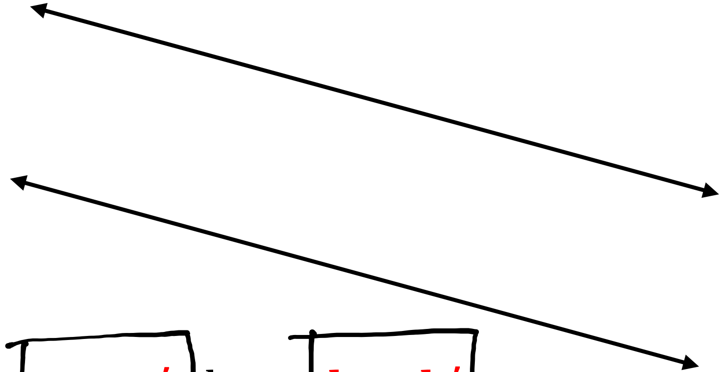
$\neq 0$



$\boxed{a = a'}$ and $\boxed{b = b'}$

$y = ax + b$

$\begin{array}{cc} \shortparallel & \shortparallel \\ a' & b' \end{array}$



$\boxed{a = a'}$ but $\boxed{b \neq b'}$

$ax + b = y = a'x + b'$

$ax - a'x = b' - b$

$0 = (a - a')x = b' - b$

0 $\neq 0$

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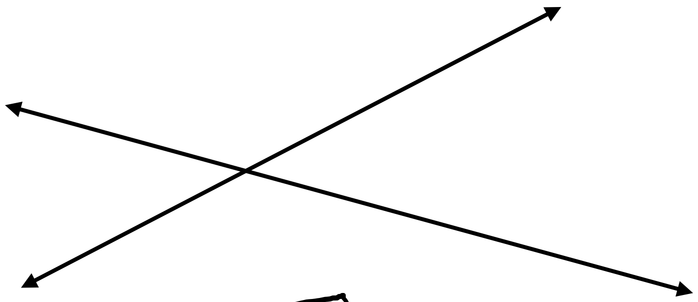

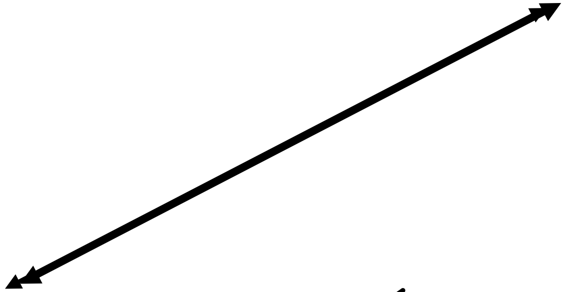

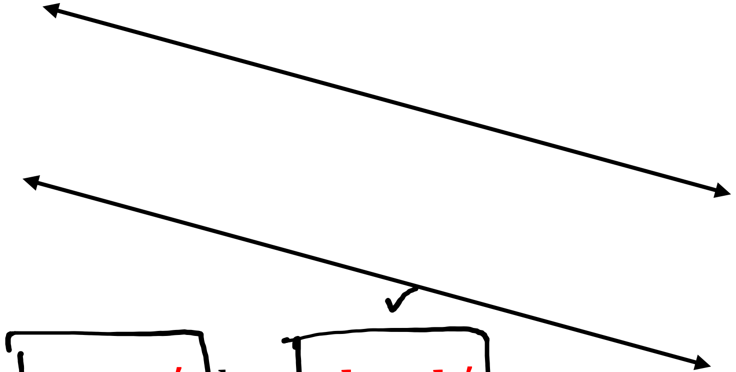

$\mathbf{a \neq a'}$ | $\mathbf{a = a'}$ and $\mathbf{b = b'}$ | $\mathbf{a = a'}$ but $\mathbf{b \neq b'}$

If given the "format" $\mathbf{ax + by + c = 0}$ and $\mathbf{a'x + b'y + c' = 0}$ instead;

$\frac{a}{a'} \neq \frac{b}{b'}$ | $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$ | $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$

$by = -ax - c$

$y = -\frac{a}{b}x - \frac{c}{b}$

$\frac{a}{b} \neq \frac{a'}{b'}$

$\frac{a}{a'} \neq \frac{b}{b'}$

$\frac{c}{b} \neq \frac{c'}{b'}$

$\frac{c}{c'} \neq \frac{b}{b'}$

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(6) What if there are more than two Variables or two Equations?

- Any of the previous methods might be used iteratively, to reduce the number of equations and the number of unknowns, simultaneously. Consider the system;

$$\mathbf{4x - 2y + 3z = 7} \quad \text{and} \quad \mathbf{3y - x + 2z = 11} \quad \text{and} \quad \mathbf{3x - z + 5y = 28}$$

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(6) What if there are more than two Variables or two Equations?

- Any of the previous methods might be used <u>iteratively</u>, to reduce the number of equations and the number of unknowns, simultaneously. Consider the system;

$$\underset{\boxed{1}}{4x - 2y + 3z = 7} \quad \text{and} \quad \underset{\boxed{2}}{3y - x + 2z = 11} \quad \text{and} \quad \underset{\boxed{3}}{3x - z + 5y = 28}$$

$$\boxed{1} + 4\cdot\boxed{2} \rightarrow (4x - 2y + 3z) + 4\cdot(3y - x + 2z) = (7) + 4\cdot(11)$$

$$\cancel{4x} - 2y + 3z + 12y - \cancel{4x} + 8z = \boxed{10y + 11z = 51}$$

$$\boxed{3} + 3\cdot\boxed{2} \rightarrow (3x - z + 5y) + 3\cdot(3y - x + 2z) = (28) + 3\cdot(11)$$

$$\cancel{3x} - z + 5y + 9y - \cancel{3x} + 6z = \boxed{14y + 5z = 61}$$

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(6) What if there are more than two Variables or two Equations?

$$4x - 2y + 3z = 7 \quad \text{and} \quad 3y - x + 2z = 11 \quad \text{and} \quad 3x - z + 5y = 28$$

$$14 \; \boxed{10y + 11z = 51} \quad \text{and} \quad -10 \; \boxed{14y + 5z = 61}$$

$$14 \cdot (10y + 11z) - 10 \cdot (14y + 5z) = 14 \cdot 51 - 10 \cdot 61)$$

$$\cancel{140y} + 154z - \cancel{140y} - 50z = 714 - 610 \rightarrow 104z = 104$$

$\rightarrow \boxed{z = 1}$ plugging back in the equations, $10y + 11 \cdot 1 = 51$ -11

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$\rightarrow \boxed{y = 4}$ and then in one of the original equations,

$$4x - 2 \cdot 4 + 3 \cdot 1 = 7 \rightarrow \boxed{x = 3}$$

$4x - 8 + 3 = 7$

$4x = 7 + 8 - 3$

$= 12$

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