(5) Functions

- A **function** relates a variable **x** (the ***independent variable***) to another variable **y** (the ***dependent variable***); $\boxed{\mathbf{f(x) =}}$ **y**, means **f** maps value **x** to value **y**.

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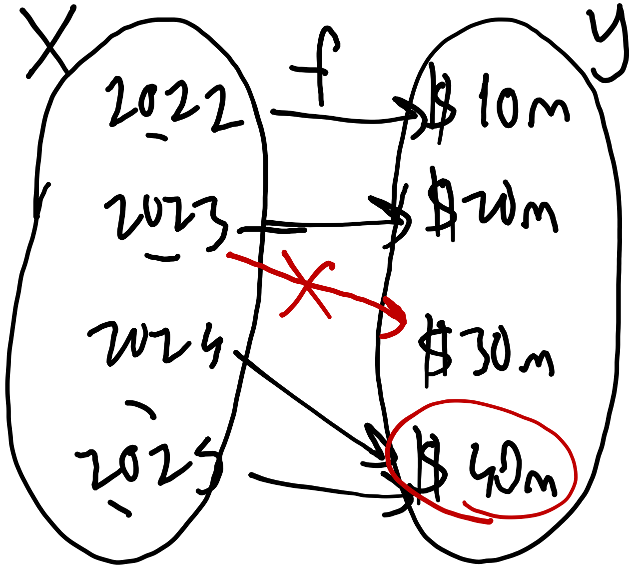
(5) Functions

- A **function** relates a variable **x** (the ***independent variable***) to another variable **y** (the ***dependent variable***); **f(x) = y**, means **f** maps value **x** to value **y**.
- For example, let **f** map the net profit of a firm over the recent years. Here, **x** is the year and **y = f(x)** is the net profit of the firm in that year.
- The possible values **x** can take is called the ***domain*** and the possible values **y** can take is called the ***range*** of the function **f.**



$f(2022) = \$10m$

$f(2025) = \$40m$

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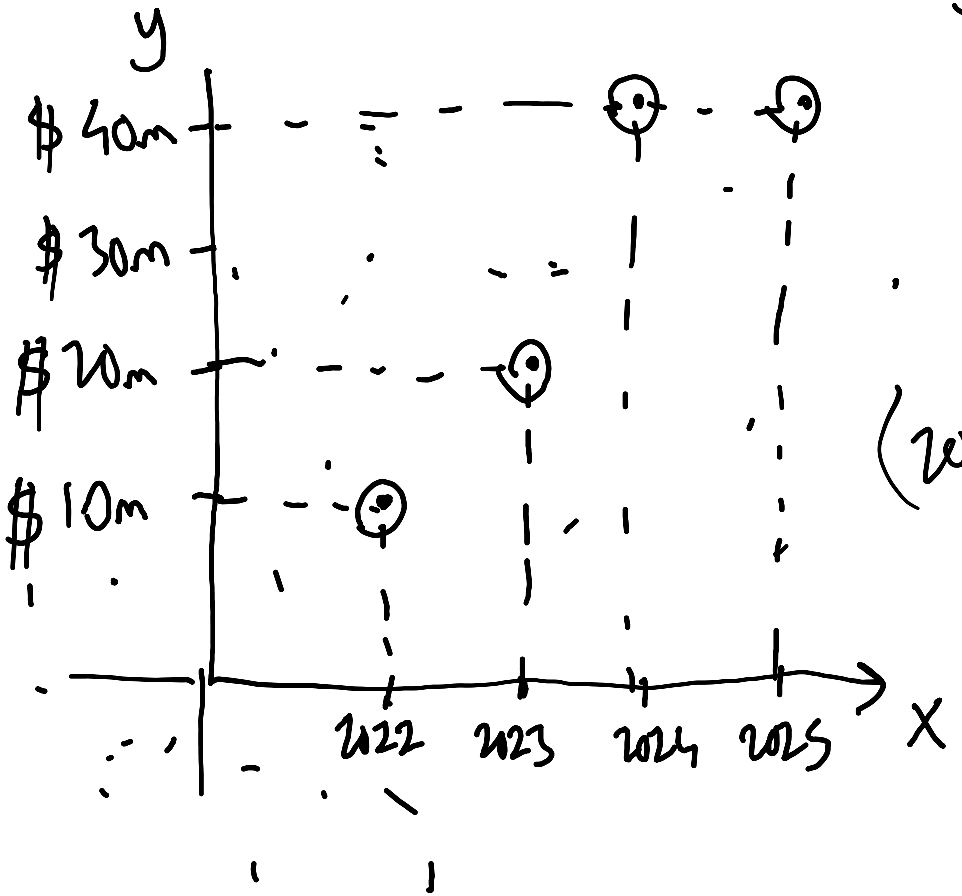
(5) Functions

- A ***graph of the function*** **f** is the set of all points **(x,y)** on the Cartesian plane that satisfy $\mathbf{y} = \mathbf{f}(\mathbf{x})$



$(2022, \$10m) = (x, y)$

$f(x) = y$

$f(2022) = \$10m$

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(5) Linear Functions

- f is a ***linear function*** if it is of the form $\mathbf{f(x)} = \boxed{\mathbf{y} = \mathbf{ax + b}}$ where **a** , **b** are given real numbers.

$y = 3x - 7$

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(5) Linear Functions

- f is a ***linear function*** if it is of the form $\mathbf{f(x) = y = ax + b}$ where **a** , **b** are given real numbers.

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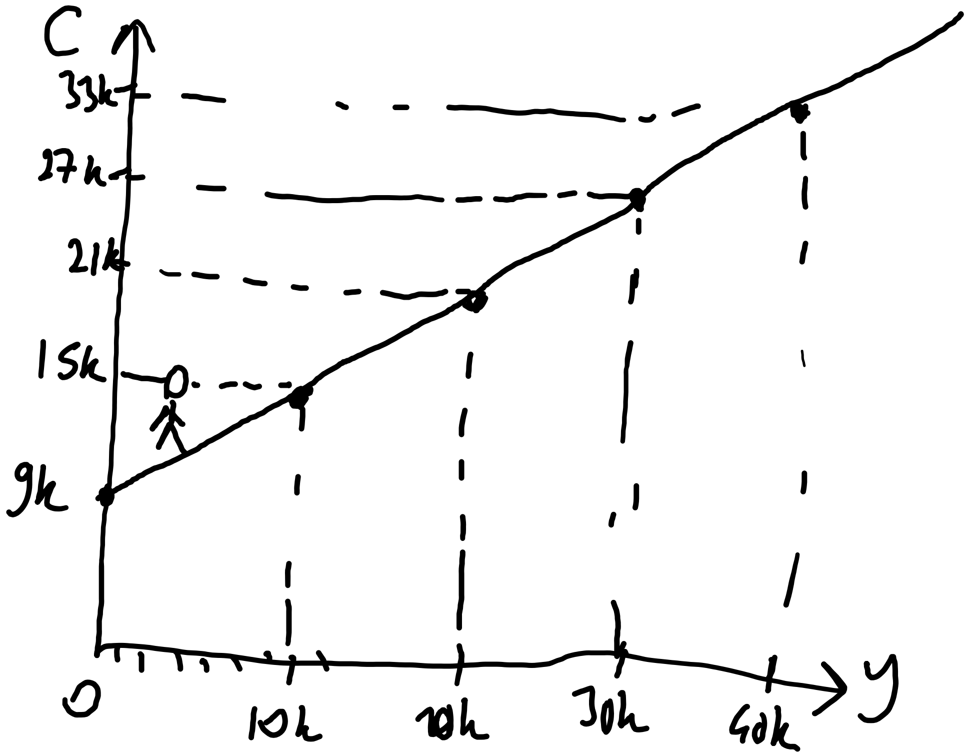
- <u>**Example:**</u> $\mathbf{f(Y) = \boxed{0.6Y + 9{,}000} = C}$, where **Y** is the average income and **C** is the average consumption of a household, both expressed in dollars.



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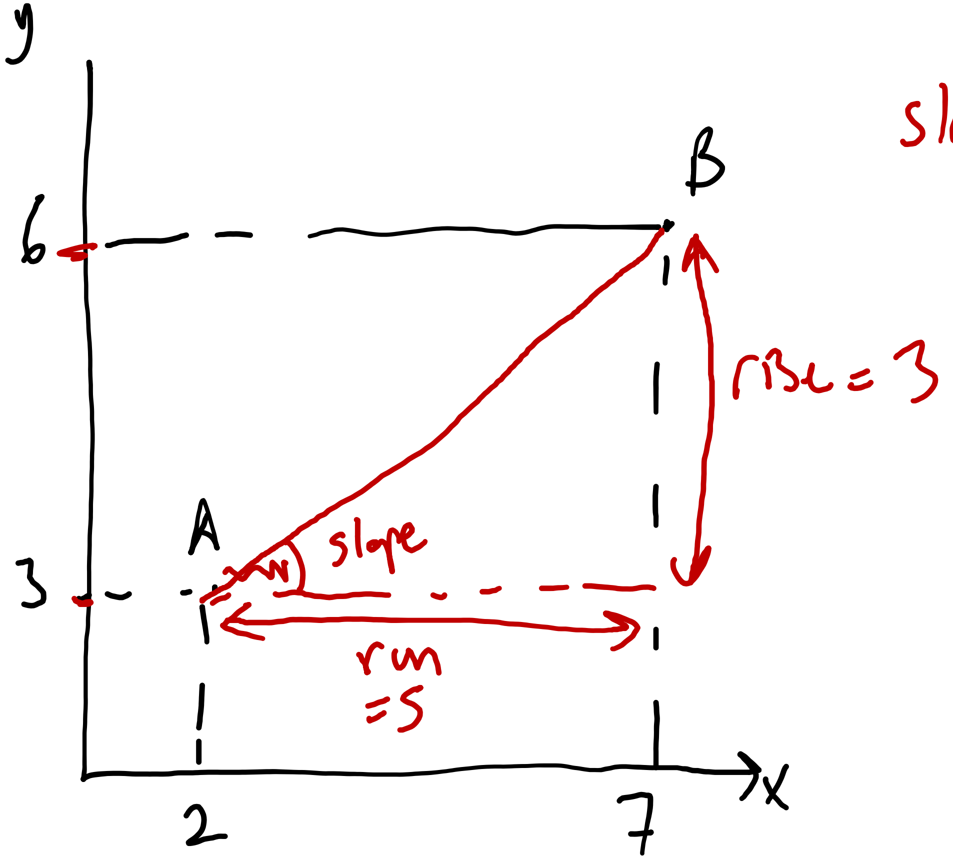
(5) Slope

- The *slope* ***between two points*** **A** $= (\mathbf{x_1}, \mathbf{y_1})$ and **B** $= (\mathbf{x_2}, \mathbf{y_2})$ is

$$\mathbf{Slope_{AB}} = \frac{\textbf{vertical rise}}{\textbf{horizontal run}} = \frac{\mathbf{y_2} - \mathbf{y_1}}{\mathbf{x_2} - \mathbf{x_1}}$$



$$slope_{AB} = \frac{rise}{run} = \frac{6-3}{7-2} = \frac{3}{5} = 0.6$$

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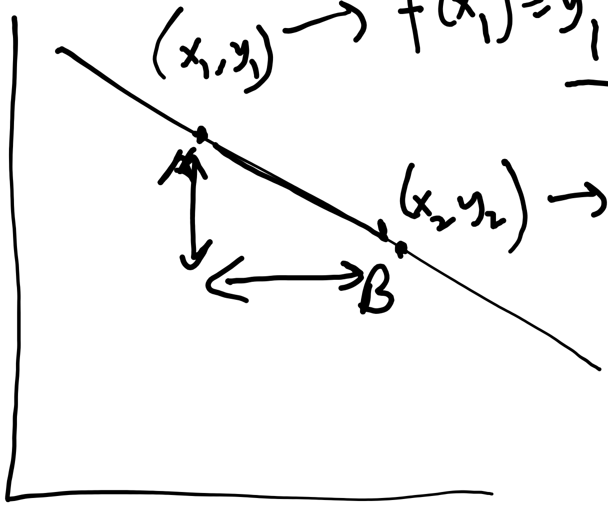
(5) Slope on a Linear Function Graph

- If **A, B** lie on the linear function's graph $\mathbf{f(x)} = \boxed{\mathbf{y = ax + b}}$, substituting

$$\mathbf{Slope_{AB}} = \frac{\mathbf{y_2 - y_1}}{\mathbf{x_2 - x_1}} = \frac{\mathbf{(ax_2 + \cancel{b}) - (ax_1 + \cancel{b})}}{\mathbf{x_2 - x_1}} = \frac{\mathbf{a\cancel{(x_2 - x_1)}}}{\mathbf{\cancel{x_2 - x_1}}} = \mathbf{a}$$

- Indeed, the slope between any two points on the linear function is equal to **a** !



$(x_1, y_1) \rightarrow f(x_1) = y_1 = ax_1 + b$

$(x_2, y_2) \rightarrow f(x_2) = y_2 = ax_2 + b$

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(5) Linear functions: Example

- **Demand function:** $f(\mathbf{p}) = \mathbf{Q} = \mathbf{120 - 2p}$ where $\mathbf{p}$ is the *market price* and $\mathbf{Q}$ is the *demanded quantity*.
- For example, if $\mathbf{p} = \mathbf{20}$ dollars, then $\mathbf{Q} = \mathbf{120 - 2p} = \mathbf{120 - 2 \cdot 20} = \mathbf{80}$

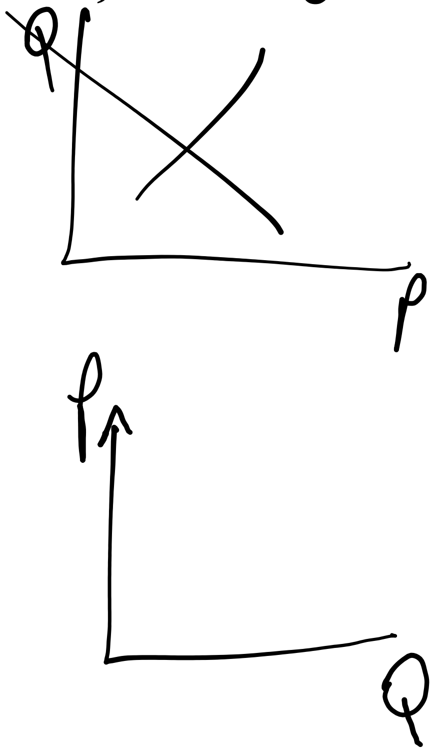
$$Q = 120 - 2p$$

$$2p = 120 - Q$$

$$\boxed{p = 60 - \frac{Q}{2}}$$

↓

Inverse demand



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(5) Linear functions

- $\boxed{\mathbf{y = ax + b}}$ and $\boxed{\mathbf{ax + by + c = 0}}$ are common "formats" for linear functions;
- **Example:** $\mathbf{y = 3x + 2}$ can also be written as $\boxed{\mathbf{3x - y + 2 = 0}}$

  $\mathbf{2x + 5y + 8 = 0}$ can be written as $\boxed{\mathbf{5y = -2x - 8}} \rightarrow \boxed{\mathbf{y = -\frac{2}{5}x - \frac{8}{5}}}$

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(5) Linear functions

- In $\boxed{ax + by + c = 0}$ format, the slope of the line is,
- $by = -ax - c \quad \rightarrow \quad y = \boxed{-\frac{a}{b}}x - \frac{c}{b}$ hence its slope is $-\frac{a}{b}$

b b

$$\frac{x}{K} + \frac{y}{L} = M$$

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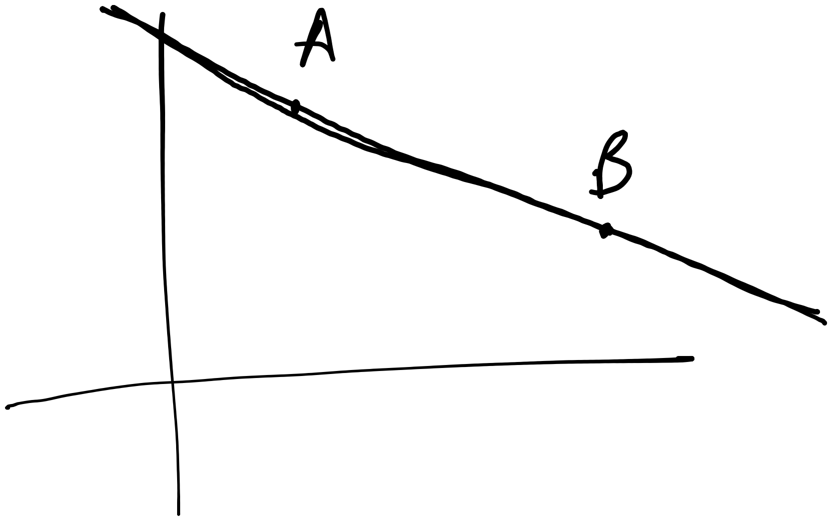
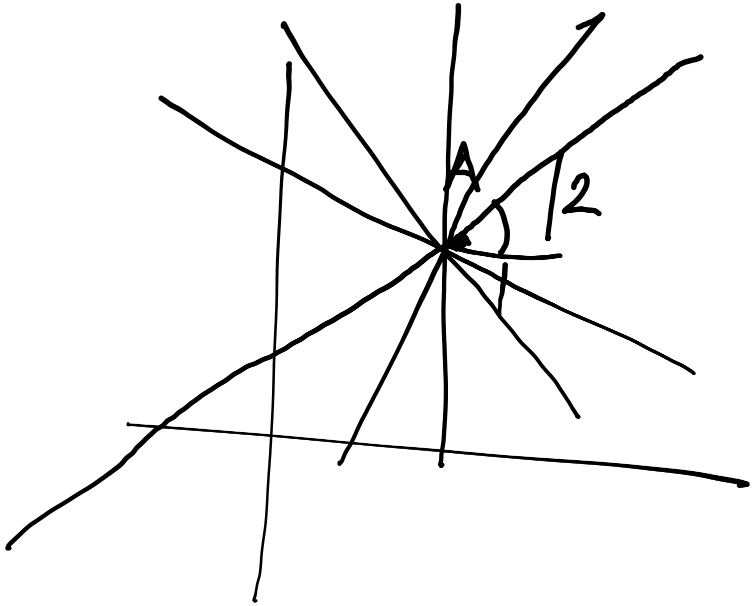
(5) Fitting a Linear function

- There is a unique line that passes through two points.
- There is a unique line passing through a point with a given slope.





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(5) Finding the Linear function passing through two given points

- If the line passes through two given points $A(x_1, y_1)$ and $B(x_2, y_2)$
- $\boxed{a}$ = slope = $\frac{y_2 - y_1}{x_2 - x_1}$, hence $\mathbf{y = ax + b = \frac{y_2 - y_1}{x_2 - x_1} x + b}$

$$y = \boxed{a}x + \boxed{b}$$

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(5) Finding the Linear function passing through two given points

- If the line passes through two given points $\mathbf{A(x_1, y_1)}$ and $\mathbf{B(x_2, y_2)}$
- $\boxed{\mathbf{a}} = \mathbf{slope} = \frac{y_2 - y_1}{x_2 - x_1}$, hence $\mathbf{y = ax + b} = \frac{y_2 - y_1}{x_2 - x_1}\mathbf{x + b}$
- and plugging in the line equation one of the points, say $\mathbf{A}$
- $\boxed{y_1 = \boxed{a}x_1 + b} = \frac{y_2 - y_1}{x_2 - x_1}x_1 + \mathbf{b}$ hence you can also solve for **b;**
- $\boxed{\mathbf{b}} = y_1 - \frac{y_2 - y_1}{x_2 - x_1}x_1$

$A(2,3)$ $B(0,-5)$

$a = \text{slope}_{AB} = \frac{-5-3}{0-2} = \frac{-8}{-2} = 4$

$y = ax + b$

$-5 = 4 \cdot 0 + b$

$-5 = b$

$\boxed{y = 4x - 5}$

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(5) Finding the Linear function given its slope and a point on it

- If the line has a given slope **a** and passes through $\mathbf{A(x_1, y_1)}$
- $\mathbf{y = a\,x + b}$ as the slope is the coefficient in front of **x**.
- As point **A** is on the line, $\mathbf{y_1 = a x_1 + b}$ hence you can also solve for **b**;

$\boxed{\mathbf{b}} = \mathbf{y_1 - a x_1}$

line that has slope -3 and passes through $(1, -4)$

$y = -3x + b$

$-4 = -3(1) + b$

$-4 = -3 + b$

$-1 = b$

$\boxed{y = -3x - 1}$

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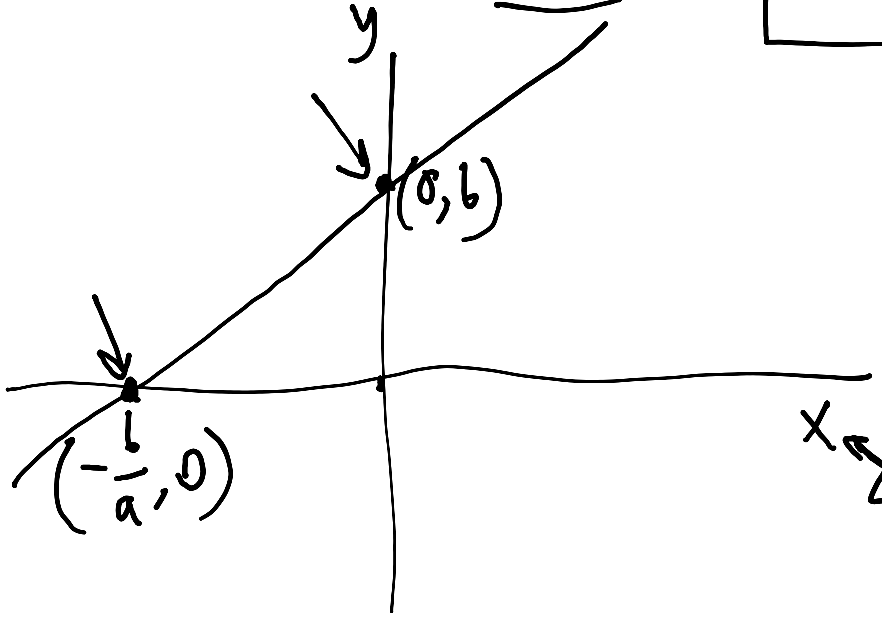
(5) Linear functions: Intercepts with axes

- A line (linear function) $\boxed{\mathbf{y = ax + b}}$ cuts the $\mathbf{x}$ and $\mathbf{y}$ axes respectively at:
- x−intercept: $\mathbf{y = \boxed{0 = ax + b}} \rightarrow \mathbf{x = -\frac{b}{a}}$ hence at the point $\left(-\frac{\mathbf{b}}{\mathbf{a}}, \mathbf{0}\right)$
- y-intercept: $\mathbf{x = 0} \rightarrow \boxed{\mathbf{y = a0 + b}} \mathbf{= b}$ hence at the point $(\mathbf{0}, \mathbf{b})$.



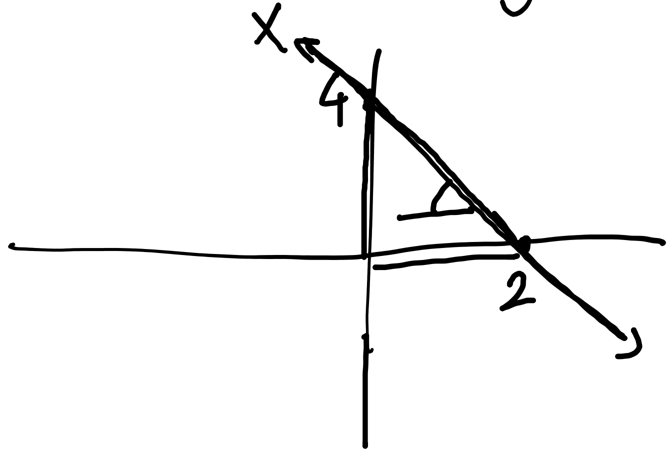
$y = -2x + 4$

$x=0 \longrightarrow y = -2 \cdot 0 + 4 = 4 \quad (0,4) \rightarrow$ y-int

$y=0 \longrightarrow 0 = -2x + 4 \quad 2x = 4$

$x = 2$

$(2,0) \longrightarrow$ x-int



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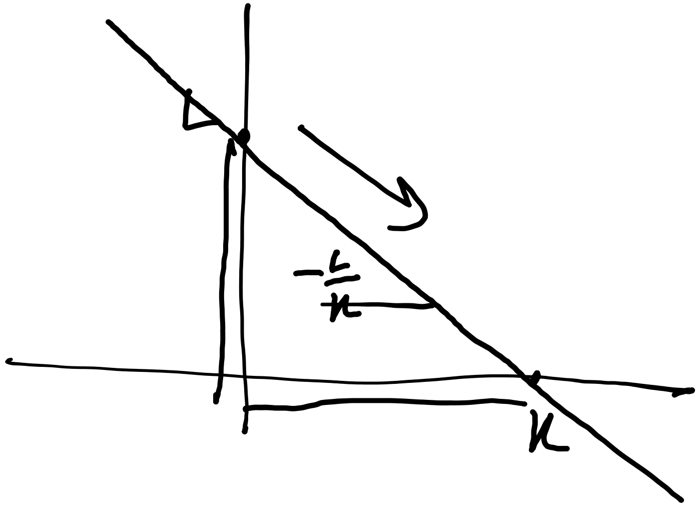
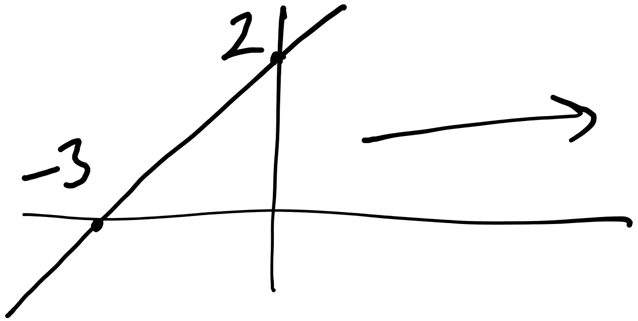
(5) Linear functions: Intercepts with axes

- The formula of a line with intercepts **(K , 0)** and **(0 , L)** is $\boxed{\dfrac{\mathbf{x}}{\mathbf{K}} + \dfrac{\mathbf{y}}{\mathbf{L}} = \mathbf{1}}$



$$\frac{y}{L} = 1 - \frac{x}{K}$$

$$y = L - \frac{L}{K}x$$



$$\frac{x}{-3} + \frac{y}{2} = 1$$

$$\frac{y}{2} = \frac{x}{3} + 1$$

$$3y = 2x + 6$$

$$y = \frac{2}{3}x + 2$$

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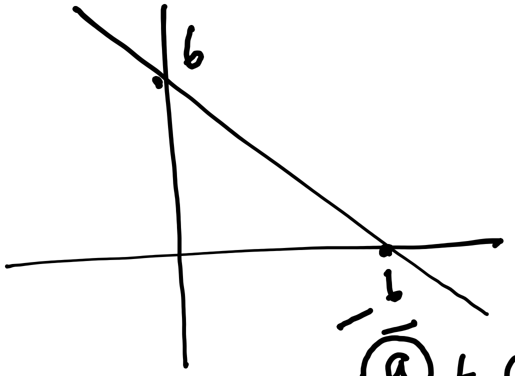
(5) Horizontal or Vertical Linear functions

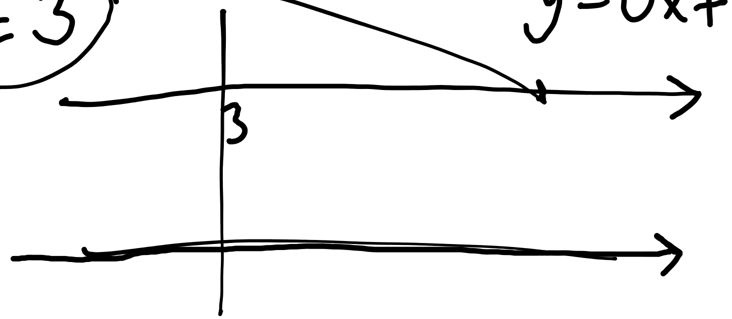
Also, don't forget these extreme cases:

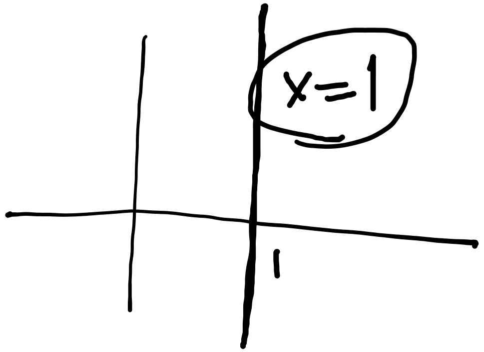
- A **horizontal line** (*slope=0*) has the form **y = c**
- A **vertical line** (*slope=±∞*) has the form **x = c**, for some value **c.**



$a \neq 0$

$y = 3$ $\quad$ $y = 0x + 3$





$y = ax + b$

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(5) Linear functions: Example

- **Budget Equation:** Assume each apple (the good **x**) costs $\$\mathbf{p_x}$ and each orange (the good **y**) costs $\$\mathbf{p_y}$, and you have $\$\mathbf{I}$ budget in total for fruits. If you end up buying **x** apples and **y** oranges; $\mathbf{p_x x + p_y y \le I}$ budget inequality should hold.

(x, y) $\quad p_x x + p_y y \le I$

$p_y y = I - p_x x \rightarrow y = -\frac{p_x}{p_y}x + \frac{I}{p_y}$

$\boxed{p_x x + p_y y = I}$

$y > ax+b$

$y = ax+b$

$y < ax+b$

$\boxed{p_x x + p_y y \le I}$

20

$-\frac{2}{3}$

30

$p_x = 2 \quad p_y = 3 \quad I = 60$

$2x + 3y = 60$

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