(4) Algebra - Simplification

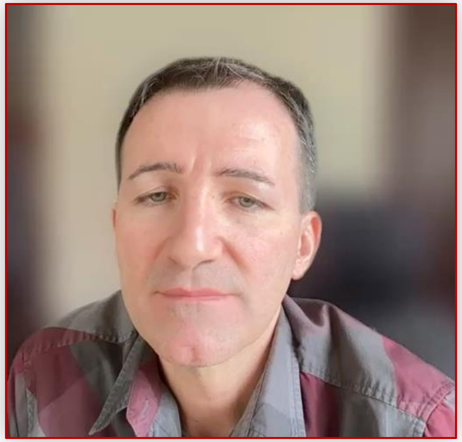
- When there are numbers and variables in an expression, you can add/subtract **similar** terms; and multiply and divide as usual:

$$8q + 10q - 3(5q + 2) = 3q - 6$$

$-15q \quad -6$

- When there are many variables, or different powers of a variable, make sure to treat them separately:

$8+10-15$



Dr. Yilmaz Kocer

(4) Algebra - Simplification

- **Example:** $(2-x)^2 + (y-4)^2 + x(y-1) - 3xy$

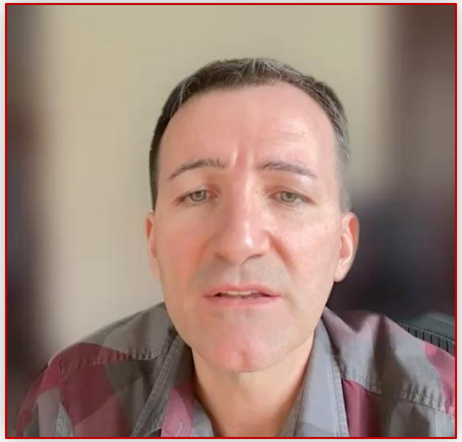
$$= (2-x)(2-x) + (y-4)(y-4) + x(y-1) - 3xy$$

$$= 4 - 2\cdot x - x\cdot 2 - x\cdot(-x) + y\cdot y - 4\cdot y - y\cdot 4 - 4\cdot(-4)$$

$$+ x\cdot y - x\cdot 1 - 3xy$$

$$= 4 - 4x + x^2 + y^2 - 8y + 16 + xy - x - 3xy$$

$$= x^2 + y^2 - 2xy - 5x - 8y + 20$$



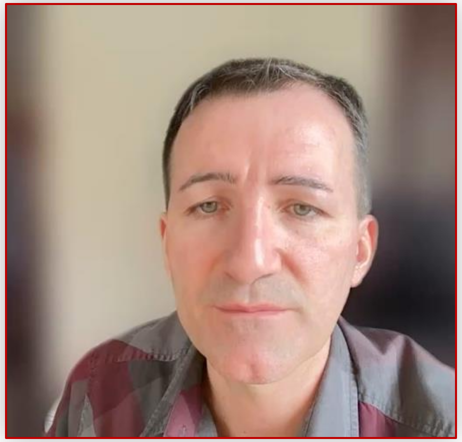
Dr. Yilmaz Kocer

(4) Algebra - Simplification

- These identities can come in handy:

$(x+y)$

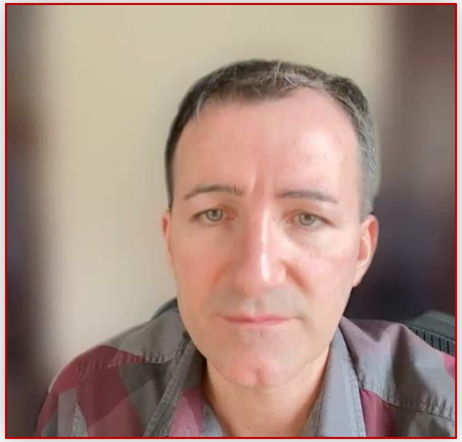
$$(\mathbf{x}+\mathbf{y})^2 = \mathbf{x}^2 + 2\mathbf{x}\mathbf{y} + \mathbf{y}^2$$
$$(\mathbf{x}-\mathbf{y})^2 = \mathbf{x}^2 - 2\mathbf{x}\mathbf{y} + \mathbf{y}^2$$
$$\mathbf{x}^2 - \mathbf{y}^2 = (\mathbf{x}+\mathbf{y})(\mathbf{x}-\mathbf{y})$$



Dr. Yilmaz Kocer

(4) Algebra - Simplification

Solving simple equations: Bring the unknown involving terms on one side of the equation and **isolate** the **x** term by **applying the same operation** (adding /subtracting /multiplying/dividing by the same number or term on both sides of the equation !)



Dr. Yilmaz Kocer

(4) Algebra - Simplification

Example: Let $\boxed{\mathbf{q = 120 - 2p + 9I}}$ be the demand equation where **q** is the quantity demanded (in *units*), **p** is the market price (expressed in *dollar units*) and **I** is the average income of the buyers (in *1,000 dollar units*).

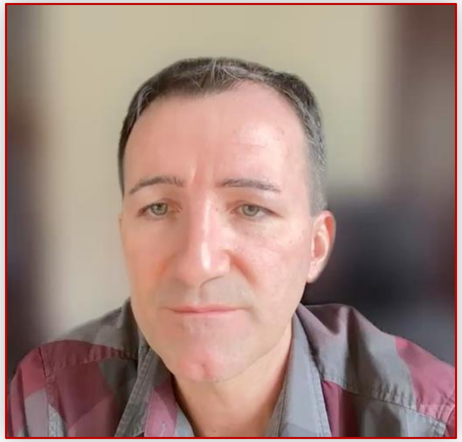
Assume we are given $\mathbf{q = 500}$ and $\mathbf{I = 80}$, then

$$\mathbf{500 = 120 - 2 \cdot p + 9 \cdot 80} \qquad -720$$

$$\mathbf{500 - 120 - 720 = -2p}\,,$$

$$\mathbf{-340 = -2p}\text{, yielding } \boxed{\mathbf{p = 170.}}$$

170   -2 -2

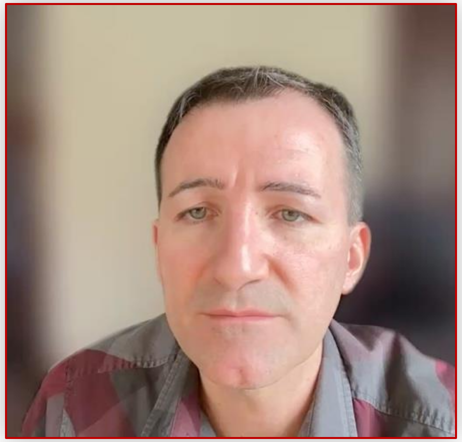


Dr. Yilmaz Kocer

(4) Algebra - Simplification

<u>**Inequalities:**</u> For example, $\mathbf{q > 120}$, $\boxed{\mathbf{p \leq 6}}$, or $\mathbf{2q - p < 2p + q}$

You can add/subtract/multiply/divide with any number or term and the inequality is preserved; <u>**except**</u> multiplying/dividing with <u>negative</u> numbers <u>**reverses the direction**</u> of the inequality:



Dr. Yilmaz Kocer

(4) Algebra - Simplification

<u>Inequalities:</u> For example, $\mathbf{q > 120}$, $\mathbf{p \le 6}$, or $\mathbf{2q - p < 2p + q}$

You can add/subtract/multiply/divide with any number or term and the inequality is preserved; **<u>except</u>** multiplying/dividing with <u>negative</u> numbers **<u>reverses the direction</u>** of the inequality:

$\boxed{\mathbf{2q - 60 > 500 - 3q}}$ which simplifies to $\mathbf{5q > 560}$ hence $\mathbf{q > 112}$.

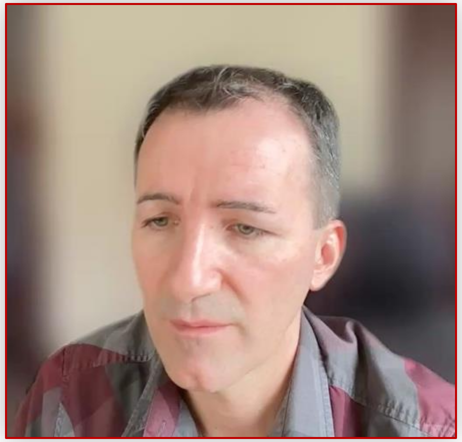
+3q +60 +60 +3q   5 5

We could instead push the $\mathbf{q}$ including terms to the right side and have

$\mathbf{-60 - 500 > -3q - 2q}$ thus

$\mathbf{-560} \boxed{>} \mathbf{-5q}$ which implies

$$\frac{-560}{-5} = 112 < \frac{-5q}{-5} = q$$



Dr. Yilmaz Kocer

(4) Algebra - Simplification

Inequalities

- When squaring (or raising to any even power), we should be careful about the sign of the initial expressions: $\mathbf{x > y}$ implies $\mathbf{x^2 > y^2}$ only when $\mathbf{x > y > 0}$.
- For example, $\mathbf{-2 > -3}$ but $(\mathbf{-2})^{\mathbf{2}} \ngtr (\mathbf{-3})^{\mathbf{2}}$

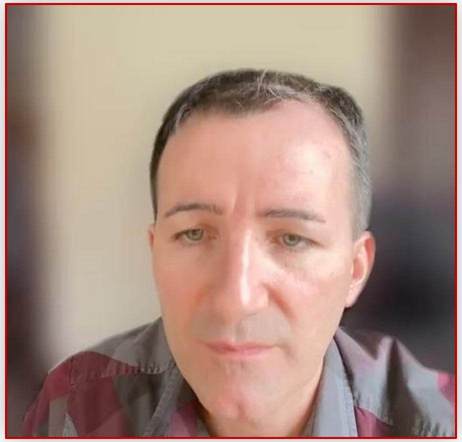
  $4 \ngtr 9$ $\qquad \neq \qquad \ngtr$

- Also, if $\mathbf{x > y > 0}$ then $\mathbf{x^{-1} < y^{-1}}$

$$\boxed{\frac{\cancel{x}}{\cancel{x}y} > \frac{\cancel{y}}{x\cancel{y}}}$$

$$\frac{1}{y} > \frac{1}{x}$$

$$y^{-1} > x^{-1}$$



Dr. Yilmaz Kocer

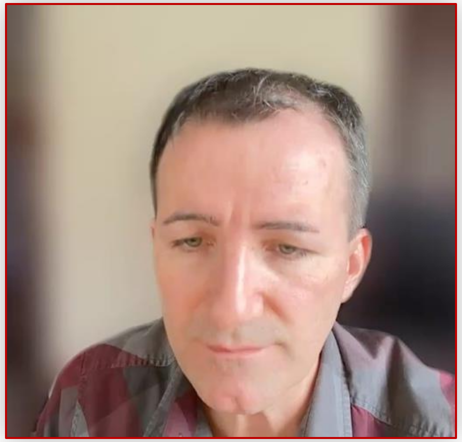
(4) Algebra - Simplification

- Suppose you are interested in whether an expression ***increases*** as a variable appearing in it ***increases***:

$2^{-1} \to 3^{-1}$

$\frac{1}{2}$ $\frac{1}{3}$

- Does $\left(\mathbf{x}^{-1} - \mathbf{x} + \frac{3}{\mathbf{x}^2}\right)$ increase as $\mathbf{x} > \mathbf{0}$ increases?
- For higher **x**, as each term falls, the whole expression falls (**decreases**)!
- Another example: what happens to the expression $(\boldsymbol{ln}(\boldsymbol{x}) - \mathbf{3x})$ as we increase $\mathbf{x} > \mathbf{0}$? It is ambiguous!



Dr. Yilmaz Kocer

(4) Algebra - Simplification

- For a fraction expression that is positive, <u>if numerator increases and denominator falls, the fraction increases</u> !!

- **Example:** $\left(\dfrac{100-x}{x^2+7}\right)$ **decreases,** as $\mathbf{0 < x < 100}$ increases.

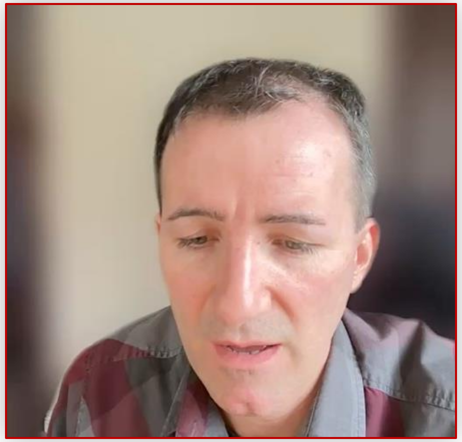
- **Example:** How about $\left(\dfrac{100-2x}{200-5x}\right)$ for $\mathbf{0 < x < 40}$?

- $\dfrac{100-2x}{200-5x} - \dfrac{2}{5} = \dfrac{5(100-2x)-2(200-5x)}{(200-5x)5} = \dfrac{100}{1000-25x} \rightarrow$ **increases**, as **x** increases !

$(200-5x)$

$\dfrac{2}{5}$

$\dfrac{3}{7}$ $\dfrac{4}{6}$



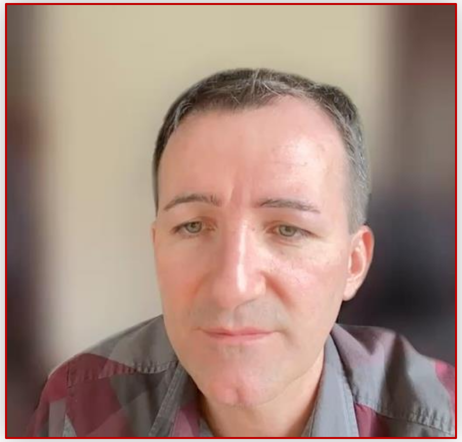
Dr. Yilmaz Kocer

(4) Algebra - Simplification

Absolute value: magnitude, distance from the point "0"; $|\mathbf{x}| = \begin{cases} x & \text{if } \mathbf{x} \geq 0 \\ -\mathbf{x} & \text{if } \mathbf{x} < 0 \end{cases}$

- For example, $|-17| = 17$, $|8| = 8$.

  Similarly, $|2x + 3| = 17$ would imply that:



Dr. Yilmaz Kocer

(4) Algebra - Simplification

Absolute value: magnitude, distance from the point "0"; $|\mathbf{x}| = \begin{cases} \boldsymbol{x} & \text{if } \mathbf{x} \geq \mathbf{0} \\ -\mathbf{x} & \text{if } \mathbf{x} < \mathbf{0} \end{cases}$

$x \geq 3$

$x < 2$

$2 \leq x < 3$

- For example, $|\mathbf{-17}| = \mathbf{17}$, $|\mathbf{8}| = \mathbf{8}$.

  Similarly, $|\mathbf{2x+3}| = \mathbf{17}$ would imply that:

  $2x = 14$

- *either* ***a)*** $\mathbf{2x+3 \geq 0}$ and hence $|\mathbf{2x+3}| = \boldsymbol{2x} + \mathbf{3} = \mathbf{17}$ -3 thus $\mathbf{x = 7}$

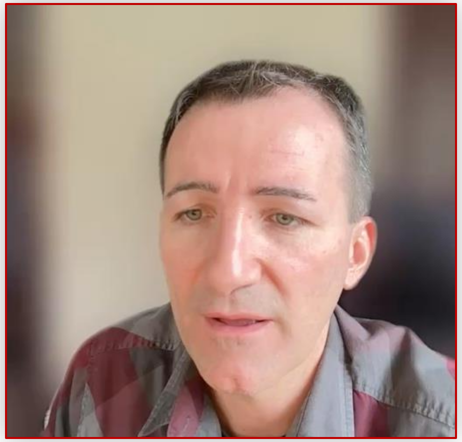
  or ***b)*** $\mathbf{2x+3 < 0}$ and hence $|\mathbf{2x+3}| = -(\boldsymbol{2x} + \mathbf{3}) = \mathbf{17}$ thus $\mathbf{x = -10}$

$2x+3 = -17$

$2x = -17-3$

$2x = -20$

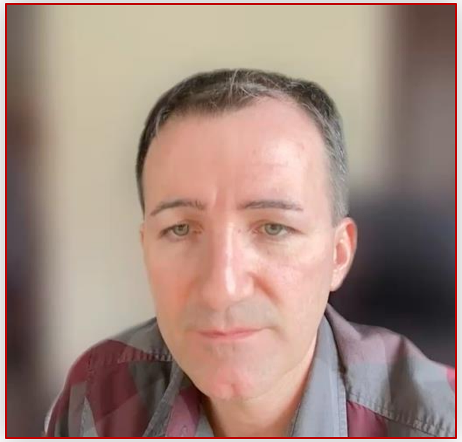
$x = -10$



Dr. Yilmaz Kocer

(4) Algebra - Simplification

<u>REMARK:</u> Equalities/inequalities can be seen as **<u>conditions</u>** (that is, **restrictions** or **constraints**) on the variables to be satisfied, hence we can <u>identify</u> an (in)equality with the corresponding "set" of solutions, i.e. allowed values for variables involved:



Dr. Yilmaz Kocer

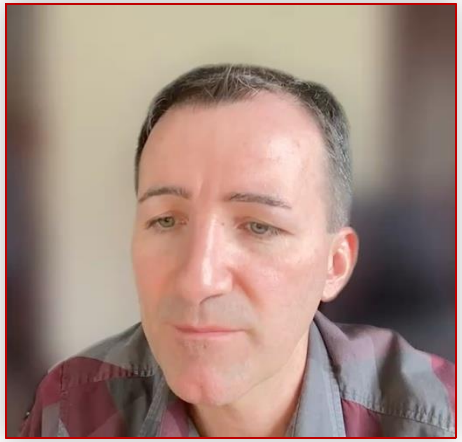
(4) Algebra - Simplification

REMARK: Equalities/inequalities can be seen as **conditions** (that is, **restrictions** or **constraints**) on the variables to be satisfied, hence we can identify an (in)equality with the corresponding "set" of solutions, i.e. allowed values for variables involved:

- In the demand example $\mathbf{p = 170}$ is the only possible value, and in the absolute

  $p \in \{170\}$

  value example $\boxed{\mathbf{x} \in \{-\mathbf{10}, \mathbf{7}\}}$ is the "**solution set**", respectively.



Dr. Yilmaz Kocer