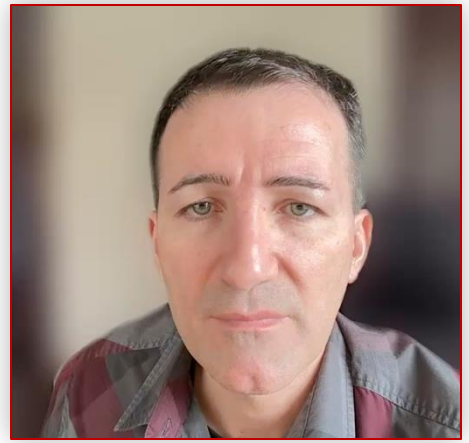
(3) Algebra - Representation and Evaluation

An example how we **represent** an economic scenario in a mathematical model:



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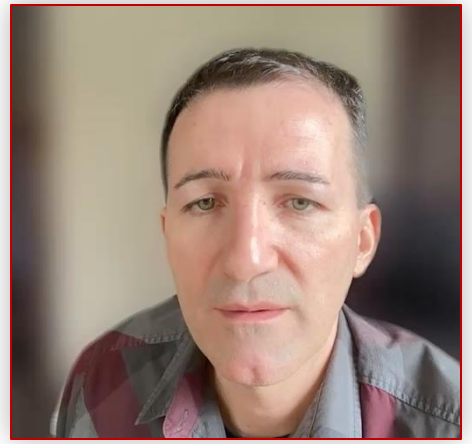
(3) Algebra - Representation and Evaluation

An example how we **represent** an economic scenario in a mathematical model:

- You are planning to grow potatoes; each **pound** requires **0.1 sqft of land**.
- You can only rent "square" shaped farmland at the rental price **$3 per sqft**.
- You also need a fence around your land and the fence costs **$8 per ft** ($8/ft)
- You can sell your potatoes at the supermarket, but you must pay **$100** for the space (independent of the pounds of potatoes you put on the shelves).
- Suppose you are planning to produce $q \geq 0$ pounds of potatoes.



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(3) Algebra - Representation and Evaluation

each **pound** requires **0.1 sqft of land**.

"square" shaped farmland at the rental price **$3 per sqft**.

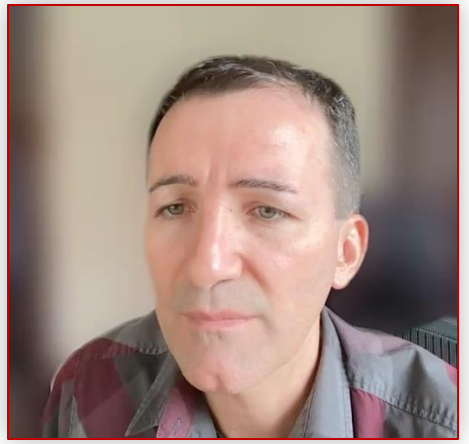
The fence around your land costs **$8 per ft** ($8/ft)

You must pay **$100** for the space in the supermarket

a) How much would the land rental cost (in terms of **q**)? How much would the required fence cost? As these depend on the target quantity **q** pounds of potatoes, the sum is called the *total variable cost* of producing and selling **q** pounds of potatoes **TVC(q)**.

$TVC(q) = \underbrace{3 \cdot 0.1 \cdot q}_{\text{rental cost}} + \underbrace{8 \cdot 4\sqrt{0.1q}}_{\text{fence cost}}$

$\sqrt{0.1q} \leftarrow \boxed{0.1q}$



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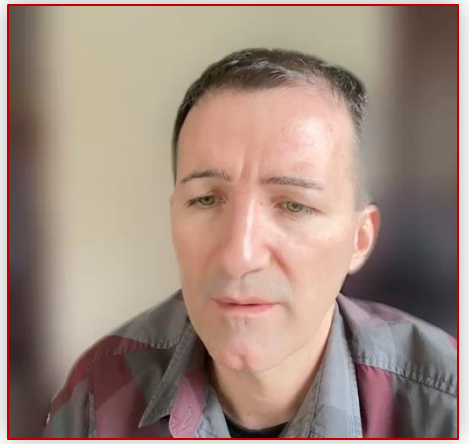
You must pay **$100** for the space in the supermarket

b) Together with the supermarket space cost of **$100** flat (it doesn't depend on the quantity **q**; hence it is called the total fixed cost, **TFC**), what is the *total cost* of producing and selling **q** pounds of potatoes **TC(q)**?

$TFC = 100$

$TC(q) = TFC + TVC(q)$

$= 100 + 0.3q + 32\sqrt{0.1q}$



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(3) Algebra - Representation and Evaluation

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"square" shaped farmland at the rental price **$3 per sqft**.

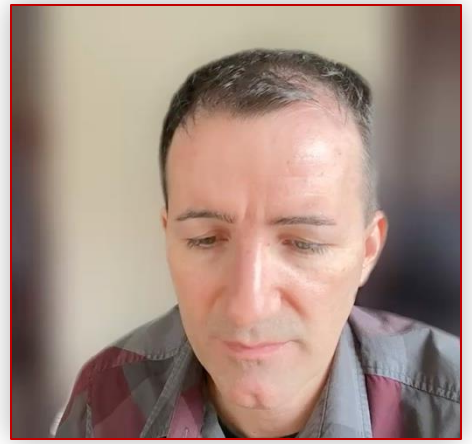
The fence around your land costs **$8 per ft** ($8/ft)

You must pay **$100** for the space in the supermarket

c) If you can sell your potatoes at **$2** per pound, what is your *revenues minus costs* (=**net profits**) if you produce and sell **q** pounds?

$$\text{profits}(q) = TR(q) - TC(q)$$

$$= 2 \cdot q - \left(100 + 0.3q + 32\sqrt{0.1q}\right)$$



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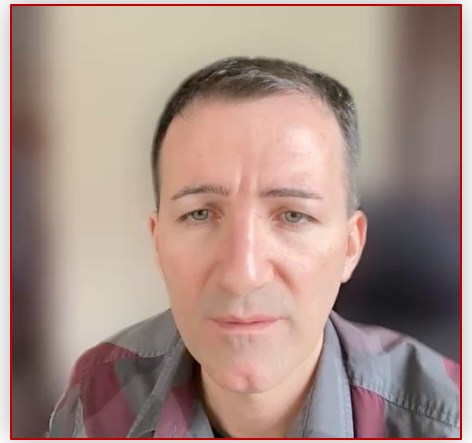
- **REMINDER:** Do not forget nonnegativity constraints: that is , **q < 0** doesn't make sense!



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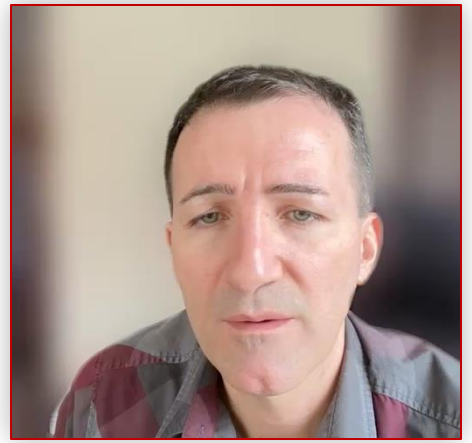
(3) Algebra - Representation and Evaluation

- **REMINDER:** Do not forget nonnegativity constraints: that is , **q < 0** doesn't make sense!
- There can be more than one "variable"(in addition to **q**, there might be the possibility of producing **t** pounds of tomatoes.)



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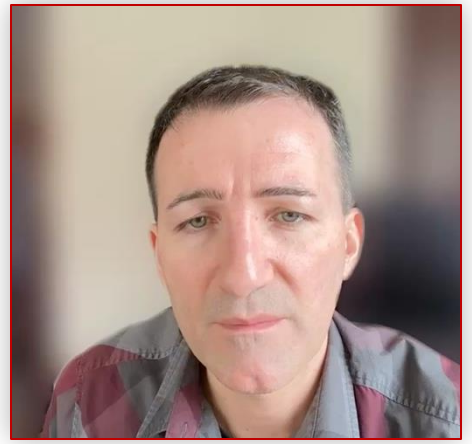
(3) Algebra - Representation and Evaluation

- **REMINDER:** Do not forget nonnegativity constraints: that is , **q < 0** doesn't make sense!
- There can be more than one "variable"(in addition to **q**, there might be the possibility of producing **t** pounds of tomatoes.)
- Suppose each apple costs **$2** and each orange costs **$3**,
  if you buy **x** apples and **y** oranges, how much do you have to pay?

  ANSWER: ***Total Payment(x,y)*** **= 2·x + 3·y**



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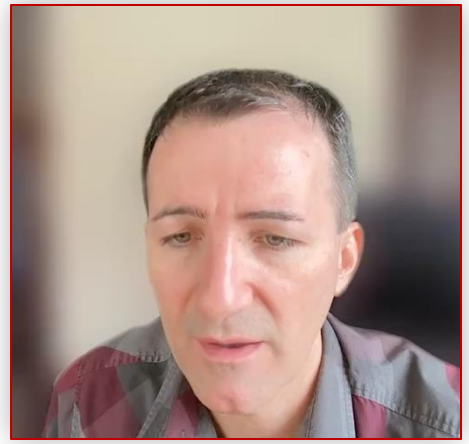
(3) Algebra - Representation and Evaluation

- **Evaluation** means "plugging in" values for the variables:
- In the apples/oranges example, **4 apples** and **7 oranges** cost

$$\textbf{Total Payments } (4,7) = 4 \cdot 2 + 7 \cdot 3 = 29$$



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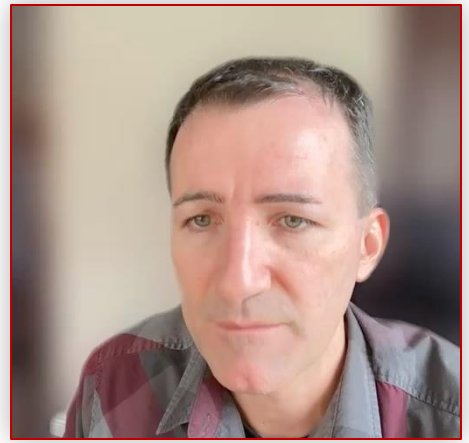
(3) Algebra - Representation and Evaluation

- **Evaluation** means "plugging in" the number of pounds (say, **q = 1000**) and calculate the value of the expression;



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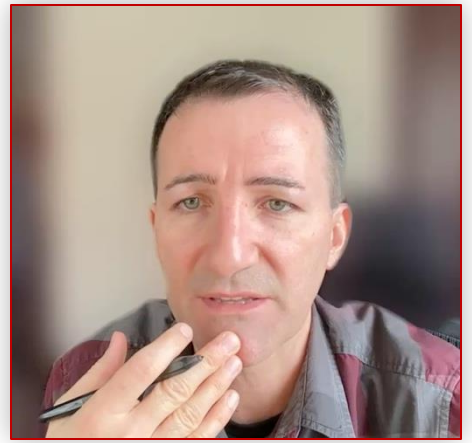
(3) Algebra - Representation and Evaluation

- **Evaluation** means "plugging in" the number of pounds (say, **q = 1000**) and calculate the value of the expression;

- $$\mathbf{TVC}(q) = 0.3q + 32\sqrt{0.1 \cdot q}$$

$$\mathbf{TVC}(1000) = 0.3 \cdot 1000 + 32\sqrt{0.1 \cdot 1000} = \$620$$



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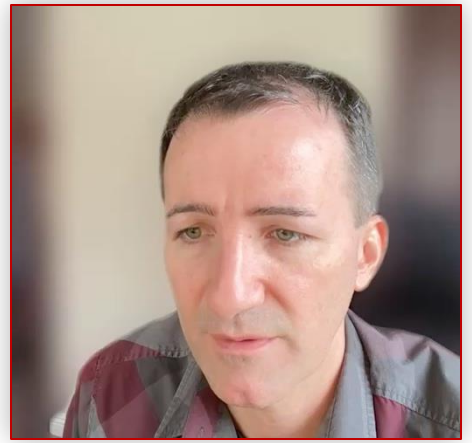
(3) Algebra - Representation and Evaluation

- **Evaluation** means "plugging in" the number of pounds (say, **q = 1000**) and calculate the value of the expression;

- $$\mathbf{TC}(q) = 100 + 0.3q + 32\sqrt{0.1 \cdot q}$$

$$\mathbf{TC}(1000) = 100 + 620 = 720$$



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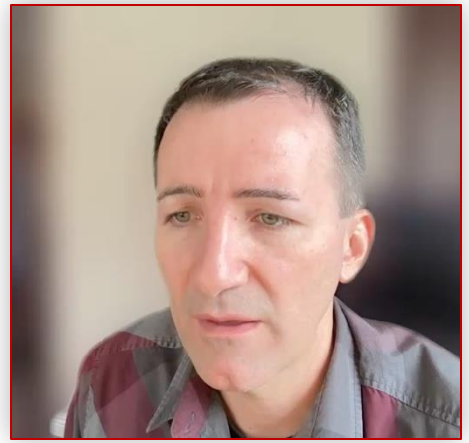
(3) Algebra - Representation and Evaluation

- **Evaluation** means "plugging in" the number of pounds (say, $q = 1000$) and calculate the value of the expression;

- $$\textbf{Profits}(q) = 2q - \left(100 + 0.3q + 32\sqrt{0.1 \cdot q}\right)$$

$$\textbf{Profits}(1000) = 1000 \cdot 2 - (720) = \$1,280.$$



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(3) Algebra - Representation and Evaluation

- **Evaluation** means "plugging in" the number of pounds (say, **q = 1000**) and calculate the value of the expression;

- $$\textbf{Profits}(q) = 2q - \left(100 + 0.3q + 32\sqrt{0.1 \cdot q}\right)$$

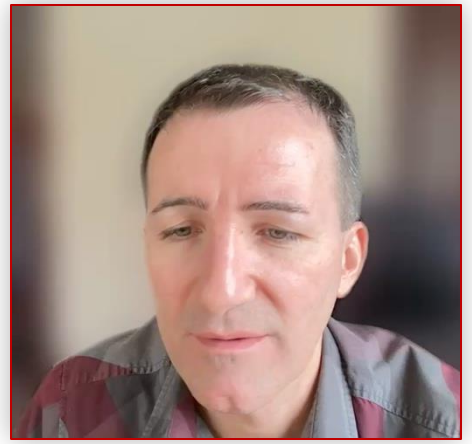
$$\textbf{Profits}(1000) = 1000 \cdot 2 - (720) = \$1,280.$$

If we consider alternative values for **q**;

q = 640 → $\textbf{Profits}(640)$

$$= 640 \cdot 2 - \left(100 + 0.3 \cdot 640 + 32\sqrt{0.1 \cdot 640}\right) = \$732.$$

q = 1,210 → $\textbf{Profits}(1210) = 1000 \cdot 2 - (720) = \$1,605.$



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