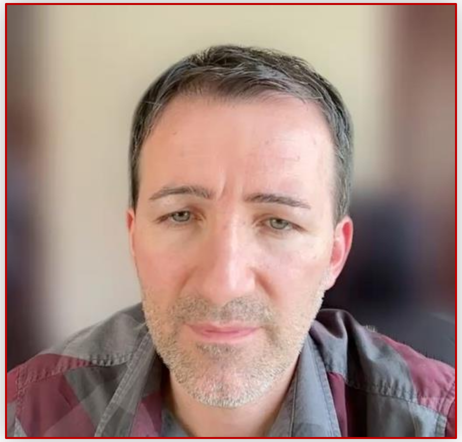
(2) Arithmetic: Powers, Roots and Logarithms

- $\mathbf{x^y}$ means "**x** multiplied **y** many times"; **x** is called the *base* and **y** the *exponent* (*power*). For example, $\mathbf{4^3 = (4)^\wedge 3 = 4 \cdot 4 \cdot 4 = 64}$

$$\underbrace{x \cdot x \cdot x \cdot \quad \cdot x}_{y}$$



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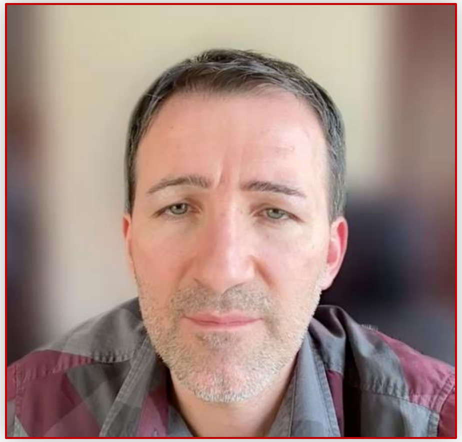
(2) Arithmetic: Powers, Roots and Logarithms

- Product of terms with the same base has $\mathbf{7^a \cdot 7^b = 7^{a+b}}$, similarly $\frac{5^a}{5^b} = \mathbf{5}^{a-b}$,

  … power has $\mathbf{7^a \cdot 5^a = (7 \cdot 5)^a = 35^a}$

  $7 \cdot 5 \quad 7 \cdot 5 \cdots 7 \cdot 5$

  a



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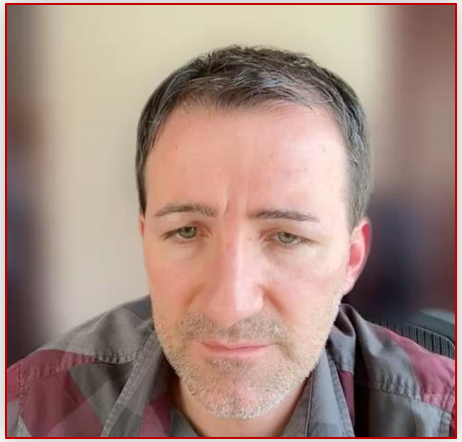
- Power of a base is taken to another power: you multiply the powers: $(3^a)^b = 3^{\boxed{a\cdot b}}$

$$\underbrace{\underbrace{(3 \cdot 3 \cdot \ldots \cdot 3)}_{a}(\qquad)(\qquad)}_{b}$$



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(2) Arithmetic: Powers, Roots and Logarithms

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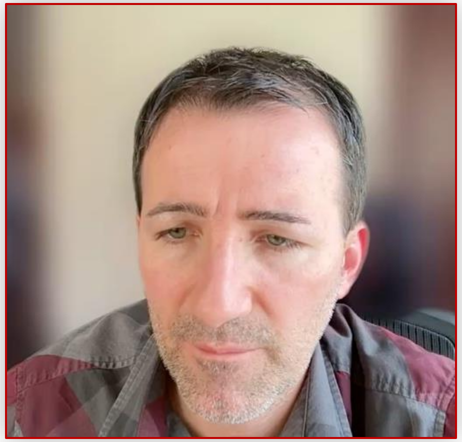
  … power has $\mathbf{7^a \cdot 5^a = (7 \cdot 5)^a = 35^a}$
- Power of a base is taken to another power: you multiply the powers: $\mathbf{(3^a)^b = 3^{a \cdot b}}$
- $\mathbf{1^a = 1}$ for any a real number **a**, $\mathbf{0^a = 0}$ except when $\boldsymbol{a} = \mathbf{0}$, which is *undefined.*
- In general, $\boldsymbol{x^{-a} = x^{0-a} = \boxed{\frac{x^0}{x^a}} = \frac{1}{x^a}}$ $0^0 = 0^{2-2} = \frac{0^2}{0^2} = \frac{0}{0}$



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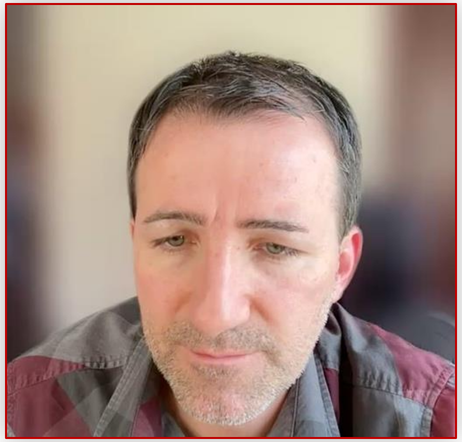
(2) Arithmetic: Powers, Roots and Logarithms

- **Roots** are *fractional* powers: $\sqrt{4} = 4^{0.5} = (2^2)^{0.5} = 2^{2 \cdot 0.5} = 2^1 = 2$

  In general, $\sqrt{a^2} = |a|$ which is the absolute value of **a.**

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

$|-3| = 3$ $\quad |+17| = 17$



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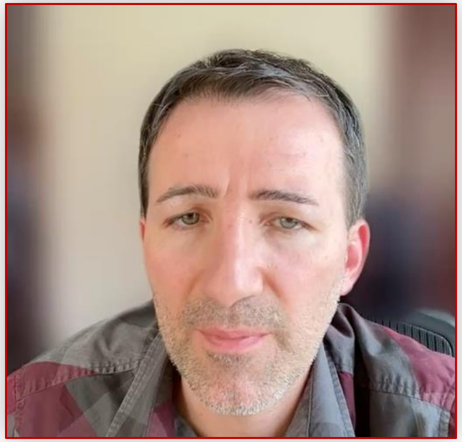
- $3^4 = 81$ means $3 = 81^{\frac{1}{4}} = \sqrt[4]{81}$.

  $\left(3^{4}\right)^{\frac{1}{4}} = (81)^{\frac{1}{4}}$

But don't forget that $\boxed{x^4 = 81}$ has solutions $\boxed{x = \pm 3}$ in real numbers.

$(-3)^4 = \underbrace{(-3)\cdot \ldots (-3)}_{4} = 3^4$



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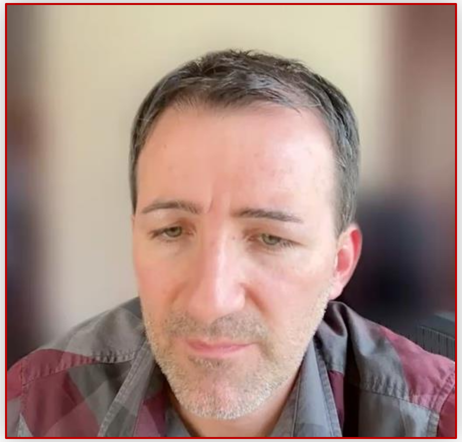
(2) Arithmetic: Powers, Roots and Logarithms

- Logarithm answers the question "to what power should I raise **a** to arrive at the value **b** ?" in the expression $\mathbf{x} = \boxed{\mathbf{\log_a b}}$. That is, it means $\boxed{\boldsymbol{a^x = b}}$.



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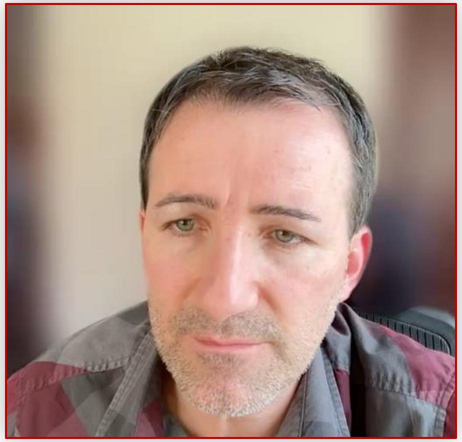
(2) Arithmetic: Powers, Roots and Logarithms

- **Logarithm** answers the question "to what power should I raise **a** to arrive at the value **b** ?" in the expression $\mathbf{x = \log_a b}$. That is, it means $\boldsymbol{a^x = b}$.
- For example, $\mathbf{\log_3 81 = 4}$ or $\mathbf{\log_{10} \boxed{1{,}000{,}000} = 6}$



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(2) Arithmetic: Powers, Roots and Logarithms

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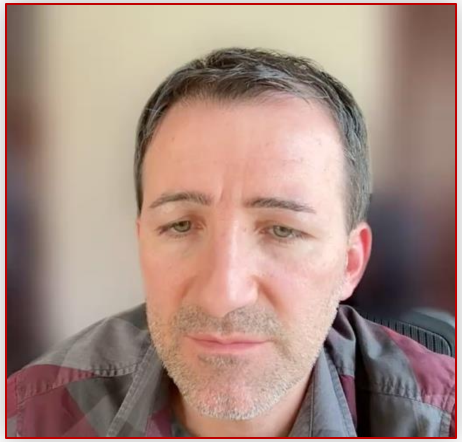
- For example, $\mathbf{log_3 81 \; = \; 4}$ or $\mathbf{log_{10} 1{,}000{,}000 = 6}$
- Common logarithm bases are **2, e, 10**
- When the base is not explicitly written, it is understood to equal **2**; for ex.

  $\mathbf{log_{2}(32) = 5}$,
- *Euler's number* $\mathbf{e = 2.718 \ldots}$; it is commonly written as

  $\mathbf{ln(x) = log_e(x)}$



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(2) Arithmetic: Powers, Roots and Logarithms

- Logarithms satisfy $\boxed{\underbrace{\log_a(b)}_{x} + \underbrace{\log_a(c)}_{y} = \log_a(bc)}$, $\log_a(b^k) = \boxed{k\cdot\log_a(b)}$

$b^k = b\cdot b \cdots b$

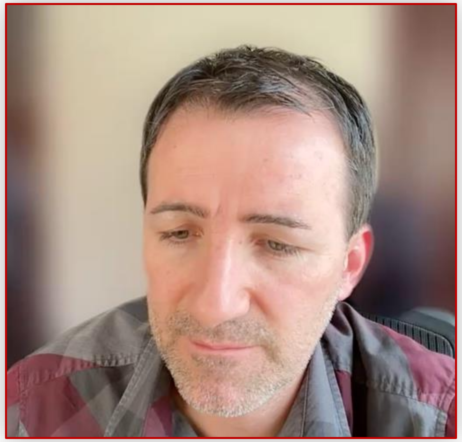
$$\underbrace{(\log_a b) + (\log_a b) + \cdots}_{k}$$

$a^x = b$

$a^y = c$

$\times$

$a^{x+y} = bc \longrightarrow \log_a bc = x+y = \log_a(b) + \log_a(c)$



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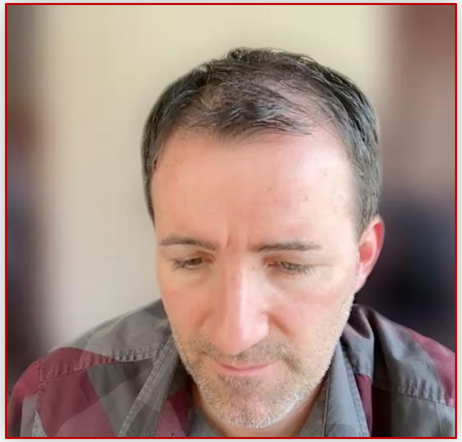
(2) Arithmetic: Powers, Roots and Logarithms

- Logarithms satisfy $\mathbf{\log_a(b) + \log_a(c) = \log_a(bc), \log_a(b^k) = k \cdot \log_a(b)}$
- Hence, $\boxed{\mathbf{4\ln(2) - 2\ln(8)}} = \mathbf{\ln\left(\frac{2^4}{8^2}\right) = \ln\left(\frac{1}{4}\right) = \ln(2^{-2}) = -2\ln(2)}$

$\ln(2^4) - \ln(8^2)$

$2^2 = 2^{-2}$



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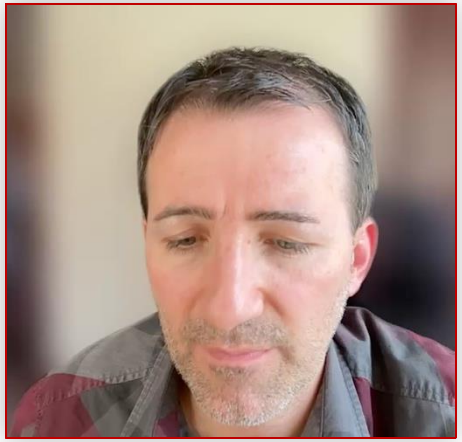
(2) Arithmetic: Powers, Roots and Logarithms

- Also, for *change of base* for logarithms we have $\frac{\log_c \boxed{a}}{\log_c \boxed{b}} = \log_{\boxed{b}} \boxed{a}$, for example;

$$\frac{\log_2 9 \ \log_3 16}{\log_4 3 \ \log_9 4} = \frac{\log(9)\ \log(16)\ \log(4)\ \log(9)}{\log(2)\ \log(3)\ \log(3)\ \log(4)} = \log_3 9 \cdot \log_3 9 \cdot \log_2 16$$

$$= 2\cdot 2\cdot 4 = 16$$

$$\frac{\frac{2\log 3}{\log 2}\ \frac{4\log 2}{\log 3}}{\frac{\log 3}{\log 4}\ \frac{\log 4}{2\log 3}} = \frac{8}{\frac{1}{2}} = 8\cdot\frac{2}{1} = 16$$



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