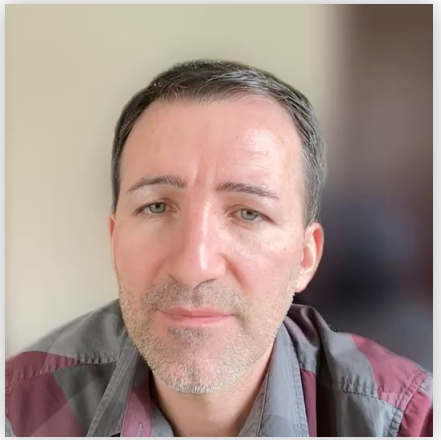
(1) Arithmetic: Order of multiple operations, Fractions and Decimals

- Integers $\mathbb{Z} = \mathbf{\ldots.-3,-2,-1,0,1,2,\ldots.}$ Natural numbers $\mathbb{N} = \mathbf{0,1,2,3,\ldots}$

Real numbers $\mathbb{R}$ are all numbers on the real line, including fractions like $\frac{7}{11}$ or decimal numbers like $\mathbf{-0.333\ldots} = -\frac{1}{3}$ and irrational numbers like $\sqrt{2}, \boldsymbol{\pi}, \mathbf{e}\ldots$

- PEMDAS parentheses (brackets) → exponents → multiplication/division → addition/subtraction:



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- PEMDAS parentheses (brackets) → exponents → multiplication/division → addition/subtraction:

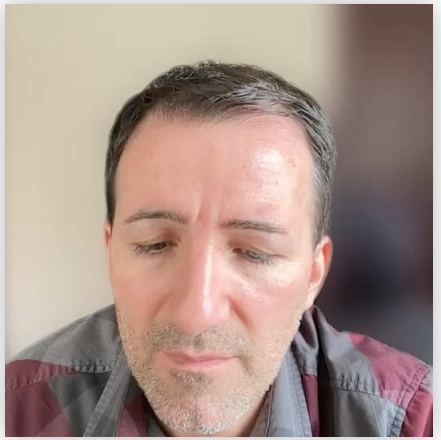
$$2^{3-2} \cdot \left(2 + 6 \cdot \frac{1}{3}\right) = 8 ,$$

Still, try to use parentheses when there is possible ambiguity

a) $8 - 4 + 2$ b) $8 \div 4 \times 2$ c) $8 \div (4 \times 2)$ d) $\frac{8}{4 \times 2}$

A more complicated example:

$$\frac{(3 + (8 - 3^{4-2}) \times 2) + 3}{5 - 2 \times 2} =$$



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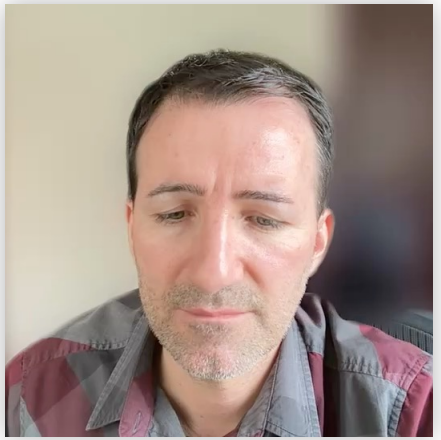
- **Multiply** = use dot "$\cdot$", not a cross "$\times$", not to confuse with the variable "**x**".

- If there is no risk of confusion, we don't use any symbols for multiplication; **ab** means $\mathbf{a \times b = a \cdot b}$. Hence $\mathbf{8 \div 4x = \frac{8}{4x} = \frac{2}{x}}$

- A minus (-) before brackets means all terms inside should be multiplied by **-1**:

$$\mathbf{2 - (5 - 10) = 2 - 1 \times (5 - 10) = 2 - 5 + 10}$$



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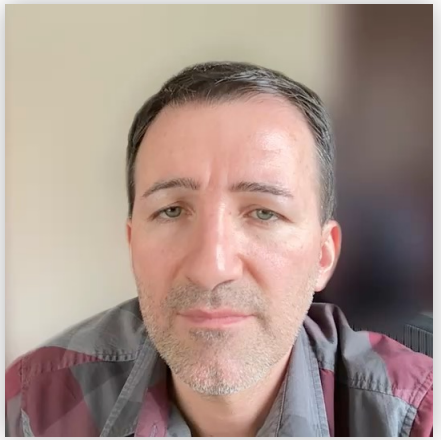
- For fraction sums, bring them to a common denominator;

$$\frac{5}{12}+\frac{3}{16}=\frac{5\cdot 4}{12\cdot 4}+\frac{3\cdot 3}{16\cdot 3}=\frac{29}{48}$$

- When multiplying/dividing, factorize fractions, and cancel out terms in the numerator and denominator to simplify, and dividing by a *fraction* means multipling by $\frac{1}{\boldsymbol{fraction}}$;

$$\frac{5}{12}\div\frac{25}{24}=\frac{\frac{5}{12}}{\frac{25}{24}}=\frac{5}{12}\times\frac{24}{25}=\frac{5\times 24}{12\times 25}=\frac{2}{5}$$



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- Try not to use fractions with whole parts, like "**3½**" ; instead use **3.5** or **7/2**.

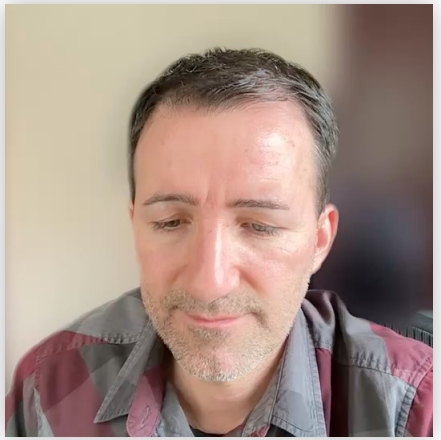
- Decimals sum/multiply like in this example;

0.12 + 0.034 = 0.154, similarly **0.12 · 0.006 = 0.00072**

+ 0.034

0.154

2 + 3 = 5



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- Percentages can be seen as a fraction or a decimal number **60% = 60/100 = 0.6**

- Rounding in intermediate stages may yield vastly inaccurate results…

$$\%60 \times 200 = \frac{60}{100} \times 200 = 120$$

$$\frac{1000}{900} = \text{🕐} \dashrightarrow 1100 - 1000\,\text{🕐} = ??$$

$$= 1.1111\ldots \rightarrow 1100 - \frac{1000 \cdot 1.1}{1100} = 0$$

$$= 1.1111 \rightarrow 1100 - \frac{1000 \cdot 1.11}{1110} = -10$$

1.285 → 1.29

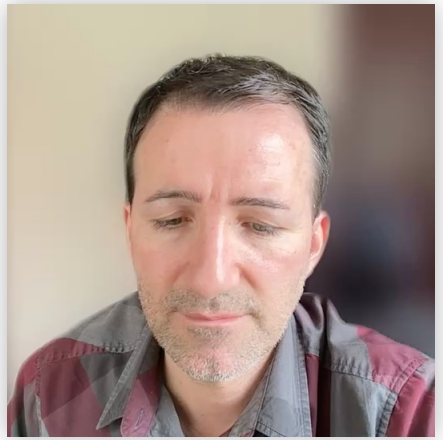
1.3

1.4565 → 1.456

1.457

0-4 rounds down

5-9 " up



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