Fractal properties of the Hofstadter butterfly, eigenvalues of the almost Mathieu operator, and topological phase transitions

Abstract: Harper's operator - the 2D discrete magnetic Laplacian - is the model behind the Hofstadter's butterfly and Thouless theory of the Quantum Hall Effect. It reduces to the critical almost Mathieu family, indexed by the phase. We will present a complete proof of singular continuous spectrum for the critical family, for all phases, finishing a program with a long history. The proof is based on a simple Fourier analysis and a new Aubry duality-type transform. We will also explain how these ideas provide for a very simple proof of zero measure of the spectrum of Harper's operator, a problem previously solved by sophisticated dynamical systems techniques, as well as progress on some other outstanding conjectures.