

Lab #7 - Joint Kinetics and Internal Forces

Purpose:

The objective of this lab is to understand how to calculate net joint forces (NJFs) and net joint moments (NJMs) from force data. Upon completion of this lab you will:

- Know how to use a step-by-step method to generate the correct free body diagrams (FBDs) and mass acceleration diagrams (MADs) for the lower extremity during the time of peak vertical reaction force of a movement.
- Understand the difference between a net joint force (NJF) and a net joint moment (NJM).
- Know how to use force plate data, along with FBDs, to calculate NJFs and NJMs at the ankle, knee, and hip.
- Know how to determine which sets of muscles are activated during a movement.

Introduction:

In human movement, there are typically multiple moments acting on the system, which can be opposed by muscle forces. A **moment** is generated when a force acts on an object at a distance from its center of mass. We want to determine the overall mechanical demand placed on the muscles that cross a joint due to these moments. In other words, we want to know how much muscular force is needed for a given movement.

I. NET JOINT MOMENTS

A **net joint moment** is the *minimum* moment required at a joint to obtain the observed kinematics. A **net joint force** is the *minimum* force required at a joint to obtain the observed kinematics.

In a static analysis, there is no motion at the joint, so the sum of the moments at the elbow must be zero. There is no rotation. *This does not mean that the NJM is zero!*

$$\Sigma M = 0$$

$$M_{\text{elbow}} = \text{NJM} - M_{\text{arm-hand}} - M_{\text{barbell}} = 0$$

In addition to the NJM, we need to know the moment arm of the muscle creating the NJM in order to determine the muscle force required for the motion. The **moment arm** of a muscle is the *perpendicular distance* between the joint and the line of action of the muscle.

Once you solve for the NJM and the moment arm of the muscle, the mechanical demand placed on the elbow muscle can be calculated.

$$\text{NJM} = F_{\text{muscle}} \times d_{\text{muscle}}$$

Note: If more than one muscle contributes to the NJM then the NJM *cannot* be attributed directly to any one muscle that crosses the joint. In this case, the contribution of each muscle to the NJM can only be estimated. The calculated muscle moment is the sum of all of the muscle actions involved. In this case, since the arm is being held in flexion, we can say that the net muscle moment is due to the action of the elbow flexors, but we cannot say exactly which elbow flexors are most involved.

II. STATIC ANALYSIS

Imagine statically holding a barbell with your forearm in a horizontal position, parallel to the ground.

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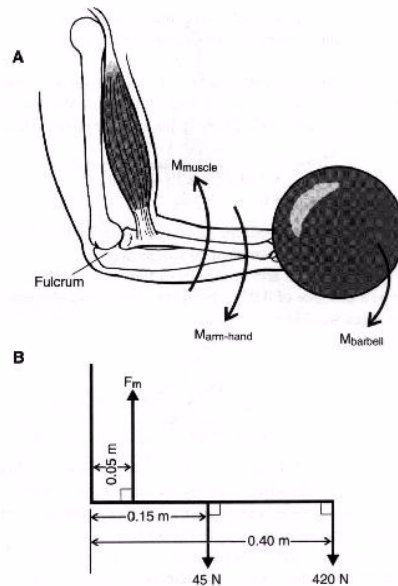


Figure 12-5. An illustration of the forearm during an instant in time of a biceps curl (a) and a free body diagram of the system (b).

If the elbow joint center is considered to be the axis of rotation, there are two negative moments acting on the system. (Remember, the right-hand rule says that counter-clockwise rotation is positive). One negative moment is the result of the weight of the forearm and hand acting through the center of mass (CM) of the forearm-hand system. The other is the result of the weight of the barbell acting on the hand. To oppose these moments, the elbow flexor muscles generate forces across the joint, which create a positive moment around the elbow joint. If the positive moment created by the muscles (counter-clockwise) is *less than* the sum of the negative moments created by the weight of the forearm-hand system and the barbell (clockwise), then there will be a net *negative* moment at the elbow, and the elbow will extend eccentrically (rotate clockwise). If the positive moment created by the muscles (counter-clockwise) is *greater than* the sum of the negative moments created by the weight of the forearm-hand system and the barbell (clockwise), then there will be a net *positive* moment at the elbow, and the elbow will flex concentrically (rotate counter-clockwise). If the system is in equilibrium, the positive moment of the muscles must be *equal to* the two negative moments, so the net moment is zero, and the elbow will not rotate.

In this system, for the sum of the moments at the elbow to be zero, the muscle force must be considerably greater than the other two forces because the moment arm for the muscle is very small compared to the moment arms for the forearm-hand or barbell.

III. FREE BODY DIAGRAMS

Free body diagrams (FBDs) and mass acceleration diagrams (MADs) of a system provide a visual representation of the cause-effect relationship between system kinetics and kinematics. The FBD (cause) illustrates all forces ($\Sigma F_x, \Sigma F_y$) that cause the linear acceleration of the system CM (a_x, a_y) and all of the moments (ΣM_{cm}) that cause the angular acceleration (α) of the system about the system CM. The MAD (effect) illustrates the result of these forces and moments, which is also dependent on the system's resistance to linear acceleration (mass) and its resistance to angular acceleration (I_{cm}).

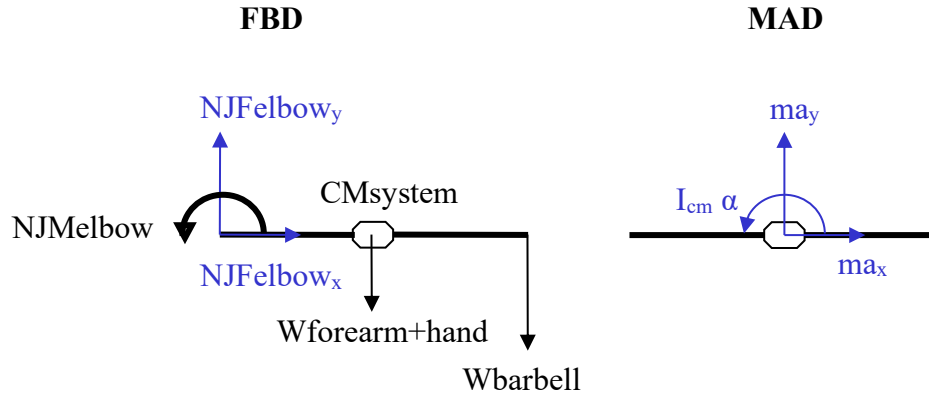


Figure 1. General FBD (cause) and MAD (effect) of a person holding a barbell.

General equations of motion can be written using the FBD and MAD. If we are doing a static analysis, the sum of the forces is zero and the sum of the moments is zero.

$$\Sigma F_x = ma_x$$

$$\Sigma F_x = NJFelbow_x = 0$$

$$\Sigma F_y = ma_y$$

$$\Sigma F_y = NJFelbow_y - Wforearm+hand - Wbarbell = 0$$

$$NJF = Wforearm+hand + Wbarbell$$

$$\Sigma M_{cm} = I_{cm} \alpha$$

$$NJM - (Wforearm+hand \times d_{forearm+hand}) - (Wbarbell \times d_{barbell}) = I_{cm} \alpha$$

$$NJM = I_{cm} \alpha + (Wforearm+hand \times d_{forearm+hand}) + (Wbarbell \times d_{barbell})$$

Where:

F = force

m = mass

M_{cm} = moment about the CM of the segment

I_{cm} = moment of inertia of the segment

α = linear acceleration of the segment center of mass

In general, the FBD and MAD of a segment could be represented as follows:

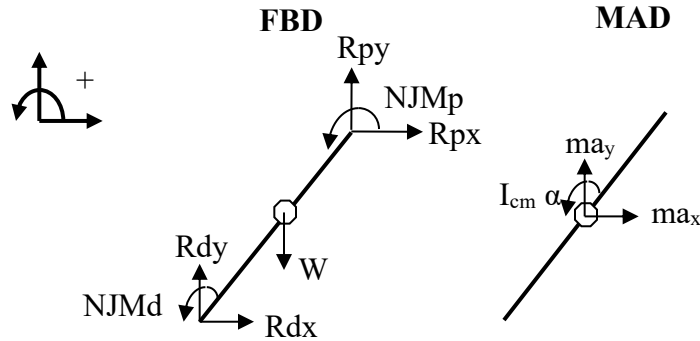


Figure 2. FBD and MAD of a segment. R_{px} and R_{py} represent the reaction forces at the proximal end of the segment; R_{dx} and R_{dy} represent the reaction forces at the distal end of the segment.

Typically, forces acting at the distal end of a segment, the weight of the segment, and the kinematics of the segment are known from direct measurement. The forces and moments at the proximal end, however, must be calculated using the equations of motion and known quantities. NJF is the net *linear* effect of all structures crossing the joint. NJM is the net *rotational* effect of all structures crossing the joint.

In this lab, you will calculate the net joint forces (NJFs) and the net joint moments (NJMs) acting at the proximal end of a segment using the information provided. Reaction force data was collected by a force plate. Segment kinematic data was calculated using digitized coordinate data of body landmarks from images recorded onto video tape. The body segment parameters are those published in the literature.

The following pages contain step-by-step examples of how to calculate NJFs and NJMs at the ankle, knee, and hip. We will start at the ankle, because it is the joint closest to the reaction force. Once we calculate the NJF and NJM for the ankle, we can use these values when we work our way up to the knee. The NJF and NJM calculations for the knee will then be used to calculate the NJF and NJM at the hip.

IV. EXAMPLE 1 ~ THE ANKLE

Imagine a runner's foot at toe-off. The foot's CM has a horizontal acceleration of 4.67 m/s^2 and a vertical acceleration of 2.87 m/s^2 . In addition, the ankle has an angular acceleration of -30.48 rad/s^2 . If the foot has a mass of 1.52 kg and a moment of inertia of $.0046 \text{ kgm}^2$, calculate the NJF's (R_x and R_y) and the NJM acting at the ankle.

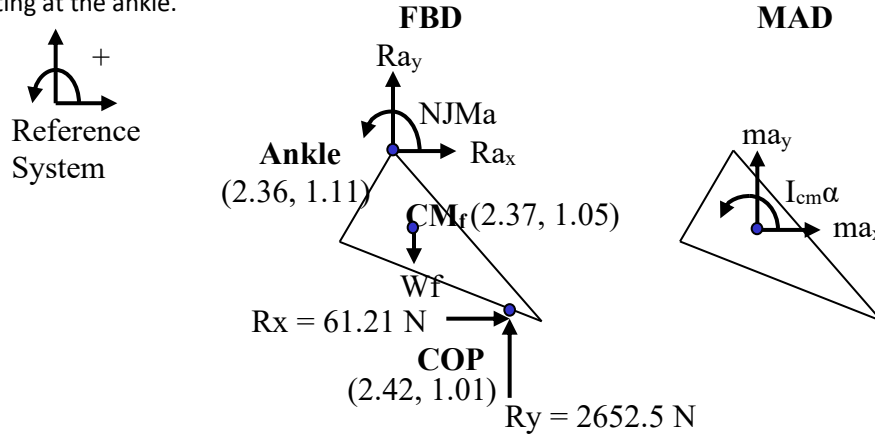


Figure 3. FBD and MAD of the foot during toe-off. In this diagram, we are assuming everything acts in a positive direction. Calculations may prove otherwise.

First, list all of the known and unknown variables.

Known

Ankle	(2.36, 1.11)	$m = 1.52 \text{ kg}$
CM _f	(2.37, 1.05)	$a_x = 4.67 \text{ m/s}^2$ $a_y = 2.87 \text{ m/s}^2$
CP	(2.42, 1.01)	$I_{cm} = 0.0046 \text{ kgm}^2$
R _x : horizontal reaction force	61.21 N	$\alpha = -30.48 \text{ rad/s}^2$
R _y : vertical reaction force	2652.5 N	$g = 9.81 \text{ m/s}^2$

Unknown

- NJMa: net joint moment at the ankle
- R_{a_x}: horizontal reaction force at the ankle
- R_{a_y}: vertical reaction force at the ankle

Second, using these values and the free body diagram above, sum the horizontal and vertical forces in order to calculate the horizontal and vertical reaction forces at the ankle.

Horizontal

$$\Sigma F_x = ma_x$$

$$\Sigma F_x = R_x + Ra_x = ma_x$$

$$R_x + Ra_x = ma_x$$

$$61.21 + Ra_x = (1.52 \times 4.67)$$

$$Ra_x = -54.11 \text{ N}$$

Vertical

$$\Sigma F_y = ma_y$$

$$\Sigma F_y = R_y - W_f + Ra_y = ma_y$$

$$R_y - (m \cdot g) + Ra_y = ma_y$$

$$2652.50 - (1.52 \times 9.81) + Ra_y = (1.52 \times 2.87)$$

$$Ra_y = -2633.23 \text{ N}$$

Now, we need to redraw the original FBD to reflect the actual directions of the forces at the ankle. *We will use this revised FBD to calculate the NJM at the ankle.*

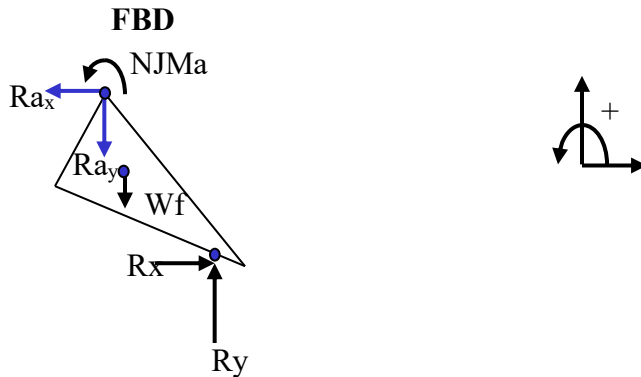


Figure 4. Revised FBD of the foot during toe-off. The forces acting at the ankle have been redrawn to reflect the calculated values (horizontal and vertical both negative).

The next step is to calculate the moment arm of each force that is acting on the foot.

$$\begin{aligned}
 R_y: d1 &= CP_x - CM_x = (2.42 - 2.37) = 0.05 \text{ m} \\
 R_x: d2 &= CM_y - CP_y = (1.05 - 1.01) = 0.04 \text{ m} \\
 W_f: d3 &= 0 \text{ (because the force of gravity acts directly through the CM)} \\
 R_{a_y}: d4 &= CM_x - Ankle_x = (2.37 - 2.36) = 0.01 \text{ m} \\
 R_{a_x}: d5 &= Ankle_y - CM_y = (1.11 - 1.05) = 0.06 \text{ m}
 \end{aligned}$$

Now that we know the values of all of the forces acting on the system and their distances from the CM, we can calculate the NJM at the ankle, or the minimum moment needed to generate the observed rotation. In this example, all of the forces create positive moments (counter-clockwise) about the ankle.

$$\Sigma M_{cm} = I_{cm}\alpha$$

$$\begin{aligned}
 & NJMa + (R_y \times d1) + (R_x \times d2) + (W_f \times d3) + (R_{a_y} \times d4) + (R_{a_x} \times d5) = I_{cm}\alpha \\
 NJMa &= I_{cm}\alpha - (R_y \times d1) - (R_x \times d2) - (R_{a_y} \times d4) - (R_{a_x} \times d5)
 \end{aligned}$$

$$\begin{aligned}
 NJMa &= (.0046 \times -30.48) - (2652.5 \times .05) + (61.21 \times .04) + (2633.23 \times .01) + (54.11 \times .06) \\
 NJMa &= -164.8 \text{ Nm}
 \end{aligned}$$

A negative NJMa means the actual direction of NJMa is clockwise. *Therefore, an ankle plantar NJM is needed to counteract the moments created by all of the forces.* Again, redraw the FBD to reflect this new information.

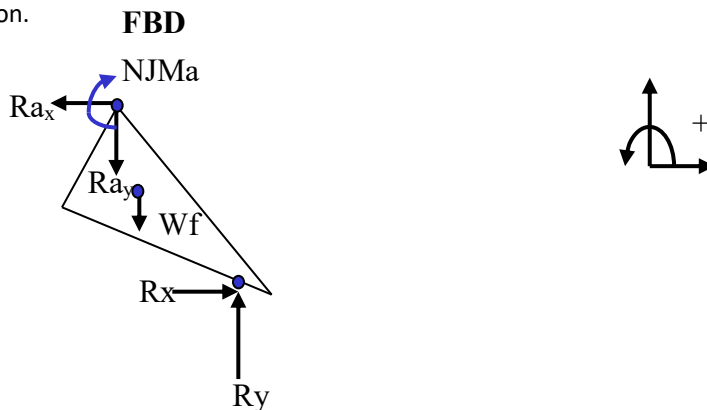


Figure 5. Actual FBD of the foot at toe-off, with actual direction of NJM and NJF's at the ankle.

V. EXAMPLE ~~ THE KNEE

We can use the NJFa's (R_{ax} and R_{ay}) and NJMa from Example 1 to move up the body and calculate the NJF's and NJM at the knee. Assume that the mass of the shank is 5.68 kg, the horizontal acceleration of the shank CM is -0.65 m/s^2 , and the vertical acceleration of the shank CM is 17.90 m/s^2 . In addition, the moment of inertia of the shank is measured as 0.0535 kgm^2 and the angular acceleration of the knee is 54.284 rad/s^2 .

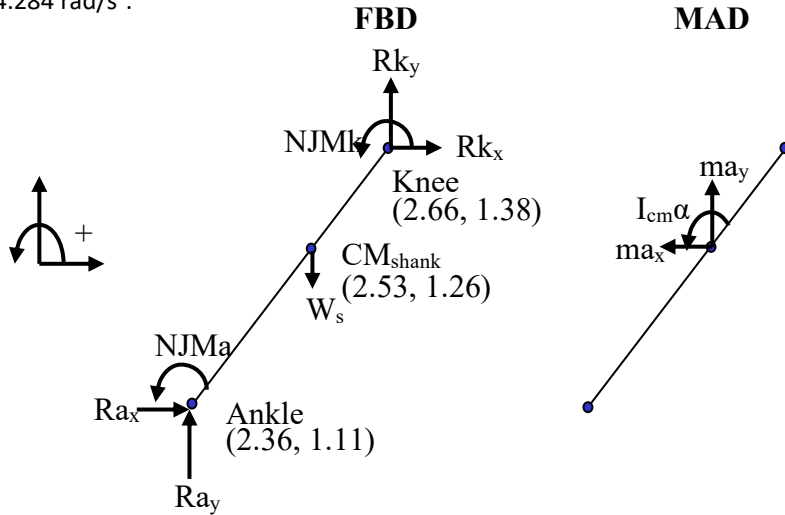


Figure 6. FBD and MAD of the shank segment at toe-off. In this diagram, we are assuming everything acts in a positive direction. Calculations may prove otherwise.

First, list all of the known and unknown variables.

Known

Ankle	(2.36, 1.11)	$m = 5.68 \text{ kg}$
CM _k	(2.53, 1.26)	$a_x = -0.65 \text{ m/s}^2$ $a_y = 17.90 \text{ m/s}^2$
Knee	(2.66, 1.38)	$I_{cm} = 0.0535 \text{ kgm}^2$
R_{ax} :	(calculated in example 1) 54.11 N	$\alpha = 54.28 \text{ rad/s}^2$
R_{ay} :	(calculated in example 1) 2633.23 N	$g = 9.81 \text{ m/s}^2$

Unknown

- NJMk: net joint moment at the knee
- R_{kx} : horizontal reaction force at the knee
- R_{ky} : vertical reaction force at the knee

Second, using these values and the FBD above, sum the horizontal and vertical forces in order to calculate the horizontal and vertical reaction forces at the knee.

Horizontal

$$\sum F_x = ma_x$$

$$\sum F_x = R_{ax} + R_{kx} = ma_x$$

$$R_{ax} + R_{kx} = ma_x$$

$$54.11 + R_{kx} = (5.68 \times -0.65)$$

$$R_{kx} = -57.80 \text{ N}$$

Vertical

$$\sum F_y = ma_y$$

$$\sum F_y = R_{ay} - W_s + R_{ky} = ma_y$$

$$R_{ay} - (m \cdot g) + R_{ky} = ma_y$$

$$2633.23 - (5.68 \times 9.81) + R_{ky} = (5.68 \times 17.90)$$

$$R_{ky} = -2475.84 \text{ N}$$

Now, we need to redraw the original FBD to reflect the actual directions of the forces at the knee. *We will use this revised FBD to calculate the NJM at the knee.*

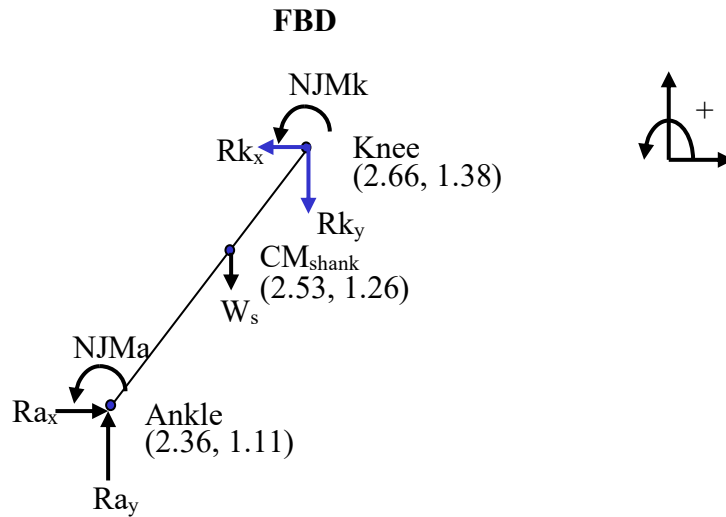


Figure 7. Revised FBD of the shank during toe-off. The forces acting at the knee have been redrawn to reflect the calculated values (horizontal and vertical both negative).

The next step is to calculate the moment arm of each force that is acting on the shank.

$$\begin{aligned}
 Ra_y: \quad d1 &= CM_x - Ankle_x = (2.53 - 2.36) = 0.17 \text{ m} \\
 Ra_x: \quad d2 &= CM_y - Ankle_y = (1.26 - 1.11) = 0.15 \text{ m} \\
 W_s: \quad d3 &= 0 \quad (\text{because the force of gravity acts directly through the CM}) \\
 Rk_y: \quad d4 &= knee_x - CM_x = (2.66 - 2.53) = 0.13 \text{ m} \\
 Rk_x: \quad d5 &= knee_y - CM_y = (1.38 - 1.26) = 0.12 \text{ m}
 \end{aligned}$$

Now that we know the values of all of the forces acting on the system and their distances from the CM, we can calculate the NJM at the knee. In this example, Ra_x , Rk_x , and $NJMa$ (equal and opposite to example 1) create positive moments (counter-clockwise) about the knee; Ra_y and Rk_y create negative moments.

$$\Sigma M = I_{cm} \alpha$$

$$\begin{aligned}
 NJMk + NJMa - (Ra_y \times d1) + (Ra_x \times d2) + (W_s \times d3) - (Rk_y \times d4) + (Rk_x \times d5) &= I_{cm} \alpha \\
 NJMk = I_{cm} \alpha - NJMa + (Ra_y \times d1) - (Ra_x \times d2) + (Rk_y \times d4) - (Rk_x \times d5)
 \end{aligned}$$

$$\begin{aligned}
 NJMk &= (.0535 \times 54.28) - 164.8 + (2633.23 \times .17) - (54.11 \times .15) + (2473.84 \times .13) - (57.8 \times .12) \\
 NJMk &= 592.3 \text{ Nm}
 \end{aligned}$$

A positive $NJMk$ indicates that you have assumed the direction of the $NJMk$ correctly on the FBD. *The knee NJM is an extensor moment.* It is working together with the ankle plantar NJM to counteract the moments created by the ground reaction force at the foot.

VI. EXAMPLE ~ THE HIP

We can use the NJF's (R_{ax} and R_{ay}) and NJMk from Example 2 to move up the body and calculate the NJF's and NJM at the hip. Assume that the mass of the thigh is 17.45 kg, the horizontal acceleration of the thigh CM is 1.54 m/s^2 , and the vertical acceleration of the shank CM is 34.35 m/s^2 . In addition, the moment of inertia of the thigh is measured as 0.2972 kgm^2 and the angular acceleration of the hip is 44.512 rad/s^2 .

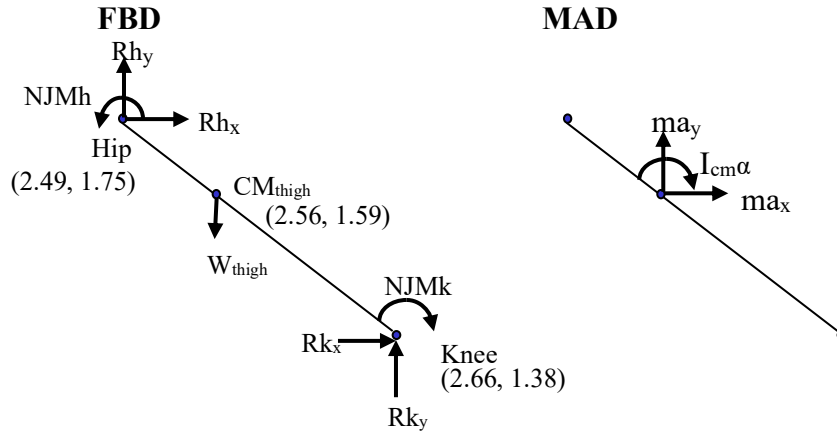


Figure 8. FBD and MAD of the thigh segment at toe-off. In this diagram, we are assuming everything acts in a positive direction. Calculations may prove otherwise.

First, list all of the known and unknown variables.

Known

Knee	(2.66, 1.38)	$m = 17.45 \text{ kg}$
CM _t	(2.56, 1.59)	$a_x = 1.54 \text{ m/s}^2$ $a_y = 34.35 \text{ m/s}^2$
Hip	(2.49, 1.75)	$I_{cm} = 0.2972 \text{ kgm}^2$
R_{kx} :	(calculated in example 2) 57.8 N	$\alpha = 44.5126 \text{ rad/s}^2$
R_{ky} :	(calculated in example 2) 2478.84 N	$g = 9.81 \text{ m/s}^2$

Unknown

- NJMt: net joint moment at the thigh
- R_{tx} : horizontal reaction force at the thigh
- R_{ty} : vertical reaction force at the thigh

Second, using these values and the FBD above, sum the horizontal and vertical forces in order to calculate the horizontal and vertical reaction forces at the knee.

Horizontal

$$\begin{aligned} \Sigma F_x &= ma_x \\ \Sigma F_x &= R_{kx} + R_{tx} = ma_x \\ R_{kx} + R_{tx} &= ma_x \\ 57.8 + R_{tx} &= (17.45 \times 1.54) \\ R_{tx} &= -30.93 \text{ N} \end{aligned}$$

Vertical

$$\begin{aligned} \Sigma F_y &= ma_y \\ \Sigma F_y &= R_{ky} - W_s + R_{ty} = ma_y \\ R_{ky} - (m \cdot g) + R_{ty} &= ma_y \\ 2478.84 - (17.45 \times 9.81) + R_{ty} &= (17.45 \times 34.35) \\ R_{ty} &= -1708.25 \text{ N} \end{aligned}$$

Again, we need to redraw the original FBD to reflect the actual directions of the forces at the hip. *We will use this revised FBD to calculate the NJM at the knee.*

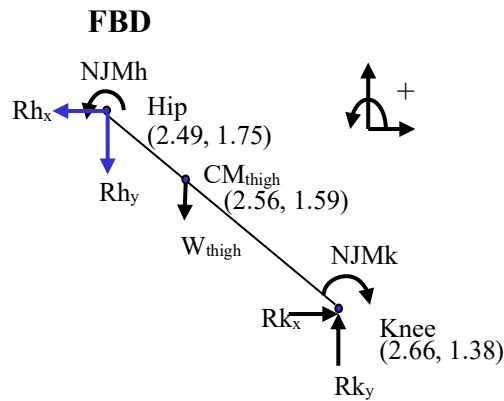


Figure 9. Revised FBD of the thigh during toe-off. The forces acting at the hip have been redrawn to reflect the calculated values (horizontal and vertical both negative).

The next step is to calculate the perpendicular distance that each force is acting from the CM of the shank.

$$R_{ky}: d1 = CM_x - Knee_x = (2.56 - 2.66) = -0.1 \text{ m}$$

$$R_{kx}: d2 = CM_y - Knee_y = (1.59 - 1.38) = 0.21 \text{ m}$$

$$W_{tt}: d3 = 0 \text{ (because the force of gravity acts directly through the CM)}$$

$$R_{hy}: d4 = hip_x - CM_x = (2.49 - 2.56) = -0.07 \text{ m}$$

$$R_{hx}: d5 = hip_y - CM_y = (1.75 - 1.59) = 0.16 \text{ m}$$

*** Because distances must be positive, take the absolute value of d1 and d4. **
 We will keep track of the negative moments in the calculations.

Now that we know the values of all of the forces acting on the system and their distances from the CM, we can calculate the NJM at the hip. In this example, R_{kx} , R_{ky} , R_{tx} , and R_{ty} create positive moments (counter-clockwise) about the hip; NJM_k (equal and opposite to example 2) creates a negative moment.

$$\Sigma M = I_{cm} \alpha$$

$$NJM_h - NJM_k + (R_{ky} \times d1) + (R_{kx} \times d2) + (W_s \times d3) + (R_{hy} \times d4) + (R_{hx} \times d5) = I_{cm} \alpha$$

$$NJM_h = I_{cm} \alpha + NJM_k - (R_{ky} \times d1) - (R_{kx} \times d2) - (R_{hy} \times d4) - (R_{hx} \times d5)$$

$$NJM_h = (.2972 \times 44.512) + 592.3 - (2479.84 \times .1) - (57.8 \times .21) - (1708.25 \times .07) - (30.93 \times .16)$$

$$NJM_h = 220.88 \text{ N}$$

A positive NJM_h indicates that you have assumed the direction of the NJM_h correctly on the FBD. *The hip NJM is a flexor moment.* It is working together with moments created by the reaction force at the hip and knee joints to counteract the knee extensor NJM .

Pre Lab:

Consider the FBD of a person statically holding a barbell, as illustrated in Figure 10. The NJM that is necessary to keep the system in static equilibrium can be calculated given the information in the free body diagram.

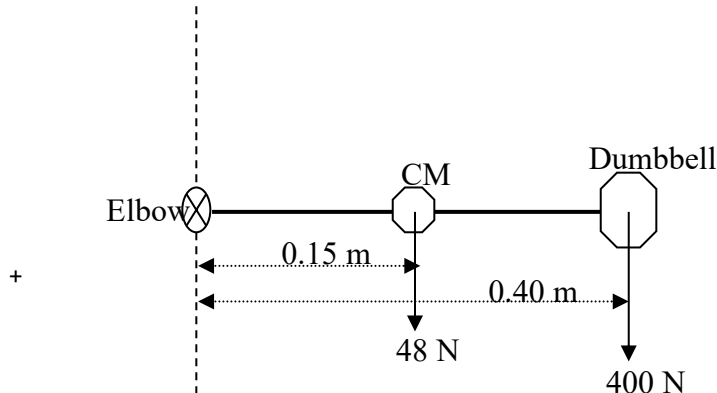


Figure 10. FBD of a forearm-hand-weight system during a barbell curl.

1. Calculate the vertical net joint force at the elbow.
2. Calculate the moment about the segment CM due to the weight of the arm-hand system.
3. Calculate the moment about the segment CM due to the barbell.
4. Calculate the moment about the segment CM due to the NJ_{elbow} .
5. Calculate the net joint moment about the elbow.
6. What effect will this elbow NJM have on action at the elbow joint?
7. Will this elbow NJM always cause an angular change at the joint in the same direction?
8. What muscle groups **MUST** be activated for this to occur?
9. Remember that the muscle moment cannot be directly attributed to any one muscle that crosses the joint. It can be stated that the NJM is due to the action of the elbow flexor muscle group, but it cannot be said exactly which members of the elbow flexor muscle group are most involved. Now, let's *ASSUME* that the biceps muscle is the only muscle contributing to the elbow NJM. We can estimate that the perpendicular distance between the elbow joint and the line of action of this biceps muscle is 0.06 m. Calculate the muscle force the biceps must generate to create the elbow NJM.
10. Compare the muscle force generated by the biceps with the forces due to the segment weight and the barbell. Explain.

Laboratory Procedure:

1. Download the data set from the web page. This data set was collected during the take-off phase of the back rotating dive.
2. Plot coordinate data to draw a general FBD of the whole body (use Sheet 2 for Subject 2)
3. Open the data set and identify the data that has been provided.
 - GRF Ground Reaction Force
 - CP Center of Pressure
 - CGP Center of Gravity Position
 - CGA Center of Gravity Acceleration
4. Use the kinematics and force data provided to draw a preliminary FBD and MAD of the foot, shank, and thigh segments. Use the given coordinate data to draw the correct orientation of the segments. Assume a positive direction for unknown variables.
5. From your FBD and MAD diagram of the foot, write the equations of motion. List which variables are known and unknown.
6. Solve for the net joint forces (horizontal and vertical) at the ankle joint.
7. Redraw your foot FBD to reflect the actual directions of the NJF's at the ankle.
8. Solve for the NJM at the ankle joint.
9. Draw the final reality FBD for the foot, with the correct directions for all forces and moments.
10. Check your reference system for +/- direction of the forces and moments. Use the information you just calculated to draw the FBD of the shank segment. Now, you should be able to draw the correct direction of the moment and forces acting at the distal end of the shank (NJMankle).
11. Solve for the reaction forces and net joint moment at the proximal end of the shank segment.
12. Repeat the same process (9-10) for the thigh segment to solve for reaction force at the proximal end of the thigh and NJM at the hip.
13. Complete the corrected FBD and MAD of three segments and label them clearly. They will help you to answer the post lab questions.

Data Analysis and Questions:

1. From the FBD of the foot segment, what is the magnitude and direction of the reaction force at the ankle joint?
2. From the FBD of the foot segment, what is the magnitude and direction of NJM at the ankle joint?
3. If the NJM acting at the proximal end of the foot rotates the foot in a clockwise direction, which set of muscles needs to be activated to create that moment?
4. From the FBD of the shank segment, what is the magnitude and direction of the reaction force at the knee joint?
5. Do the resultant reaction forces at the proximal and distal ends of the shank work together to rotate the shank in a clockwise direction? If no, what do they do to the shank segment?
6. From the FBD of the shank segment, what is the magnitude and direction of the NJM at the knee?
7. If the NJM at the knee is positive, which set of muscles need to be activated to generate that moment?
8. Does NJM at the knee rotate the shank in a counter-clockwise direction, against the moments created by the reaction force at the proximal and distal end of the shank segment? If no, what does it do to the shank segment?
9. From the FBD of the shank segment, does the NJM at the ankle work together with the NJM at the knee to rotate the shank segment in the same direction?
10. What is the magnitude and direction of the reaction force at the hip joint?
11. From the FBD of the thigh segment, what is the magnitude and direction of the NJM at the hip?
12. According to your reference system, if the NJM at the hip is positive, which set of muscles need to be activated to generate that moment?
13. Does the NJM at the hip rotate the thigh in a counter-clockwise direction, against the moments created by the reaction force at the proximal and distal end of the thigh segment? If no, what does it do to the thigh segment?

14. From the FBD of the thigh segment, does the NJM at the knee work together with NJM at the hip?
15. What is the magnitude of the shear and compression force at the knee joint on the shank?
16. Please turn in your FBDs and MADs of all three segments.