## Lab \#5 - Angular Impulse and Momentum

## Purpose:

The objective of this lab is to understand the angular impulse/momentum relationship. Upon completion of this lab you will:

- Understand and know how to calculate positive and negative impulse using both the trapezoidal rule in Excel and the grid block method.
- Understand and know how to calculate moment arms, moments of horizontal and vertical forces, and angular momentum.
- Be able to plot moment-time curves from a data set.


## Introduction:

Many athletic movements, such as the takeoff of a dive, require athletes to generate both linear and angular impulse. The linear impulse will translate the total body center of mass (TBCM) in the direction of the net linear impulse relative to the TBCM. Similarly, the angular impulse will rotate the TBCM in the direction of the net angular impulse relative to the TBCM.

## I. IMPULSE

Angular impulse represents the effect of a moment (force acting at a distance from the TBCM) on a system. It is defined as the moment of force acting over a specified period of time.

$$
\text { Angular Impulse }=\Sigma M \Delta t=\text { area under moment-time curve }
$$

Figure 2. Illustration of a moment-time curve.


A net positive angular impulse indicates that the system will rotate in a counter clockwise direction.

## II. MOMENTUM

Angular momentum describes the quantity of angular motion. It is defined as the moment of linear momentum.

$$
\mathrm{H}=\mathrm{I}_{\mathrm{cm}} * \omega
$$

where $\mathrm{H}=$ angular momentum
$I_{c m}=$ moment of inertia about the center of mass
$\omega$ = angular velocity

## III. PRINCIPLE OF IMPULSE AND MOMENTUM

The principle of impulse and momentum is a useful concept for understanding the cause-effect relationship between kinetics (forces) and kinematics (motion). Newton's Second Law illustrates the basic relationship that

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$\boldsymbol{\Sigma F}=\mathbf{m a}$ (cause = effect). From this relationship we are able to substitute and rearrange terms to come up with other useful relationships.

$$
\begin{aligned}
& \Sigma F=m a \\
& \text { substitute: } a=\left(v_{f}-v_{i}\right) / \Delta t \\
& \Sigma F=m\left(\left(v_{f}-v_{i}\right) / \Delta t\right) \\
& \Sigma F \Delta t=m\left(v_{f}-v_{i}\right) \\
& \Sigma F \Delta t=m v_{f}-m v_{i}
\end{aligned}
$$

Here, we see that the linear impulse created by forces acting on an object is equal to the change in momentum of the object.

Rearranging the terms a different way,

$$
m v_{f}=m v_{i}+\Sigma F \Delta t
$$

Here, the final momentum of an object is equal to an object's initial momentum and the impulse created by the forces acting on the object over a specified time period.

A similar strategy can also be applied for the angular motion of an object.

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\(\Sigma \mathrm{M}=\mathrm{I}_{\mathrm{cm}} * \alpha\)
    substitute: \(\alpha=\left(\omega_{f}-\omega_{i}\right) / \Delta t\)
\(\Sigma \mathrm{M}=\mathrm{I}_{\mathrm{cm}} *\left(\left(\omega_{\mathrm{f}}-\omega_{\mathrm{i}}\right) / \Delta \mathrm{t}\right)\)
\(\Sigma\left(\mathrm{F}^{*} \mathrm{~d}\right)=\mathrm{I}_{\mathrm{cm}} *\left(\left(\omega_{\mathrm{f}}-\omega_{\mathrm{i}}\right) / \Delta \mathrm{t}\right)\)
    where \(d\) is the moment arm from the point of linear force application to the TBCM.
\(\Sigma\left(\mathrm{F}^{*} \mathrm{~d}\right)^{*} \Delta \mathrm{t}=\mathrm{I}_{\mathrm{cm}}{ }^{*}\left(\omega_{\mathrm{f}}-\omega_{\mathrm{i}}\right)\)
\(\Sigma\left(\mathrm{F}^{*} \mathrm{~d}\right)^{*} \Delta \mathrm{t}=\mathrm{I}_{\mathrm{cm}} \omega_{\mathrm{f}}-\mathrm{I}_{\mathrm{cm}} \omega_{\mathrm{i}}\)
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Just as in the linear case, the angular impulse created by forces acting at a distance to the TBCM is equal to the change in angular momentum of the object.

Rearranging the terms in a different way,

$$
\mathrm{I}_{\mathrm{cm}} \omega_{\mathrm{f}}=\mathrm{I}_{\mathrm{cm}} \omega_{\mathrm{i}}+\Sigma\left(\mathrm{F}^{*} \mathrm{~d}\right)^{*} \Delta \mathrm{t}
$$

## Angular Impulse

Lab \#5 - Laboratory Procedure:

1. Looking at the videos, fill out the table by determining whether the TBCM is Anterior or Posterior to where you estimate the COP to be:

| Event | Back Timer | Back | Reverse |
| :--- | :--- | :--- | :--- |
| High |  |  |  |
| Low |  |  |  |
| Joint Extension |  |  |  |
| Departure |  |  |  |

2. Save the Excel file named "Angular Impulse" for your subject to the desktop. Open the file from within Excel and save it as a new file. This is the force data from the end of the load period until plate departure.
3. On the back timer sheet, locate the following data: Fx, Fy, COPx, COPy, TBCMx, TBCMy. (COP is the center of pressure, which indicates the location of force application.)
4. Calculate $d_{x}$ (moment arm of the force in $x$ direction) by using COPx and TBCMx. Define $\mathrm{dx}=$ COPx.- TBCMx Anytime this value goes from positive to negative (or vice versa) draw a picture and record the time phases this corresponds to when the Mcm due to Rv is positive or negative!
5. Calculate $d_{y}$ by using COPy and TBCMy data. $d y=T B C M y-C O P y$. Record the time phases that the horizontal reaction force is positive and when it is negative.
6. Calculate moment of horizontal force ( Mcm due to Fx ) as $\mathrm{Fx} * \mathrm{dy}$.
7. Calculate moment of vertical force (Mcm due to Fy) as Fy * dx.
8. Sum the moments of the vertical and horizontal force ( $\mathrm{\Sigma M}$ ).
9. Plot the moment-time curves of this data set. The graph should include Mcm due to Fx, Mcm due to Fy, and $\Sigma \mathrm{M}$ data on the y axis and time on the x axis.
10. Calculate the angular impulse due to the horizontal force by calculating the area under the Mcm due to Fxtime curve. Use the trapezoidal rule to calculate impulse for each time step. Then, sum this column to calculate the net impulse due to the horizontal force.
11. Calculate the angular impulse due to the vertical force by calculating the area under the Mcm due to Fy-time curve. Use the trapezoidal rule. Then, sum this column to calculate the net impulse due to the vertical force.
12. Calculate the net angular impulse by calculating the area under the $\Sigma \mathrm{M}$-time curve. Use the trapezoidal rule. Then, sum this column to calculate the net impulse due to the both horizontal and vertical forces.
13. Sum the angular impulse due to Mcm due to Fx and angular impulse due to Mcm due to Fy at every instant in time and take a sum of this column. Confirm that this value is the same as the value found in step 12.
14. Copy and paste your formulas onto the back and reverse sheets to repeat the calculations from steps 2-13. Because each dive has a different number of frames, you will have to make some adjustments to how far down your formulas go and what rows you are summing to get the impulse values.
15. Plot the moment-time curves for the back and reverse. The graph should include Mcm due to Fx, Mcm due to Fy, and $\Sigma \mathrm{M}$ data on the y axis and time on the x axis.

## Lab \#5 - Analysis and Questions:

1. Create a table that includes Angular Impulse: due to Fx, due to Fy, and Net for the Back, Back Timer, and Reverse Dives.

|  | Ang. Imp. due to Fx | Ang. Imp. Due to Fy | Net Ang. Imp |
| :--- | :--- | :--- | :--- |
| Back Timer |  |  |  |
| Back |  |  |  |
| Reverse |  |  |  |

2. What is the overall rotational effect of the horizontal force observed during the back timer rotating dive?
3. What is the overall rotational effect of the vertical force observed during the back timer rotating dive?
4. What is the overall rotational effect of the horizontal force observed during the back rotating dive?
5. What is the overall rotational effect of the vertical force observed during the back rotating dive?
6. What is the overall rotational effect of the horizontal force observed during the reverse rotating dive?
7. What is the overall rotational effect of the vertical force observed during the reverse rotating dive?
8. What would be a major risk if the diver reduces his/her horizontal reaction force during the reverse rotating dive takeoff?
9. Does the back timer replicate the angular impulse generation requirements of a back dive: load tip, push?
10. Compare the angular impulse generated in the reverse and back dives. Discuss rotational and how it is related to the mechanical objective of the dives.
11. Draw the ways these cases can be accomplished in free body diagrams: moment due to Rv is positive, moment due to $R v$ is zero, moment due to $R v$ is negative, moment due to $R h$ is positive, moment due to $R h$ is negative. In a table, write down the times that this is occurring based on when dx changes from positive/negative and when Rh changes from positive to negative.

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12. The back timer should have a negative net angular impulse. Why does this make sense even though there is no negative rotation?
13. What would be the performance outcome if the diver reduces his vertical reaction force during the back rotating dive takeoff? Consider linear and angular effects. Include free body diagrams.
14. What would be the performance outcome if the diver reduces his vertical reaction force during the reverse dive takeoff? Consider linear and angular effects. Include free body.

