All-Units Discounts as a Partial Foreclosure Device

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Abstract

We investigate the strategic effects of volume-threshold based all-units discounts (AUDs) used by a dominant firm in the presence of a capacity-constrained rival. As compared to linear pricing, AUDs adopted by a dominant firm are shown to lead to “partial foreclosure” of an equally or more efficient rival, in the sense that the rival’s profits, sales volume, and market share are strictly reduced. When the rival’s capacity level is in the range of low values relative to the demand size, AUDs reduce the buyer’s surplus and increase total surplus. When the rival becomes more efficient, AUDs may reduce total surplus.

The intuition for our findings is that, due to the limited capacity of the rival, the dominant firm that has a “captive” portion of the buyer’s demand for a single product is able to use the AUD to leverage its market power from the “captive” portion to the “competitive” portion of the demand, much like the tied-in selling strategy in the context of multiple products. Our analysis applies to other similar settings, in which the dominant firm has some captive market when it offers “must-carry” brands or a wider range of products.

Keyword: all-units discounts, captive demand, partial foreclosure

JEL code: L13, L42

1 Introduction

All-units discounts (AUDs) are pricing schemes that lower a buyer’s per-unit price on every unit purchased when the buyer’s purchase exceeds or is equal to a pre-specified volume threshold. The AUDs and related conditional rebate schemes are commonly observed in both final-goods and intermediate-goods markets, and their adoption by dominant firms has become a prominent antitrust issue. For instance, in the well-known

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Tomra and Michelin II cases, “individualised retroactive rebate schemes” used by Tomra, and quantity rebates used by Michelin, were found to be exclusionary. The European Commission also has found loyalty discounts adopted by dominant firms to be anticompetitive in several other cases.

In all these antitrust cases, the dominant firm holds market power over part of the buyer’s demand, which is “captive” to the dominant firm. On the other hand, there is a “competitive” part of the buyer’s demand, for which the dominant firm faces competition. The major concern about the AUD scheme and its variations is their potential foreclosure effects on the “competitive” portion of the market. Intuitively, a larger firm may take advantage of its “captive” portion of the demand so to induce the buyer to purchase a significant portion of her requirements. This may cause small rivals to become even smaller because AUDs tend to limit their growth possibilities. Such a logic has been pointed out in some of the above cases, as well as by the European Commission. However, to the best of our knowledge, it has not been formalized in economic theory yet. In other words, we are still unclear about the mechanism through which the AUD scheme forecloses small rivals when it is adopted by a dominant firm, although intuition may suggest so.

Here, we propose a model to formalize the foreclosure idea and examine how AUDs can affect competition when a dominant firm has “captive” demand. In reality, the existence of a “captive” market depends on a variety of factors. For instance, competitors often have capacity constraints, as in the cases of Tomra, Michelin II and Intel; the dominant firm usually offers a “must-carry” brand to customers, as in the cases of Intel and 3M; and the dominant firm has a wider range of products than competitors could offer, as in the cases of 3M and Canada Pipe. Regardless of where the “captive” demand comes from, the important fact is that the small rival cannot compete for the entire demand of the buyer. To model a “small” rival in a simple way, we introduce capacity constraints on the rival, as this is an intuitive way of giving rise to the “captive” portion of the demand for a dominant firm. However, our results are robust to other factors that

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3 To name a few, Hoffmann-La Roche (Case 85/76, Hoffmann-La Roche & Co. AG v. Commission of the European Communities, Judgment of the Court of 13 February 1979), British Airways (Case C-95/04, British Airways plc v. Commission of the European Communities supported by Virgin Atlantic Airways Ltd., Judgment of the European Court of Justice, March 2007), and Intel (Case COMP/C-3/37.990——Intel (2009); Docket No. 9341, In the Matter of Intel Corporation (2010)).

4 For instance, “(a)ll RVM suppliers, apart from Tomra, at the time when the investigation took place were very small companies with a small number of employees and were active only in one country or a small number of EEA Contracting Parties” (Paragraph 20, Case COMP/E-1/38.113, Prokent-Tomra, Commission Decision 2006).

5 “At least in the short run, most if not all of the major OEMs must engage significantly with Intel because AMD is too small to service all their needs” (Paragraph 63, AMD v. Intel Complaint 2005).


9 LePage’s Inc. v. 3M, 324 F.3d 141 (3rd Cir. 2003), cert. denied, 124 S. Ct. 2932 (2004).

constrain the ability of a small rival to serve the buyer’s entire demand.

In particular, we investigate the strategic effects of volume-threshold based AUDs used by a dominant firm in the presence of a capacity-constrained rival. We consider a three-stage game with complete information in which the dominant firm and its rival have identical marginal costs and make sequential price offers to a buyer before the buyer purchases. We find that AUDs always increase the dominant firm’s profits, sales volume, and market share over linear pricing (LP) or two-part tariff (2PT). At the same time, AUDs adopted by the dominant firm lead to “partial foreclosure” of the rival, in the sense that the rival’s profits, sales volume, and market share are strictly reduced compared to LP. These results hold for any capacity level of the small rival. Moreover, even when the rival has a lower marginal cost than the dominant firm, the dominant firm still can partially foreclose the more efficient rival by using AUDs. When the rival’s capacity level is in the range of low values relative to the demand size, AUDs reduce the buyer’s surplus and increase total surplus. When the rival becomes more efficient, AUDs may reduce total surplus. Our analysis applies to other similar settings, in which the dominant firm has some captive market when it offers “must-carry” brands or a wider range of products.

The intuition for our findings is that, due to the limited capacity of the rival, the dominant firm that has a “captive” portion of the buyer’s demand for a single product is able to use AUDs to leverage its market power from the “captive” to the “competitive” portion of the demand, much like the tied-in selling strategy in the context of multiple products. Under AUD, the dominant firm always sets the quantity threshold so that it exceeds the “captive” demand size, together with a per-unit discount as an incentive. This beyond-captive-demand threshold encroaches on the competitive demand, and thus conflicts with the buyer’s intention to buy all competitive demand from the cheaper source (the small rival) only. The dominant firm is able to tie all the competitive demand to its captive demand. However, it is not in the dominant firm’s best interest to leverage its market power from the captive demand to the whole competitive demand, because doing so would leave the small rival no choice but to undercut to its marginal cost. That explains why we get the partial foreclosure result. By leaving part of the competitive demand to the rival, the dominant firm can induce more favorable responses from it, and earn higher profit than by fully excluding the rival. To ensure the leverage only affects part of, but not all of, the competitive demand, the dominant firm’s pricing scheme must entail a quantity threshold, above which and below which prices are different. Thus, the dominant firm can behave very aggressively before the buyer meets the quantity threshold but relatively softly after the threshold is met. This ensures the buyer purchases more than the captive demand, but still leaves some competitive demand to the small rival. Such strategic use of the dominant firm’s leverage is impossible under LP or 2PT, because under either pricing scheme there is only a single per-unit price applying to all units.

In AUDs, the quantity threshold and the corresponding discounted per-unit price act as a quasi-fixed fee with minimum quantity requirement. This feature leads to several interesting effects that are absent in LP or 2PT. First, AUDs have a quantity expansion effect. The quasi-fixed fee enables the dominant firm to extract surplus from the buyer more efficiently, and hence gives it more incentive to expand supply. Such a quantity expansion is carried out by a quantity threshold larger than the dominant firm’s captive demand. That quantity threshold encroaches on the competitive demand, which hurts the small rival but may increase
total surplus and buyer’s surplus. Second, the quasi-fixed fee has a surplus extraction effect. Because the dominant firm can extract buyer’s surplus better under AUD, AUDs may hurt the buyer. Depending on the competitive pressure from the small rival, e.g., the small rival’s capacity level, the surplus extraction effect may dominate the quantity expansion effect and reduce buyer’s surplus, and vice versa. Third, when the small rival is more efficient, the quantity expansion from the dominant firm can have a negative effect on social welfare: because the dominant firm sells more than its captive demand and forces the more efficient rival to supply less, more output is supplied by the less efficient dominant firm under AUD. Thus, the quantity expansion from the dominant firm may harm total surplus.

The literature on AUDs is sparse. Kolay, Shaffer and Ordover (2004)\cite{Kolay2004} study the price discrimination effect of AUDs offered by a monopolist when the downstream buyer has private information. They show that a menu of AUDs can generate higher profits for the monopolist than a menu of 2PTs. In a successive, bilateral monopolies setting, O’Brien (2013)\cite{O'Brien2013} shows that AUDs can facilitate non-contractible investments. Conlon and Mortimer (2013)\cite{Conlon2013} use data from Mars Inc. to empirically quantify welfare effects of AUDs. Feess and Wohlschlegel (2010)\cite{Feess2010}, in the spirit of Aghion and Bolton (1987)\cite{Aghion1987}, show that AUDs can shift the rent from the entrant to the coalition between the incumbent and the buyer. The crucial element needed for this rent-shifting idea to work is that the adversely affected third party must be absent from the bilateral contracting stage. However, the order of sequential moves in this standard literature of rent-shifting and exclusion might not be consistent with some well-known antitrust cases, where the alleged victims of the exclusionary strategies were already active in the market and could make counteroffers before the buyer could make any purchase.

By contrast, we consider a model in which the competitor is already active in the market and can respond to the dominant firm’s pricing scheme with a counteroffer before the buyer makes her purchase decision. In particular, we consider a model with two firms, firms 1 and 2, in the upstream market, producing identical products with the same marginal cost. There is a representative buyer in the downstream. We assume complete information, between firms and the buyer, to prevent price discrimination from being a plausible explanation for AUDs. The game is a three-stage sequential-move game in which firms 1 and 2 make offers to the buyer sequentially, and the buyer does not make any binding purchase decision until the last stage. This order of moves automatically excludes the rent shifting possibility between the buyer and any seller, because neither contract is binding unless the buyer purchases from it in the last stage. We provide a new rationale for AUDs in the absence of price discrimination, incentivizing investment, or rent-shifting motives in the literature.

There is a small body of literature on exclusionary contracts with competition between asymmetric firms. Greenlee, Reitman and Sibley (2008)\cite{Greenlee2008} study bundled loyalty discounts, which requires complete loyalty from consumers when they purchase the tied good in their settings. We consider a single-product model, and our optimal AUDs only require certain amount, not all, of buyer’s purchases. Ordover and Shaffer (2013)\cite{Ordover2013} consider exclusionary discounts in a two-period model, where one firm is financially constrained, and the buyer incurs switching costs after her first period purchase. They find that the unconstrained firm can exclude the constrained firm by locking in the buyer with a below-cost price for their second period demand. Our model departs from theirs because we consider a one-time purchase from the buyer, and thus there is no
switching cost or externality across periods. DeGraba (2013)\cite{10} considers naked exclusive contracts when a dominant firm competes against a small rival with downstream competition. He shows that the large firm can bribe downstream firms for exclusivity, provided that the size difference between the large firm and the small firm is sufficiently large. We consider a different model with no downstream competition and do not allow upstream firms to pay the buyer directly for exclusivity. And we find that AUDs can have a partial foreclosure effect for any capacity difference between the large firm and small firm.

Another related literature is the market-share discounts, where discounts are conditional on a seller’s percentage share of a buyer’s total purchases, instead of an absolute quantity. Majumdar and Shaffer (2009)\cite{19} explain how the market-share discounts can create countervailing incentives for a retailer with private information on demand, when it buys from a dominant firm and competitive fringes. Inderst and Shaffer (2010)\cite{16} point out that the market-share discounts can dampen both intra- and inter-brand competition at the same time. Mills (2010)\cite{22} suggests the market-share discounts can induce non-contractible effort from retailers when their sizes are different, but optimal effort levels are proportional to their sizes. Calzolari and Denicolo (2013)\cite{5} show that the market-share discounts can be anticompetitive when buyers have private information. Chen and Shaffer (2013)\cite{7} study exclusionary contracts with minimum-share requirements. They find that the less than 100% share requirement may be more effective in deterring entry than a 100% naked exclusionary contract. The game in Chen and Shaffer (2013)\cite{7} proceeds as in Rasmusen et al. (1991)\cite{27} and Segal and Whinston (2000)\cite{28}, where the incumbent and buyers can sign contracts before the potential entrant enters. Our model differs from theirs in two important respects. First, we abstract away from downstream competition. Second, in our model, the small firm is already in the market, and it can make a counteroffer before the buyer makes her purchase decision. As a complement to those mentioned above, our article suggests that we should place a cautious eye on those volume- or share-threshold based contracts when they are adopted by a dominant firm.

The remainder of the article is organized as follows. In Section 2, we set up the model. Section 3 presents two benchmark cases, in which the leading firm can only offer LP or a 2PT. Section 4 offers an intuition of how AUDs could improve the leading firm’s profits by tying captive portion to competitive portion. Sections 5 presents the equilibrium analysis of AUDs. In Section 6, we use linear demand examples to illustrate properties of the equilibria. In Section 7, we discuss some assumptions of the model. The article closes in Section 8 with some concluding remarks. All proofs are relegated to the Appendix.

2 Model Setting

In our model, we consider simple AUDs which consist of a triple \((p_0, Q, p_1)\) with \(p_0 > p_1\) and \(Q > 0\). Here \(p_0\) is the per-unit price when the quantity purchased is less than the quantity threshold \(Q\), and \(p_1\) is the per-unit price for all units once the quantity purchased reaches \(Q\). In other words, the AUDs are pricing schemes that reward a buyer for purchasing some threshold quantity from a firm\footnote{In practices, multiple volume thresholds are often observed, but we focus on a single volume threshold case. This is because we consider a complete information setting, and it is unnecessary to offer more than one threshold in equilibrium.}. In particular, the total

\[\text{Total Profit = Revenue - Cost} = (p_0 \times \min(Q, Q_{\text{threshold}}) + p_1 \times (Q - \min(Q, Q_{\text{threshold}}))) - (\text{variable cost})\]

\[\text{where } Q_{\text{threshold}} = \text{the minimum threshold quantity for which } p_1 \text{ applies.}\]
payment schedule under AUD is

\[ T(q) = \begin{cases} 
  p_0 \cdot q & \text{if } q < Q \\
  p_1 \cdot q & \text{if } q \geq Q
\end{cases} \]

It is illustrated in Figure 1, where the horizontal axis represents quantity and the vertical axis represents total payment from the buyer to the seller. Note that the AUD schedule is initially increasing in quantity, has a sharp drop right at the threshold quantity, and then increases with quantity again at a lower rate.

There are two firms, say firm 1 and firm 2, in the upstream market that produce identical products with the same marginal cost \( c \geq 0 \). In order to examine strategic effects of AUDs when a dominant firm competes against a smaller firm, we introduce an asymmetry between the two firms into the model—capacity constraints for the small firm. Specifically, firm 1 has full capacity to serve the whole demand of the buyer, whereas firm 2 is capacity-constrained in the sense that it can produce at marginal cost \( c \) up to its capacity \( k \). Note that the “capacity constraints” here do not have to be interpreted literally as the production capacity limit. The small rival can be constrained because of a “must-have” brand or a wider product line from the dominant firm.

In the downstream, there are a large number of buyers, each of whom is a local monopoly in selling to final consumers, due to local brand names or other attributes of product differentiation. Although each buyer is a local monopoly, none of them has monopsony power. This is because either each of them has only a small share of the whole market, or the number of upstream supplies is quite limited compared with the downstream demand. Moreover, we assume complete information about the demands in every market,

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7“Tomra’s rivals, including those who had the potential to become strong competitors, were all small or very small companies, with a very low turnover and very few employees” (Paragraph 85, Case COMP/E-1/38.113, Prokent-Tomra, Commission Decision 2006). In ZF Merito v. Eaton case, “even if an OEM decided to forgo the rebates and purchase a significant portion of its requirement from another supplier, there would still have been a significant demand from truck buyers for Eaton product. Therefore, losing Eaton as a supplier was not an option.” (D.C. No. 1-06-cv-00623). In the Intel case, it is widely known that AMD is capacity constrained, because large computer manufacturers have to carry a significant proportion of their CPU requirements from Intel.

8Such market structure, where there are a large number of buyers whereas only few sellers, is consistent with many antitrust cases in which contracts offered by the dominant upstream firm give rise to abuse of dominance concern, because otherwise the large buyer power can be a countervailing force to discipline upstream suppliers’ abuse of power. Mathewson and Winter (1987) made such an assumption when studying exclusive dealing.
and two manufacturers make customized offers to each local monopoly retailer. Therefore, without loss of generality, we can consider a representative buyer with a gross utility function denoted as $u(q)$.

This set up has the following interpretations. As our objective here is to see if AUDs can have any strategic effects purely coming from upstream competition, we want to rule out any other motives as best as we can. The assumption of one representative buyer helps us to abstract away from strategic interactions resulting from downstream competition. In addition, the complete information assumption in the model prevents price discrimination from being a plausible explanation for AUDs. Our point is that, even in the absence of downstream competition or asymmetric information, AUDs have some strategic effects on competition, and their competitive effects can be different and significant, depending on the rival firm’s capacity level.

We model the interactions between the firms and the buyer as a sequential-move game with three stages. In the first stage, firm 1 offers a pricing scheme to the buyer, which could be LP, a 2PT, or the AUDs. In the second stage, after observing the pricing scheme from firm 1, firm 2 sets its per-unit price for the buyer. In the third stage, the buyer decides where and how many units to purchase. In our setting, we assume firm 2 can only use LP in order to capture the fact that smaller firms in reality usually cannot match the pricing scheme as complex as offered by a dominant firm. It is worth noting that the buyer here can purchase from both firms. For completeness, we assume that in the event of a tie when the two firms offer the same surplus to the buyer, the buyer will buy from firm 2 with an attempt to fulfill $Q$ (if any) if possible. The game’s timeline is described in Figure 2.

![Figure 2: The Timeline of the Game](image)

The assumption that the buyer does not make any decision until two competing offers are on the table is to capture the contestable conditions in favor of the buyer. It is worth noting that the nature of the sequential-move game in our model is different from that first introduced by Aghion and Bolton (1987) and then extended by Marx and Shaffer (2004). In their models, the buyer has to decide whether to accept firm 1’s offer or not before seeing firm 2’s offer. Once firm 1’s offer is accepted, it becomes binding for both firm 1 and the buyer. This is crucial for rent-shifting, which is from firm 2 to firm 1 and the buyer, to occur. The sequence of moves in their setting allows the buyer to commit to pay firm 1 even if there is no trade between them, and hence such a payment is credible when the buyer meets firm 2 after accepting firm 1’s offer. So the absence of firm 2 or its inability to make a counteroffer before the buyer accepts firm 1’s is

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9.“As it is the case in relation to quantity commitments the rebate schemes applied by Tomra constitute individualised targets rather than a generalised grid” (Paragraph 133, Case COMP/E-1/38.113, Prokent-Tomra, Commission Decision 2006).

10.We are not aware of any evidence that small firms used AUD contracts in the Tomra case or other cases involving dominant firms using loyalty discounts. There likely are several reasons for this empirical fact, such as monitoring cost, management cost, enforcement cost.

11.This tie-breaking rule is used to avoid the need to consider a situation in which the follower charges a price arbitrarily close to, but below the leader’s price.
where the contracting externality in their models comes from. However, such an order of sequential moves is inconsistent with some well-known antitrust cases, such as in Tomra, Michelin II, Intel, Canada Pipe, Microsoft, and 3M cases, where the alleged victims of the exclusionary strategies were already active in the market and could make counteroffers before the buyer making any purchase. On the contrary, the order of moves in our setting automatically excludes the possibility of rent-shifting between the buyer and any firm, because neither contract is binding for the buyer until the buyer purchases from it in the last stage. And we allow the small firm to respond to the dominant firm’s pricing scheme with counteroffers before the buyer makes a purchase decision.

In addition, the equilibrium strategies are renegotiation-proof by nature of the timing because the buyer doesn’t commit to any contract before both manufacturers make offers. Our article shows that even in this substantially competitive environment at upstream level, AUDs still have some interesting strategic effects.

We assume the buyer’s utility function \( u(q) \) is increasing and concave for any quantity below a satiation quantity \( q^S \), where \( u'(q^S) = 0 \) and \( 0 < q^S \leq \infty \), and \( u'(0) > c \). Let the optimal quantity demanded by the buyer at per-unit price \( p \) be \( q(p) \equiv \arg\max_{q > 0} [u(q) - p \cdot q] \). With \( u''(q) < 0 \), \( q(p) \) exists and is uniquely determined by \( u'(q) = p \) for \( c \leq p \leq u'(0) \). Let \( v(p) \equiv u(q(p)) - p \cdot q(p) \) be the buyer’s surplus when she purchases optimally at per-unit price \( p \).

We assume that, firm 2’s capacity level is strictly less than the socially efficient level of quantities, i.e., \( 0 < k < q(c) \), implying that firm 2 cannot serve the whole demand of the buyer when two firms compete à la Bertrand. We can consider \( q(c) - k \) as firm 1’s “captive” demand. It is the residual demand for firm 1 when firms compete in prices up to the marginal cost. This is also the maximum demand left for firm 1 if firm 2 supplies at its full capacity \( k \). Correspondingly, the “competitive” portion is \( k \), for which both firms compete.

Let the monopoly profit at per-unit price \( p \) be \( \pi(p) \equiv (p - c) \cdot q(p) \). To facilitate our analysis, we assume the monopoly profit function to be concave. Denote \( p^m \equiv \arg\max_p \pi(p) \) as the monopoly price, and \( q^m \equiv q(p^m) \) as the monopoly quantity. In addition, let

\[
\pi^R(Q) \equiv \max_p (p - c) \cdot [q(p) - Q]
\]

for \( 0 \leq Q \leq q(c) \) be the maximum profit based on the residual demand \( q(p) - Q \). One can readily verify that \( \pi^R(Q) \) is strictly decreasing and convex in \( Q \in [0, q(c)] \). From the concavity of \( \pi(p) \) and the fact that \( \pi'(c) = q(c) \geq Q \) and \( \pi'(p^m) = 0 \leq Q \), it follows that there exists a unique \( p^R(Q) \equiv \arg\max_p (p - c) \cdot [q(p) - Q] \in [c, p^m] \) for \( 0 \leq Q \leq q(c) \), where

\[
\pi'(p^R) = Q,
\]

and \( p^R(Q) \) is strictly decreasing in \( Q \).

In the rest of our article, we will determine the subgame perfect equilibrium outcome of the sequential-move game, allowing the dominant firm to choose LP, a 2PT, and the AUDs, respectively, and we will

\footnote{Spector (2011)[29] emphasized this, too, when considering exclusive contracts. In a setting with economies of scale, he showed eviction can occur even if the excluded firm is present and can make counteroffers. We do not assume economies of scale.}
compare the equilibrium outcomes.

3 Two Benchmarks

In the first benchmark, the dominant firm can offer LP only.

**Proposition 1 (LP vs LP Equilibrium)** (i) The LP equilibrium is uniquely characterized by $p_1^{LP} = p_2^{LP} = p^R(k) \in (c, p^m)$, where $p^R(\cdot)$ is given by (1).

(ii) In the LP equilibrium, firm 1 earns $\pi_1^{LP} = \pi_R(k)$ with sales $q_1^{LP} = q(p^R) - k$; firm 2 earns $\pi_2^{LP} = (p^R - c) \cdot k$ with sales $q_2^{LP} = k$; the buyer’s surplus is $BS^{LP} = v(p^R)$.

This proposition states that, when firm 1 is restricted to LP, it has to leave firm 2 its capacity $k$ and only focuses on the residual demand $q(p) - k$. The per-unit price from firm 1, which is available for the buyer’s whole demand, forces firm 2 to undercut it, because otherwise firm 2 would have no sales. Once firm 2 undercuts, the buyer will consider firm 1’s supply only after exhausting firm 2’s capacity.

An immediate result following from Proposition 1 is the comparative statics below.

**Corollary 1** For $k \in (0, q(c))$, as $k$ increases, $p^R(k)$ decreases, $BS^{LP}$ increases, and $\pi_1^{LP}$ decreases.

As firm 2’s capacity $k$ increases, competition becomes more intensive, from which the buyer benefits and firm 1 gets hurt. However, firm 2’s profit is not necessarily monotonic in $k$, because there are two opposing effects on its price and sales, respectively: $p^R$ falls whereas $k$ rises. Indeed, firm 2’s profit increases with $k$ when $k$ is small, whereas it decreases with $k$ when $k$ is large.

In order to see whether AUDs can outperform a classical form of nonlinear price, we consider the second benchmark in which the dominant firm offers a 2PT, say a pair $(T_1, p_1)$, where $T_1$ is a fixed fee and $p_1$ is a per-unit price.

**Proposition 2 (2PT vs LP Equilibrium)** (i) The 2PT equilibrium is uniquely characterized by

$$p_1^{2PT} = c, \quad T_1^{2PT} = v(c) - [u(k) - c \cdot k]; \quad p_2^{2PT} = c.$$

(ii) In the 2PT equilibrium, firm 1 earns $\pi_1^{2PT} = v(c) - [u(k) - c \cdot k]$ with sales $q_1^{2PT} = q(c) - k$; firm 2 earns $\pi_2^{2PT} = 0$ with sales $q_2^{2PT} = k$; the buyer’s surplus is $BS^{2PT} = u(k) - c \cdot k$.

This proposition says that, when firm 1 uses a 2PT, it still leaves firm 2 its full capacity $k$, as in the LP equilibrium. The difference is that firm 1 now can extract all the surplus from the residual demand through the fixed fee. Therefore, firm 1 has an incentive to ensure that the total surplus is maximized so that the incremental surplus for it to extract is maximized, too.

In both LP and 2PT equilibrium, because the per-unit prices from both firms are applicable to any unit without any restriction, the buyer can freely allocate her purchases between the two firms, and thus will
never resort to the more expensive source unless the cheaper source is exhausted. This forces two firms to compete for the first \( k \) units. Thus, firm 2, as a follower, always undercuts and serves the first \( k \) units of the buyer’s demand, and firm 1 supplies the residual demand after \( k \) in both benchmarks.

4 Intuition for the AUDs

Due to an extra instrument of fixed fee, 2PT allows firm 1 to extract more surplus than LP does. Indeed, from Proposition\(^2\), the equilibrium outcome under 2PT is efficient, and firm 1 has extracted the full surplus from its captive portion \( q(c) - k \). It seems that it is impossible for firm 1 to do better, given that total pie has been maximized and firm 1 has grabbed all the surplus over its captive portion. Can firm 1 do even better than using 2PT in the current setting?

Note that in the 2PT equilibrium, the sum of firm 2’s profit and buyer’s surplus is \( \pi_{2PT} + BS_{2PT} = u(k) - c \cdot k \). Given that total pie is already maximized under 2PT at \( v(c) \), if firm 1 wants to gain more profit than \( \pi_{2PT} = v(c) - [u(k) - c \cdot k] \), then the sum of firm 2’s profit and buyer’s surplus must be strictly less than \( u(k) - c \cdot k \). Notice that whenever the buyer purchases \( k \) units from firm 2, the surplus sum they can achieve must be at least \( u(k) - c \cdot k \). Consequently, in order to further increase its profit over a 2PT, it is necessary for firm 1 to encroach into the competitive portion and prevent firm 2 from selling at its full capacity \( k \).

To prevent firm 2 from selling its full capacity, firm 1 must induce the buyer to buy firm 2’s product only after buying certain amount from firm 1, because otherwise the buyer won’t buy from firm 1 until exhausting firm 2’s capacity \( k \). As a result, firm 1 must commit to a minimum quantity requirement more than its captive portion so that the residual demand for firm 2 is less than \( k \). For such a quantity requirement to be accepted by the buyer, firm 1 must tie its captive portion to competitive portion, and design its pricing scheme in such a way that the buyer cannot afford to lose firm 1 as a supplier. Consequently, firm 2 now, instead of firm 1, becomes a supplier for residual demand.

So the intuition for how the AUD scheme works is through its distinct element—–quantity threshold, compared with LP or a 2PT. Given that a buyer has no choice but to purchase some portion, although not all, of her requirement from the dominant firm, the dominant firm can set its quantity threshold above its captive portion and induce the buyer to buy more from it and less from its rival.

A Simple Example (Step-function Demand)

In what follows, we provide a simple example to illustrate how tying captive portion to competitive portion can help firm 1 to achieve higher profits and partially foreclose its rival.

Suppose the buyer demands at most 10 units, with willingness-to-pay (WTP) for the 1st unit as 10, for the 2nd unit as 9, for the 3rd unit as 8, and so on (see Figure 3). Assume two firms produce identical products with zero marginal cost. Firm 1 can serve at least 10 units, and firm 2 can produce at most \( k = 2 \) units. It is easy to see that the total surplus is \( 10 + 9 + 8 + \ldots + 1 = 55 \).

\(^{13}\)This is equivalent to say that, under LP and 2PT, the buyer’s demands are fulfilled with efficient rationing because this rationing maximizes her surplus. Efficient rationing is assumed in Kreps and Scheinkman (1983)\(^{18}\), and random rationing is assumed in Boyer and Moreaux (1988\(^{3}\), 1989\(^{4}\)). For a detailed discussion on different rationing rules, see Davidson and Deneckere (1986)\(^{9}\).
Under LP, firm 2 undercuts firm 1’s per-unit price and serves the first 2 units of the buyer’s demand. As a result, firm 1 sets a monopoly per-unit price over the residual demand with WTPs starting from 8 to 1, which is 4 (or 5). In equilibrium, firm 1 sells 5 (or 4) units, firm 2 sells 2 units. Firm 1 earns a profit of 20, and firm 2 earns 8 (or 10). The buyer gets 21 (or 15).

Under 2PT, firm 2 undercuts firm 1’s per-unit price and serves the first 2 units of the buyer’s demand as under LP. But now firm 1 can use a fixed fee to extract all the surplus from its sales. Its potential sales are the remaining 8 units with total WTPs of $8 + 7 + 6 + \ldots + 1 = 36$. To maximize the surplus it can extract, firm 1 will set its per-unit price at marginal cost 0 to maximize its sales and then use a fixed fee to extract all the surplus 36 from the last 8 units. Hence, firm 2 has to set its per-unit price at 0 and earns 0 profit. The buyer gets surplus 19 from the first 2 units. Note that in both LP and 2PT equilibria, firm 2 sells at its full capacity $k = 2$, and the 2PT equilibrium is efficient.

Now, suppose firm 1 uses AUDs with a volume threshold $Q = 9$, $p_o = 10$, and $p_1 = \frac{36.5}{9}$. Given the AUDs and $p_2$ from firm 2, the buyer has to make a choice between meeting $Q$ and not meeting $Q$. Meeting $Q$ means that the buyer will buy 9 units from firm 1 and the last one unit from firm 2, which results in buyer’s surplus $BS^{DS} = (10 + 9 + 8 + \ldots + 1) - \frac{36.5}{9} \cdot 9 - p_2 = 18.5 - p_2$. Not meeting $Q$ implies that the buyer has to rely on firm 2 only, because it is not worth buying at $p_o = 10$ from firm 1, which leaves buyer’s surplus $BS^{SS} = (10 + 9) - 2p_2 = 19 - 2p_2$. Clearly, the buyer will meet $Q$ if and only if $BS^{DS} = 18.5 - p_2 \geq 19 - 2p_2 = BS^{SS}$, i.e., $p_2 \geq 0.5$. As a result, if firm 2 wants to sell at its full capacity $k = 2$, then it has to undercut below 0.5. So the maximal profit it can achieve when selling at 2 units is $0.5 \times 2 = 1$. Nevertheless, if firm 2 sets $p_2 = 1$, then it still can get $1 \times 1 = 1$, although it only sells one unit. So inducing the buyer to meet its quantity threshold $Q$ is possible for firm 1. In equilibrium, firm 1 will set $Q = 9$ with per-unit price $p_o = 10$ and discounted per-unit price $p_1 = \frac{36.5}{9}$, and it earns a profit of 36.5, which exceeds what it earns if it chooses either LP or 2PT. Correspondingly, firm 2 sets its per-unit price $p_2^{AUD} = 1$ and earns a profit of 1, which is lower than what it earns in the case of LP. On the other hand, the buyer gets 17.5, which is lower than what she receives in the case of LP or 2PT.

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14Here DS is short for dual sourcing, and SS stands for single sourcing.
This example illustrates that, by setting a quantity threshold above its captive portion together with a carefully chosen per-unit price, firm 1 can successfully encroach on the competitive portion and earn a higher profit than both LP and 2PT. Meanwhile, firm 2 is forced to undersupply, which never occurs under LP or 2PT.

5 Equilibrium Analysis of the AUDs

In this section, we characterize the equilibrium when firm 1 offers the AUD scheme. We find that the AUDs always increase firm 1’s profit and market share, and induce firm 2 to undersupply below its capacity level. There exists a threshold of capacity level below which both firm 2 and the buyer are worse off under AUD than under LP.

We solve our sequential-move game by backward induction. It turns out that the determination of the leader’s optimal AUDs can be reduced to a mechanism design problem. In particular, by judiciously choosing a quantity threshold together with a payment structure, the leading firm induces the buyer to reach the threshold and firm 2 to be indifferent between supplying the residual demand at a higher price and being a sole supplier by undercutting (firm 2 and the buyer’s outside option). Through this way, the leading firm can leverage its market power in its captive market to the competitive part for which the smaller firm would otherwise be interested in competing.

Below we will first present several lemmas, which offer a set of necessary conditions for equilibrium. The logic is supported by iterated elimination of dominated strategies using firm 1 and firm 2’s forward thinking. We will then formulate firm 1’s maximization problem and characterize the equilibrium.
5.1 Buyer’s Problem: Single-Sourcing or Dual-Sourcing

We begin with analyzing the buyer’s purchase decisions in the last stage of the game. Given the AUDs \((p_o, Q, p_1)\) offered by firm 1, and a uniform price \(p_2\) from firm 2, the buyer’s maximization problem

\[
\max_{q_1, q_2 \leq k} [u(q_1 + q_2) - T(q_1) - p_2 \cdot q_2]
\]

can be decomposed into the following two maximization problems. The first one is given by

\[
\max_{q_1 < Q, q_2 \leq k} [u(q_1 + q_2) - p_o \cdot q_1 - p_2 \cdot q_2], \tag{2}
\]

which represents the case when the buyer does not meet firm 1’s volume threshold \(Q\). The second one is given by

\[
\max_{\Delta \geq 0, q_2 \leq k} [u(Q + \Delta + q_2) - p_1 \cdot (Q + \Delta) - p_2 \cdot q_2], \tag{3}
\]

which represents the case when the buyer meets firm 1’s volume threshold \(Q\). The buyer chooses one of the two options that gives her higher surplus.

**Single Sourcing from Firm 2.** In order for the AUDs to improve firm 1’s profit over LP, the buyer must meet firm 1’s volume threshold \(Q\) in the AUD equilibrium. This is because the outcome of (2) can always be achieved by LP \((p_o)\) vs LP \((p_2)\). Therefore, firm 1 does not want the buyer to choose (2) in equilibrium, and it is without loss of generality to restrict our attention to \(p_o = \infty\). In what follows, we use \((Q, p_1)\) to denote the AUD scheme.

As a result of sufficiently high \(p_o\), (2) is reduced to

\[
\max_{q_2 \leq k} [u(q_2) - p_2 \cdot q_2], \tag{SS}
\]

which represents *single-sourcing (SS)* when the buyer does not meet firm 1’s volume threshold and thus purchases from firm 2 only. That is, under AUD, if the buyer decides not to meet \(Q\), she essentially chooses SS from firm 2.

The solution to (SS) problem serves as an outside option for firm 2 as well as for the buyer. Denote the buyer’s demand under SS as \(\bar{q}(k, p_2) \equiv \min\{k, q(p_2)\}\). We can write the buyer’s surplus under SS as

\[
BS_{SS}(p_2) = u(\bar{q}(k, p_2)) - p_2 \cdot \bar{q}(k, p_2). \tag{4}
\]

The two firms’ profits under SS are \(\pi_1 = 0\) and

\[
\pi_2 = (p_2 - c) \cdot \bar{q}(k, p_2). \tag{5}
\]

\[15\] Here \(p_o\) does not have to be \(\infty\), literally. In fact, we only need \(p_o\) to be above a certain level in equilibrium, ensuring that any amount below \(Q\) from firm 1 is never optimal for the buyer.

\[16\] Note that there is another kind of SS in which the buyer only purchases from firm 1. However, as will be shown in the proof of Lemma 1, introducing the buyer SS from firm 1 only can at most give firm 1 the 2PT equilibrium profit.
Dual Sourcing. Now we study (3) carefully, as this is the case that emerges in equilibrium.

Under (3), after the buyer meets firm 1’s volume threshold, she continues to buy from the cheaper source, as long as her marginal utility is above the corresponding price. Thus, in order to have positive sales, firm 2 as a follower must always set \( p_2 \leq w \equiv \min\{p_1, u'(Q)\} \) as long as \( c < w \). As a result, the buyer buys exactly \( Q \) units from firm 1 and her residual demand from firm 2. With \( p_2 \leq w \), (3) will be reduced to

\[
\max_{q_2 \leq k} [u(Q + q_2) - p_1 \cdot Q - p_2 \cdot q_2], \tag{DS}
\]

which represents dual-sourcing (DS) when the buyer meets firm 1’s volume threshold and continues to purchase her remaining demand from firm 2.

Under DS, firm 1 would never allow the buyer to have the freedom to purchase \( k \) units from firm 2 without interfering with meeting its \( Q \) requirement, when firm 2 simply matches firm 1’s price \( p_1 \). That is, we cannot have \( p_1 \leq u'(Q + k) \), because \( q(p_1) \geq Q + k \) and \( p_2 \leq w \) together imply that the buyer can meet \( Q \) even after purchasing \( k \) units from firm 2 first, which cannot be a profitable improvement over a 2PT for firm 1. Hence, we must have \( u'(Q + k) < p_1 \), and it follows that \( u'(Q + k) < w \).

Because \( u'(Q + k) < w \), the buyer’s purchase when \( p_2 \leq w \) will be \( q(Q + k, p_2) = \min\{Q + k, q(p_2)\} \).

So the buyer’s surplus in (3) is

\[
BS_D(p_2) = \begin{cases} 
  u(Q + p_2) - p_1 \cdot Q - p_2 \cdot q_2 & \text{if } p_2 \leq w \\
  u(q(w)) - p_1 \cdot q(w) & \text{if } w < p_2
\end{cases} \tag{6}
\]

The two firms’ profits from (3) are

\[
\pi_1 = \begin{cases} 
  (p_1 - c) \cdot Q & \text{if } p_2 \leq w \\
  (p_1 - c) \cdot q(w) & \text{if } w < p_2
\end{cases}, \tag{7}
\]

and

\[
\pi_2 = (p_2 - c) \cdot [\bar{q}(Q + k, p_2) - Q] \tag{8}
\]

for \( p_2 \leq w \), and 0 otherwise.

Single Sourcing or Dual Sourcing?

As firm 1 would have no sales under SS in order for firm 1 to earn positive profit, it must ensure the buyer to choose DS under AUD. The following lemma shows that the buyer will meet firm 1’s quantity threshold \( Q \) in the AUD equilibrium, and firm 2 will supply too, but at a level strictly below its capacity \( k \).

**Lemma 1 (Firm 1 must induce DS and firm 2 undersupplies)** In the AUD equilibrium, (i) \( q_1 = Q \in (0, q(c)) \); (ii) \( 0 < q(p_2) - Q < k \).

Lemma 1 tells us that, in the AUD equilibrium, the buyer will buy from both firms—\( Q \) from firm 1 and \( q(p_2) - Q \) from firm 2. So firm 2 becomes a residual demand supplier after \( Q \). Note that after the buyer fulfills firm 1’s threshold \( Q \), firm 2 will always set \( p_2 < u'(Q) \), because otherwise the buyer would never buy anything from firm 2 in DS. So \( Q < q(p_2) \) indicates that firm 1 will leave some demand for firm 2 under
AUD. But at the same time firm 1 contains firm 2. \( q(p_2) - Q < k \) implies that in the AUD equilibrium, firm 2 strictly undersupplies as a residual demand supplier. This contrasts remarkably with the case of LP or a 2PT, where firm 2 always supplies its full capacity.

We now discuss two price constraints imposed by the equilibrium AUDs. First, due to the availability of \( p_1 \) for incremental demand, firm 2 faces one more constraint \( p_2 \leq p_1 \). Second, in the AUD equilibrium, \( p_1 \) cannot be set too high, i.e., \( p_1 < u'(k) \), because otherwise the buyer always chooses SS when \( p_2 \leq p_1 \). They are highlighted in the lemma below.

**Lemma 2 (Price Constraints Under AUD)** *The equilibrium AUDs \((Q, p_1)\) need to satisfy the following two constraints:*

\[
p_1 < u'(k), \tag{C1}
\]

*and*

\[
p_2 \leq p_1. \tag{C2}
\]

### 5.2 Firm 2’s Implied Threat Price

From (4) and (6), the buyer’s surplus curves under both SS and DS weakly decrease with \( p_2 \), and \( BS_S \) curve as a function of \( p_2 \) is everywhere no flatter than \( BS_D \) curve. Intuitively, the impact of \( p_2 \) on \( BS_S \) is larger than that on \( BS_D \), because firm 2 is the sole supplier under SS whereas firm 1, as a substitute supplier, becomes available under DS.

![Figure 5: Buyer’s Surpluses under AUD](image-url)

If \( BS_D \) is everywhere below \( BS_S \), then the buyer would never choose DS. But if \( BS_D \) is everywhere above \( BS_S \), it is not optimal for firm 1, either. Note that \( BS_D \) decreases with \( p_1 \cdot Q \). Whenever \( BS_D \) is everywhere above \( BS_S \), although the buyer will choose DS, firm 1 can always increase its profit by increasing \( p_1 \cdot Q \). Hence, \( BS_D \) and \( BS_S \) must cross once, as shown in Figure 5. Such a unique crossing point is firm 2’s threat price to undercut and induce SS.
Lemma 3 (Firm 2’s equilibrium threat price) In the AUD equilibrium, there exists a unique \( x \in (u'(Q + k), w) \) determined by

\[
u(k) - x \cdot k = v(x) + (x - p_1) \cdot Q,
\]

such that \( BS_S(p_2) \sube BS_D(p_2), \forall p_2 \leq x. \)

The left-hand side (LHS) of (9) is \( BS_S \) at \( p_2 = x \) when buying \( k \) from firm 2 only. The right-hand side (RHS) of (9) is \( BS_D \) at \( p_2 = x \) when buying \( Q \) from firm 1 and residual demand \( q(x) - Q \) from firm 2. The condition (9) uniquely determines such \( x \) at which the buyer is indifferent between SS and DS, given \((Q,p_1)\).

Given the AUDs \((Q,p_1)\) from firm 1, firm 2 can always induce the buyer to choose SS by undercutting sufficiently. The upper bound of such an undercutting threshold for SS is threat price \( x \). That is, if firm 2 charges a price below \( x \), the buyer will choose SS from firm 2 only for \( k \). If firm 2’s price is above \( x \), the buyer will choose DS.

Now we can see firm 2’s trade-offs introduced by the AUDs. Such trade-offs are absent under LP or a 2PT. Under LP or a 2PT, firm 2’s only viable option is to undercut or match firm 1’s per-unit price \( p_1 \), as \( p_1 \) is uniformly applied to all units supplied by firm 1. Nonetheless, with the quantity requirement \( Q \), firm 1 commits to supply only \( Q \) units with a payment \( p_1 \cdot Q \) as long as \( p_2 \leq w \), and thus creates trade-offs for firm 2: undercuts below \( x \) to be a monopoly supplier, or instead charges a price above \( x \) to supply the residual demand beyond \( Q \). So the most firm 1 can extract using \( p_1 \cdot Q \) is the incremental surplus the buyer and firm 1 as a coalition can gain over the buyer’s outside option of SS from firm 2 only, when firm 2 undercuts at \( x \).

From (9), the total payment \( p_1 \cdot Q \) to firm 1 is determined as

\[
p_1 \cdot Q = v(x) + x \cdot Q - [u(k) - x \cdot k].
\]

5.3 Firm 2’s Pricing Decision

Lemma 3 tells us that, if firm 2 sets its \( p_2 \) below the cutoff \( x \), then it will be a monopoly supplier for \( k \); if it sets its \( p_2 \) above \( x \) but below \( w \), then it will supply the residual demand \( q(p_2) - Q \). As a result, firm 2’s profit can be written as

\[
\pi_2(p_2) = \begin{cases} 
(p_2 - c) \cdot k & \text{if } p_2 < x \\
(p_2 - c) \cdot \{q(p_2) - Q\} & \text{if } x \leq p_2 \leq w \\
0 & \text{if } w < p_2 
\end{cases}
\]

Note that there is a discontinuous drop at \( x \) in firm 2’s profit curve, which is shown as the red curves in Figure 6.
From its profit curve, we can clearly see the trade-offs firm 2 faces: undercutting below $x$ with its limited capacity $k$ and making itself a monopoly supplier, or giving up part of the competitive portion by leaving $Q$ units to firm 1 but charging a higher price between $x$ and $w$. Accordingly, firm 1’s profit is

$$\pi_1 = \begin{cases} 0 & \text{if } p_2 < x, \\ (p_1 - c) \cdot Q & \text{if } x \leq p_2 \leq w, \\ (p_1 - c) \cdot q(w) & \text{if } w < p_2. \end{cases}$$

Note that firm 2 would never choose $p_2 > w$, because it would earn zero in that case. But setting $p_2 < x$ would leave zero profit for firm 1. Thus, for a profitable improvement, firm 1 must ensure $x \leq p_2 \leq w$, instead of $p_2 < x$. That is,

$$\max_{p_2 < x} (p_2 - c) \cdot k = (x - c) \cdot k \leq \max_{x \leq p_2 \leq w} (p_2 - c) \cdot [q(p_2) - Q],$$

which says being a residual demand supplier is at least as profitable as being an undercutting monopoly.

Because there is a discontinuous drop at $x$ in firm 2’s profit curve, firm 2 would prefer $p_2 < x$ if $p_2 = x$ is the optimal solution to the RHS problem in (11). Thus, firm 2’s optimal price $p_2$ must be an interior solution. We can further show that the inequality (11) must be binding in equilibrium.

**Lemma 4 (Firm 2’s Choices)** In the AUD equilibrium,

$$(x - c) \cdot k = \pi^R(Q),$$

and $p_2 = p^R(Q) \in (x, w]$, that is,

$$\pi'(p_2) = Q.$$
The LHS of (12) is firm 2’s profit when it supplies \( k \) as an undercutting monopoly. The RHS of (12) is firm 2’s maximum profit when it supplies the residual demand and undersupplies. Recall from (10) that \( p_1 \cdot Q \) increases with \( x \), as \( u'(Q + k) < x \). So whenever the LHS of (12) is smaller than the RHS of (12), firm 1 can always increase its profit by increasing \( p_1 \cdot Q \), thereby increasing threat price \( x \). Lemma 4 demonstrates that in equilibrium, firm 1 will design its AUDs to induce firm 2 to be just satisfied as a residual demand supplier, rather than an undercutting sole supplier. In the AUD equilibrium, firm 2 undersupplies and sets its price \( p_2 \) above threat price \( x \) to maximize the residual profit.

5.4 Firm 1’s Optimal AUDs

Note that firm 1’s choice of the AUD scheme can be reduced to an incentive contract design problem in which firm 1 chooses \((Q, p_1)\) to maximize its profit such that (i) the buyer prefers DS to SS and (ii) firm 2 chooses its uniform price \( p_2 \) optimally and yet is indifferent between undersupplying at a higher price and selling its full capacity at a lower price. From the discussion in Sections 5.1~5.3, firm 1’s optimization problem is

\[
\max_{(Q, p_1)} \pi_1^{AUD} = (p_1 - c) \cdot Q \\
\text{s.t. } (9), (12), (13), (C1), (C2) \\
u'(Q + k) < x < p_2 < u'(Q) 
\]

To better understand strategic roles of the quantity threshold, we now denote all variables in terms of \( Q \). For \( 0 \leq Q \leq q(c) \), let \( x(Q) \) satisfies (12). Using (10), the profit function of firm 1 can be expressed as

\[
\pi_1^{AUD}(Q) = \underbrace{v(x) + (x - c) \cdot Q}_{\text{Sum of surpluses for firm 1 and the buyer under DS at } x} - \underbrace{[u(k) - x \cdot k]}_{\text{BS under SS at } x},
\]

where \( x = x(Q) \) is determined by (12). From this profit expression, in the AUD equilibrium, firm 1 extracts all the incremental surplus over the buyer’s outside option at threat price \( x \). Note that when \( x = c \), the profit above is \( v(c) - [u(k) - c \cdot k] \), which is firm 1’s profit in the 2PT equilibrium. As will be shown later, \( x = c \) satisfies all equality constraints, except for (14), and in the AUD equilibrium, (14) is never binding. So the AUDs can at least reach the 2PT equilibrium profit by choosing \( Q = q(c) \).

Note that

\[
\frac{d\pi_1^{AUD}}{dQ} = \frac{\partial\pi_1^{AUD}}{\partial Q} + \frac{\partial\pi_1^{AUD}}{\partial x} \cdot x'(Q) \\
= x - c + \underbrace{\{k - [q(x) - Q]\} \cdot x'(Q)}_{\text{Indirect Effect}}.
\]

Clearly, when \( Q \) increases by one unit, firm 1 has to incur an extra per-unit production cost \( c \) whereas it saves \( x \), because \( x \) is the amount of per-unit payment to firm 2 for a coalition of firm 1 and the buyer. The
difference $x - c$ is thus the direct effect of setting a higher $Q$. There is an indirect effect of increasing $Q$. It is through its impact on the most profitable undercutting price $x(Q)$. Recall that from \(10\), the most firm 1 can extract using $p_1 \cdot Q$ is the incremental surplus the buyer and firm 1 as a coalition can gain over the buyer’s outside option of $SS$ from firm 2 only, when firm 2 undercuts at $x$. By the Envelope theorem, an increase in $x$ reduces $BS$ under $SS$ by $k$. This helps firm 1, as it needs to compensate less to the buyer when inducing $DS$.

Meanwhile, the higher $x$ means the sum of surpluses for firm 1 and the buyer under $DS$ is reduced, thanks to the greater payment to firm 2. By the Envelope Theorem, the magnitude of such reduction in surplus (or the increased payment to firm 2) is the residual demand purchased from firm 2 under $DS$ at $x$, i.e., $q(x) - Q$. This hurts firm 1’s profit. Consequently, the overall impact from increasing $x$ is $k - [q(x) - Q] \cdot x'(Q)$. To maximize its profit, firm 1 will balance these two effects, taking into account inequality constraints (C1), (C2) and (14).

From (12), we get $x - c = \pi^R(Q)/k$ and $x'(Q) = \pi^{R'}(Q)/k = -(p_2 - c)/k$. Substituting them into (15) yields

$$
d\pi^{AUD}_{1} = \frac{p_2 - c}{k} \cdot \{[q(p_2) - Q] - [k - (q(x) - Q)]\}.
$$

So (15) becomes

$$q(p_2) - Q = k + Q - q(x). \quad \text{(FOC)}$$

That is, firm 1 sets its volume threshold to balance the direct effect measured by the residual demand $q(p_2) - Q$ and the indirect effect measured by the difference $k - [q(x) - Q]$.

To ensure the sufficiency and the uniqueness of (FOC) for the optimum and facilitate our comparative statics analysis, we assume $q''(p) \leq 0, \forall p \in [c, u'(0)]$. The concave demand guarantees that $\pi^{AUD}_{1}(Q)$ is single-peaked in $Q$, and thus (FOC) characterizes the optimal solution. It is satisfied by generalized linear demand such as $q(p) = 1 - p^r \ (r \geq 1)$. The following proposition summarizes our equilibrium analyses.

**Proposition 3 (AUD Equilibrium)** The AUD equilibrium exists with $p_o = \infty$ and is characterized as follows. There exists a unique $k \in (0, q(c))$ such that

- when $k \in (0, \hat{k})$, the equilibrium outcome $(Q, p_1, p_2)$ along with threat price $x$ is jointly determined by (9), (12), (13), and (FOC); 
- when $k \in [\hat{k}, q(c))$, the equilibrium outcome $(Q, p_1, p_2)$ along with threat price $x$ is jointly determined by (9), (12), (13), and the binding (C2).

We now provide further intuition for how the AUDs work in equilibrium. As we discussed before, under both LP and 2PT, firm 2 always undercut and sells at its full capacity. So the competitive portion $k$ becomes firm 2’s turf. Accordingly, the best firm 1 can do is to use a fixed fee to extract the incremental surplus from its captive demand. Such incremental surplus is maximized at the efficient outcome under 2PT, and thus firm 1 extracts its marginal contribution to the efficiency $v(c) - [u(k) - c \cdot k]$. How can the AUDs further increase firm 1’s profit over a 2PT, given that the 2PT equilibrium outcome is efficient and firm 1 has already extracted the full surplus from its captive portion $q(c) - k$? The crux is to leverage its market power from the captive portion to the competitive portion, and at the same time prevent firm 2 from undercutting.
The unique component of the AUDs, compared with LP or a 2PT, is the quantity requirement \( Q \). Under AUD, firm 1 now can take the initiative to dictate a quantity target beyond its captive portion, and commit not to supply any amount other than that. By doing so, the buyer faces trade-offs between SS and DS—if she buys from firm 2 at \( p_2 \) for \( k \), she would not be able to meet firm 1’s quantity requirement, and thus is forced to rely on firm 2’s limited supply only; instead, if she meets firm 1’s quantity target, her residual demand does not allow her to enjoy firm 2’s lower price up to firm 2’s full capacity. So with the quantity target instrument, firm 1 acts more aggressively and encroaches on the competitive portion. It induces the buyer to treat firm 2, instead of firm 1, as a residual demand supplier.

Correspondingly, under AUD, firm 2 now faces trade-offs that are missing under LP or 2PT. Recall that under LP or 2PT, firm 2’s only option to survive is to undercut and hence sell its full capacity. Facing the AUDs, firm 2 has two options—undercut low enough to be a sole supplier, or set a high price serving the residual demand only. Hence, the quantity target creates another option other than undercutting for firm 2, so that preventing undercutting that is implausible under LP or 2PT becomes possible now.

As a result, in the AUD equilibrium, firm 1 judiciously designs the quantity requirement subject to two incentive constraints. One is from the buyer. Firm 1 has to ensure that the buyer will meet the quantity target rather than miss it and rely on firm 2 only. It is guaranteed by inducing firm 2 to set \( x < p_2 \), where \( x \) is given by (9). The other incentive constraint is from firm 2. Firm 1 has to induce firm 2 to be satisfied as a residual demand supplier instead of undercutting to be a sole supplier, as stated by (12).

Recall from Lemma 2 that the AUDs need to satisfy two price constraints \( (C1) \) and \( (C2) \). We find that the former one is never binding, whereas the latter one \( p_2 \leq p_1 \) might be binding, depending upon \( k \). When \( k \) is small, firm 1 can extract surplus without worrying too much about competition. It will set a large requirement \( Q \), and its average price for the \( Q \) units \( p_1 \) will be high, too. From (13), the large \( Q \) squeezes firm 2’s residual demand and forces its optimal price \( p_2 \) to be low. So \( (C2) \) is not binding in this case. On the contrary, when \( k \) is large, the market becomes more competitive as firm 2’s capacity grows. The competitive pressure forces firm 1 to set a small \( Q \) as well as a low average price for the \( Q \) units. The small \( Q \) results in a high \( p_2 \) from (13). That is, as \( k \) increases, \( p_1 \) is forced to fall whereas firm 2’s optimal price rises. Then the constraint \( (C2) \) becomes binding and, in equilibrium, firm 2 will just match \( p_1 \) by setting \( p_2 = p_1 \). So the equilibrium condition (FOC) is replaced with \( (C2) \) when \( k \) is large.

### 5.5 Properties of the AUD Equilibrium

In this subsection, we present some major properties of the AUD equilibrium and compare the AUD equilibrium outcome with the LP and 2PT equilibrium outcomes.

The corollary below illustrates the quantity expansion effect of the AUDs.

**Corollary 2 (Quantity Expansion of the AUDs)** *In the AUD equilibrium, \( Q > q(c) - k \) for any \( k > 0 \).*

Under AUD, firm 1 will expand its quantity requirement so much that the buyer would not be able to absorb firm 2’s full capacity, even if firm 2 undercuts towards marginal cost \( c \). Note that \( Q > q(c) - k > q(p_2) - k \) for any \( p_2 > c \). So Corollary 2 is stronger than Part (ii) of Lemma 1. Such a significant quantity expansion squeezes the buyer’s demand for firm 2’s product to a level that it is strictly below its full capacity.
for any above-cost price it can charge. This illustrates how the dominant firm can leverage its market power from its captive portion to the competitive portion of the demand. By setting \( p_c \) to a prohibitively high level and the quantity threshold above its captive demand, the leverage is realized through a refusal-to-deal threat if the buyer’s purchase is less than the threshold.

Define the total surplus \( TS \) as the sum of both firms’ profits and the buyer’s surplus. The following corollary summarizes how the equilibrium outcomes change as \( k \) varies, when (C2) is not binding.

**Corollary 3 (The Impacts of Limited Capacity)** For \( 0 < k \leq \hat{k} \), as \( k \) increases, the followings hold:

(i) the equilibrium quantity threshold \( Q \) and the total output decrease;

(ii) the equilibrium \( p_2 \) (and also \( x \)) increases;

(iii) the equilibrium profit \( \pi_1^{AUD} \) decreases, and \( \pi_2^{AUD} \) increases;

(iv) \( TS^{AUD} \) decreases.

As \( k \) increases, firm 2’s competitive position becomes stronger. Therefore, firm 1, when designing its quantity target, has to leave more room for firm 2, in order to prevent firm 2 from undercutting. So the equilibrium \( Q \) decreases as \( k \) increases. Other comparative statics follow from the pattern of \( Q \). The results that \( \pi_1^{AUD} \) decreases whereas \( \pi_2^{AUD} \) increases when \( k \) increases are easy to understand. Hence, if we allow firm 2 to change its capacity level, it has a strong incentive to increase its capacity when \( k \) is small. It is due to two effects: one is that a larger capacity implies it can sell more in equilibrium; the other is the strategic effect through impacting firm 1’s equilibrium AUD design. This is in stark contrast with traditional IO literature. In traditional IO literature, the incumbent often uses idle capacity to deter entry, whereas in our model it is the small firm that can gain competitive advantage from its “forced” idle capacity.\(^{17}\)

When \( k \) is above \( \hat{k} \), (C2) becomes binding, and we are unable to perform the comparative statics for \( k \) in the range of high values. In Section 6, we use an example with linear demand to illustrate the comparative statics of AUDs for a full range of values of \( k \).

In the corollary below, we provide a comparison of LP and AUD equilibria. Note that the LP equilibrium price \( p^R \) decreases with \( k \), whereas the AUD equilibrium price \( p_2^{AUD} \) increases with \( k \) as long as (C2) is not binding. Because \( p_2^{AUD}(0) = c < p^R(0) = p^m \), there must be a cutoff \( k_0 > 0 \) such that \( p_2^{AUD}(k_0) = p^R(k_0) \).

**Corollary 4 (Comparison with LP)** (i) Prices: \( p_2^{AUD} < p_2^{LP} \), \( \forall 0 < k < \min\{k_0, \hat{k}\} \);

(ii) Quantities: \( q_1^{LP} < q_1^{AUD} \), \( q_2^{AUD} < q_2^{LP} = k \); \( \forall 0 < k < q(c) \);

(iii) Profits: \( \pi_1^{LP} < \pi_1^{AUD} \); \( \forall 0 < k < q(c) \); \( \pi_2^{AUD} < \pi_2^{LP} \); \( \forall 0 < k < \min\{k_0, \hat{k}\} \);

(iv) Buyer’s Surpluses: there exists a \( k_1 \in (0, q(c)) \) such that \( BS^{AUD} < BS^{LP} \) for \( 0 < k < k_1 \);

(v) Total Surpluses: there exists a \( k_2 \in (0, q(c)) \) such that \( TS^{AUD} > TS^{LP} \) for \( 0 < k < k_2 \).

Hence, when \( k \) is relatively small, firm 1 gains from the AUDs, firm 2 gets hurt in terms of profits, volume sales and market shares, and the buyer gets hurt, compared with LP equilibrium. In the limiting case when \( k \) goes to 0, the equilibrium price under LP is close to the monopoly price since the rival does not put any competitive pressure on the dominant firm. On the other hand, under AUD, the equilibrium quantity

\(^{17}\)We thank Joseph Farrell for this comment.
threshold is close to the efficient quantity level $q(c)$ and the dominant firm can almost extract all the surplus from the buyer. As a result, the buyer is worse off under AUD than under LP when $k$ is small. In the next section, we provide examples to illustrate that $k$ does not have to be really small in order for the results in Corollary 4 to hold. So under AUD, we have partial foreclosure in the sense that firm 2 under-supplies strictly below its capacity and its profit is reduced. If firm 2 has certain fixed cost, then the AUDs adopted by a dominant firm can partially deny firm 2’s profitable access to the otherwise competitive market, and it may induce firm 2 to exit. Our results support the antitrust concern on AUDs when $k$ is relatively small.

Compared with LP, the AUDs have a quasi-fixed fee $T = p_1Q$ at the quantity threshold $Q$. Such a quasi-fixed fee leads to two effects: quantity expansion effect and surplus extraction effect. On the one hand, because firm 1 can extract incremental surplus using this quasi-fixed fee, it has an incentive to push the equilibrium outcome towards a more efficient one. Such a quantity expansion effect tends to increase total surplus and buyer’s surplus. On the other hand, because of the very quasi-fixed fee, firm 1 can extract surplus from the buyer more efficiently. Such a surplus extraction effect reduces buyer’s surplus. Of course, firm 1’s surplus extraction is constrained by the competitive pressure from firm 2. For relatively small $k$, competition does not concern firm 1 that much, and the quasi-fixed fee under AUD extracts most of the buyer’s surplus. Thus, the surplus extraction effect dominates the quantity expansion effect, which results lower buyer’s surplus. As $k$ increases, because the buyer’s outside option becomes better, the surplus extraction effect will be limited and the buyer may not be worse off under AUD.

Last, we provide a comparison between the AUDs and 2PT equilibria.

**Corollary 5 (Comparison with a 2PT)** Compared with a 2PT, the AUDs increase both firm 1 and firm 2’s profits, reduce firm 2’s volume sales, and decrease the buyer’s surplus and total surplus.

Under AUD, firm 1 always gains more in profits as well as volume sales than it does under 2PT, whereas firm 2 under-supplies all the time. The buyer’s surplus under AUD is always below that under 2PT. This is because the 2PT equilibrium outcome is efficient, and the buyer enjoys its outside option—buying $k$ from firm 2 at marginal cost $c$. Nevertheless, the realized AUD equilibrium price $p_2$ firm 2 charges is above marginal cost $c$. Essentially, relatively to the 2PT, the AUD scheme is used as a collusive mechanism which allows both firms to gain at the expense of the buyer, and the outcome is less efficient.

### 5.6 Partial Foreclosure of A More Efficient Competitor

We now show that our major findings regarding the AUDs also hold when firm 2 has a lower marginal cost. This suggests that the AUDs can be an effective tool to squeeze firm 2’s profit even when firm 2 is more efficient up to its capacity level.

Suppose firm 2’s marginal cost $c_2$ is no higher than firm 1’s marginal cost $c_1$, i.e., $c_2 \leq c_1$. We consider a case in which $0 < k < q(c_1)$. This means that firm 2 cannot serve the whole demand of the buyer when firm 1 undercuts to its marginal cost $c_1$. Denote firm 2’s profit as $\pi(p; c_2) \equiv (p - c_2) \cdot q(p)$ and its monopoly price as $p^{m_2}(c_2) \equiv \max_p \pi(p; c_2)$. The following proposition shows that our analysis of AUDs works with differential marginal costs when the difference is not too large.
Proposition 4 (With More Efficient Rival) Suppose

\[ c_2 \leq c_1 < p^m(c_2) \text{ and } k + (c_1 - c_2) \cdot q'(c_1) > 0, \]  

(16)

the AUD equilibrium outcome is characterized by Proposition 3 with adaptations of \( c = c_2 \) in (12), (13), and (FOC) being replaced by

\[ q(p_2) - Q = k + Q - q(x) + \frac{c_1 - c_2}{p_2 - c_2} \cdot k. \]  

(17)

Even when facing a more efficient rival up to its capacity limit, as long as the rival’s cost advantage is within a certain range, the AUD scheme is an effective competition instrument to improve firm 1’s profit over LP and a 2PT. Other comparative statics also hold. Particularly, the buyer’s surplus under AUD is lower than the one under LP when \( k \) is relatively small. Proposition 4 implies that, in the presence of a more efficient competitor, the AUDs may still lead to partial foreclosure of the competitor (and full foreclosure is likely if there are fixed costs).

6 Illustrative Examples

To illustrate our analyses above and gain further insights on how the limited capacity affects the equilibrium, in this section, we use examples to investigate competitive effects of capacity constraint. We consider a linear demand function \( q(p) = 10 - p \), which is generated by the gross utility function \( u(q) = q(10 - q/2) \). \( k \) is normalized to be in the range of \((0, 10)\).

Let’s take a quick look at some examples of the partial foreclosure effect of AUDs. Table 1 shows the LP and AUD equilibrium outcomes with identical zero costs. Using AUDs, firm 1 expands its volume sales dramatically by offering a lower price upon a large threshold. It forces firm 2 to reduce its price by a big percentage in order to stay competitive for the residual demand. As a result, the total surplus is increased due to quantity expansion at lower prices. However, firm 2 loses significantly in volume sales, market shares and profits, and the buyer is induced to buy more than before and receives a lower surplus. For example, at \( k = 1 \), firm 1’s quantity sales are more than double of that under LP, whereas firm 2’s quantity sales are less than 40% of that under LP. Firm 1’s profit under LP is about half of that under AUD, whereas firm 2’s profit under LP is 30 times of that under AUD. The buyer’s surplus under LP is about 1.6 times of that under AUD.
Table 1: A Linear Demand and Identical Costs

<table>
<thead>
<tr>
<th></th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>BS</th>
<th>TS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
<td>1</td>
<td>20.25</td>
<td>4.5</td>
<td>15.13</td>
<td>39.87</td>
</tr>
<tr>
<td>AUD</td>
<td>4.39</td>
<td>0.38</td>
<td>9.24</td>
<td>0.38</td>
<td>40.55</td>
<td>0.15</td>
<td>9.24</td>
<td>49.93</td>
</tr>
<tr>
<td>$\Delta%$</td>
<td>-2</td>
<td>-92</td>
<td>+105</td>
<td>-62</td>
<td>+100</td>
<td>-97</td>
<td>-39</td>
<td>+25</td>
</tr>
</tbody>
</table>

$c_1 = c_2 = 0$ and $k = 2$

<table>
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<th>$p_2$</th>
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<th>$q_2$</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>BS</th>
<th>TS</th>
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</thead>
<tbody>
<tr>
<td>LP</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>8</td>
<td>18</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>AUD</td>
<td>3.80</td>
<td>0.76</td>
<td>8.47</td>
<td>0.77</td>
<td>32.18</td>
<td>0.58</td>
<td>16.94</td>
<td>49.71</td>
</tr>
<tr>
<td>$\Delta%$</td>
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<td>-81</td>
<td>+112</td>
<td>-62</td>
<td>+101</td>
<td>-93</td>
<td>-6</td>
<td>+18</td>
</tr>
</tbody>
</table>

When the small rival is more efficient, Table 2 confirms Proposition 4 that the AUDs can still effectively foreclose the small rival. Compared with identical costs cases in Table 1, a new effect emerges from firm 1’s quantity expansion when firm 1 is less inefficient. Due to the shift of output from firm 2 to firm 1 under AUD, more outputs are produced using a less efficient technology. This production inefficiency can dominate the quantity expansion effect, resulting in lower total surpluses, as shown in Table 2.

Table 2: A Linear Demand and Different Costs

<table>
<thead>
<tr>
<th></th>
<th>$p_1$</th>
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<th>$q_1$</th>
<th>$q_2$</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>BS</th>
<th>TS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>AUD</td>
<td>3.99</td>
<td>3.98</td>
<td>2.04</td>
<td>3.98</td>
<td>2.02</td>
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<td>18.11</td>
<td>35.96</td>
</tr>
<tr>
<td>$\Delta%$</td>
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<td>-0.5</td>
<td>+104</td>
<td>-20</td>
<td>+102</td>
<td>-21</td>
<td>+0.6</td>
<td>-7.8</td>
</tr>
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</table>

$c_1 = 3 > c_2 = 0$ and $k = 5$

<table>
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<th>$p_2$</th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>BS</th>
<th>TS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>4.5</td>
<td>4.5</td>
<td>0.5</td>
<td>5</td>
<td>0.25</td>
<td>22.5</td>
<td>15.13</td>
<td>37.88</td>
</tr>
<tr>
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<td>4.498</td>
<td>1.01</td>
<td>4.50</td>
<td>0.50</td>
<td>20.23</td>
<td>15.14</td>
<td>35.87</td>
</tr>
<tr>
<td>$\Delta%$</td>
<td>-0.03</td>
<td>-0.06</td>
<td>+101</td>
<td>-10</td>
<td>+101</td>
<td>-10</td>
<td>+0.08</td>
<td>-5.3</td>
</tr>
</tbody>
</table>

Both tables above show that in the short run, the AUDs always hurt firm 2, and could lower buyer’s surplus, or lower total surplus, as compared to LP. In the long run, firm 2 might be induced to exit, depending on the magnitude of fixed costs involved, and firm 1 would monopolize the market, resulting in lower buyer’s surplus and lower total surplus.

Now we perform our comparative statics analyses for the full range of $k \in (0, 10)$ with $c_1 = c_2 = 0$, by directly applying Propositions 1-3. The computed results are listed in Tables A1 and A2 in the Appendix. It is easy to compute the cutoff at which (C2) to be binding is $\widetilde{k} \approx 5.354$.

■ Firm 2’s Volume Sales and Profits. The equilibrium volume sales for firm 2 under LP and AUD schemes are shown in Figure 7. Firm 2’s volume sales are severely hurt by the AUDs. As firm 2 will supply
to its full capacity $k$ under LP, the difference between the blue line and red line tells us the idle capacity of firm 2 $k - [q(p_2) - Q]$.

As shown in Figure 8, firm 2’s profit is reduced dramatically when firm 1 adopts the AUDs, and this is true for the full range of $k$. So firm 2 gets partially foreclosed by the dominant firm’s AUDs for all levels of $k$. This result may raise antitrust concerns when a dominant firm competes against a capacity-constrained competitor and the dominant firm uses the AUDs.

**Buyer’s Surpluses.** The equilibrium buyer’s surpluses under LP and AUD equilibria are shown in Figure 9. Note that $BS_{AUD}^{AUD}$ crosses $BS_{LP}^{LP}$ from below at $k \simeq 2.3$. So when $k < 2.3$, $BS_{AUD}^{AUD} < BS_{LP}^{LP}$; when $k \geq 2.3$, $BS_{AUD}^{AUD} \geq BS_{LP}^{LP}$. This shows two effects of the AUDs on the buyer. First, the AUD scheme is a more efficient surplus extraction tool than LP, which in principle hurts the buyer. Second, the adoption
of the AUDs intensifies competition by pushing firm 2 to set a lower price. As shown in Figure 9, when 
k\ is relatively small, the former effect dominates the latter because the competitive pressure from firm 2 is
limited due to its small capacity; when \( k \) is relatively large, the latter effect dominates the former, for more
intensified competition becomes significant when firm 2’s capacity is large.

![Figure 9: Buyer’s Surpluses](image)

From numerical examples above, we find that when \( k \) is relatively small, both the competitor and the
buyer are hurt by the dominant firm’s adoption of the AUDs. This observation appears to be consistent
with antitrust concerns put forward in a number of recent cases. Moreover, when \( k \) is relatively large, the
buyer may not be hurt by the adoption of the AUDs in the short run, but the competitor is always partially
foreclosed as when \( k \) is small. So if there is any fixed costs, such limited profit level as well as not enough
growth opportunity under AUD may induce the competitor to exit the market. Hence, the buyer’s welfare
may get hurt due to the adoption of the AUDs by the dominant firm in the long run.

7 Discussions

We now discuss some assumptions of the model. In the first subsection, we consider a game in which two
firms make offers simultaneously. In the second subsection, we offer some thoughts on whether our results
would be affected if firm 2’s feasible contract set is expanded.

- **Simultaneous Move.** For the timing of the game, in practice, the nonlinear pricing schemes, such as
AUDs, become an antitrust concern only when the firm adopting it enjoys a dominant position in the market.
When there is a dominant firm, it is the dominant firm that often moves first, and the number of moves is low.
The literature on price leadership shows that the dominant firm will emerge as the price leader\(^{18}\). Thus, we
model firm 1 as the dominant firm due to which it moves first and offers a more complicated pricing scheme

\(^{18}\)For literature on price leadership, see Deneckere and Kovenock (1992)\(^{11}\) and van Damme and Hurkens (2004)\(^{30}\).
than the follower firm 2 does. Basically, this sequential-move nature captures the commitment power the dominant firm has in preventing renegotiation.

With a simultaneous move, given firm 1’s offer, firm 2’s best response will be exactly the same as in our previous analysis. However, firm 1 with no capacity constraint will behave as a full-capacity firm 2 in our previous setting. More importantly, firm 1 can use nonlinear contracts such as a 2PT, or the AUDs to extract any incremental surplus. From firm 1’s perspective, inducing SS now weakly dominates inducing DS from the buyer. This is because the most firm 1 can extract under DS is its incremental surplus given firm 2’s price $p_2$, but it can get at least the same amount by supplying what firm 2 supplies under DS simply through undercutting $p_2$ a bit and extracting the buyer’s surplus $v(p_2) - [u(\bar{q}(k, p_2)) - p_2 \cdot \bar{q}(k, p_2)]$ using a 2PT, or the AUDs. Such undercutting reasoning drives $p_2$ towards marginal cost $c$, and hence firm 1 earns $v(c) - [u(k) - c \cdot k]$, whereas firm 2 gets zero. Therefore, equipped with a 2PT, or the AUDs, firm 1 will always undercut and maximize the joint profit between it and the buyer. The equilibrium outcome will be efficient, as in the settings of common agency when there is complete information and nonlinear contracts are allowed (see O’Brien and Shaffer (1997) [25], Bernheim and Whinston (1998) [2]).

In the simultaneous move game, firm 1’s equilibrium profit $v(c) - [u(k) - c \cdot k]$ when using a 2PT, or the AUDs is the same as its profit when it moves first and uses a 2PT in Proposition 2. With a sequential move, we have shown that firm 1 can improve its profit over a 2PT by adopting the AUDs, and firm 2 can also earn positive profits. This implies that firm 1 is better off by moving first and hence has incentives to make such a commitment. In addition, in this setting, firm 2 also prefers to be a second mover rather than moving simultaneously.

**Nonlinear Counteroffer.** In the analysis up to this point, we have restricted our attention to the equilibrium when firm 2 can use LP only. This is to capture the fact that small firms in practice usually cannot offer contracts as complicated as offered by a dominant firm. One reason could be that the buyer considers the dominant firm’s product as a “must carry” one, and thus she is reluctant to sign another AUD contract with a small supplier. Moreover, due to the lack of experience, small firms often do not have sufficient information on market demand compared to the dominant firm. Even if allowing them to offer the AUDs, setting proper threshold requirements and the corresponding payments would be hard for small firms, not to mention their insufficient ability in monitoring and enforcing those complicated nonlinear contracts.

With complete information, when both firms can use nonlinear contracts such as a 2PT or the AUDs, it is well known in the common agency literature that the equilibrium outcomes are efficient, when two principals can both offer complicated enough nonlinear contracts (see O’Brien and Shaffer (1997) [25], Bernheim and Whinston (1998) [2], and Marx and Shaffer (2004) [20]). This is in stark contrast with our equilibrium outcome when firm 2 can only offer LP; our AUD equilibrium is not efficient. Because the AUDs are

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19 A formal proof of this result in our setting is available upon request. Besides, when both firms use complex contracts, the surplus division between firm 2 and the buyer is not uniquely determined. Such a multiplicity of surplus division between firm 2 and the buyer could cause mis-coordination or uncertainty for both of them, and full foreclosure is possible. Hence, it is not in firm 2’s interest to use nonlinear counteroffer, because this might result in more intensive competition.

In our Example 1 (Step-function Demand), it is easy to see that, when firm 2 uses a 2PT, the equilibrium outcome is $\pi_{1}^{AUD+2PT} = 36, \pi_{2}^{AUD+2PT} + BS^{AUD+2PT} = 19$. This is because if firm 1 does not leave firm 2 and the buyer at least 19 surplus, then firm 2 can always induce the buyer to SS with firm 2 and use the fixed fee $T_2$ to grab certain amount from the firm2-buyer coalition surplus 19. So the maximum firm 1 can extract is $(10 + 9 + \ldots + 1) - 19 = 36$. Firm 1 can fully exclude firm 2 by setting $Q = 10, p_1 = 3.6$, which will force firm 2 to set $p_2 = T_2 = 0$. 

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often observed in practice, our analysis provides a theoretic explanation for their prevalence in the strategic context, complementary to the existing common agency literature.

8 Conclusion

The use of AUDs by a dominant firm has become a hotly debated topic in antitrust economics and competition policy enforcement. A key feature in some of the antitrust cases involving AUDs is that a dominant firm’s competitors often are capacity-constrained and thus cannot economically match the dominant firm’s AUD offer to serve a customer’s whole demand requirement. Although the existing literature has thus far focused on interpreting AUDs as a price discrimination tool, investment incentive program, or rent-shifting tool, the antitrust concerns on AUDs are often on its plausible exclusionary effects.

In absence of asymmetric information, downstream competition, or contract externality, we establish strategic effects of AUDs when a dominant firm competes against an equally efficient (or more efficient) but capacity-constrained competitor. In our setting, we find that the dominant firm is able to use AUDs to partially foreclose its competitor’s access to the otherwise competitive portion of the market, when the competitor’s capacity is limited. Our finding appears to be consistent with the following logic of the European Commission:

Intel is an unavoidable trading partner. The rebate therefore enables Intel to use the inelastic or “non-contestable” share of the demand of each customer, that is to say the amount that would anyhow be purchased by the customer from the dominant undertaking, as leverage to decrease the price of the elastic or “contestable” share of demand market to lower the price in the contestable market, that is to say the amount for which the customer may prefer and be able to find substitutes.


In our single-product model, the dominant firm can use AUDs to leverage its market power from its captive portion of the market to the competitive portion. The idea is much like the tied-in selling strategy in the context of multiple products. We conjecture that the foreclosure mechanism in this article might work for other conditional discounts, such as quantity forcing, three-part tariff, bundled loyalty discounts and so on.

We leave this for future research.

\[^{20}\text{Chao (2013) studies the three-part tariff in a duopoly model. But in his setting, the rival has full capacity to serve the whole market, and competing products are differentiated.}\]
Appendix

Proof of Proposition 1 First, firm 2 must set $p_2 \leq p_1$ unless $p_1 < c$, because otherwise firm 2 would have no sales and thus zero profit. But $p_1 < c$ can be ruled out as it gives firm 1 negative profit. So $p_2 \leq p_1$.

Second, firm 1 must set $p_1 < u'(k)$. This is because, due to $p_2 \leq p_1$, the buyer always buys from firm 2 first, and $u'(k) \leq p_1$ would result in no sale for firm 1.

Hence, with $p_2 \leq p_1 < u'(k)$, the buyer buys $k$ from firm 2 at $p_2$ and $q(p_1) - k$ from firm 1 at $p_1$. Firm 2’s profit is $(p_2 - c) \cdot k$ and firm 1’s profit is $(p_1 - c) \cdot [q(p_1) - k]$. It is easy to see that firm 2 must set $p_2 = p_1$ and firm 1 will set $p_1 = p^R(k)$. ■

Proof of Proposition 2 First, firm 2 must set $p_2 \leq p_1$ unless $p_1 < c$, because otherwise firm 2 would have no sales and thus zero profit. Note that in the 2PT equilibrium, firm 1 must ensure that the buyer accepts the 2PT. Then $p_1 < c$ can be ruled out, because if so then $\pi_1 = T_1 + (p_1 - c) \cdot q(p_1)$ such that the buyer is better off by buying from firm 1 only even if firm 2 undercuts to marginal cost $c$, i.e., $v(p_1) - T_1 \geq u(k) - c \cdot k$.

But then $\pi_1 \leq v(p_1) + (p_1 - c) \cdot q(p_1) - [u(k) - c \cdot k]$, which is maximized at $p_1 = c$.

By the same argument in the proof of Proposition 1, we must have $p_2 \leq p_1 < u'(k)$, and firm 2 must set $p_2 = p_1$. Then $\pi_1 = T_1 + (p_1 - c) \cdot [q(p_1) - k]$ subject to $v(p_1) - T_1 \geq u(k) - p_1 \cdot k$. So $\pi_1 \leq v(p_1) + (p_1 - c) \cdot q(p_1) - [u(k) - c \cdot k]$, which is maximized at $p_1 = c$. The claim follows. ■

Proof of Lemma 1 (i) Under AUD, if $q_1 < Q$, then $q_1 = 0$ and $\pi_1 = 0$ because $p_o = \infty$; if $q_1 > Q$, then it is equivalent to LP ($p_1$) vs. LP ($p_2$), and the AUDs cannot improve firm 1’s profit.

We now show $Q < q(c)$. Suppose not, i.e., $u'(Q) \leq c$. Then under DS firm 2 would have no sales, and it would try its best to undercut until $c$ in order to induce SS if possible. To ensure the buyer meets $Q$, firm 1 must make $u(Q) - p_1 \cdot Q \geq u(k) - ck$, i.e., $p_1 \cdot Q \leq u(Q) - [u(k) - ck]$. Thus,

$$
\pi_1 = (p_1 - c) \cdot Q \\
\leq u(Q) - cQ - [u(k) - ck] \\
\leq v(c) - [u(k) - ck] = \pi_1^{2PT}.
$$

So in order to have a strictly profitable improvement over a 2PT, we must have $Q < q(c)$.

(ii) $Q < q(p_2)$ follows from the fact that $c < u'(Q)$ and $u'(Q) \leq p_2$ would result in no sales for firm 2.

We now show $q(p_2) < Q + k$. Suppose not, i.e., $p_2 \geq u'(Q + k)$. It follows that $\pi_2 = (p_2 - c)k \leq [u'(Q + k) - c]k$. Then firm 2 can always increase its profit without losing any sales, as long as $p_2 < u'(Q + k)$. Next, we rule out the case of $p_2 = u'(Q + k)$. Suppose $BS_D(u'(Q + k)) \geq BS_S(u'(Q + k))$, i.e., $u(Q + k) - p_1 \cdot Q - u'(Q + k)k \geq u(k) - u'(Q + k)k$. So $p_1 \cdot Q \leq u(Q + k) - u(k)$. Then

$$
\pi_1 = (p_1 - c) \cdot Q \\
\leq u(Q + k) - u(k) - cQ \\
\leq v(c) - [u(k) - ck] = \pi_1^{2PT}.
$$

For $\pi_1 > \pi_1^{2PT}$, we must have $BS_D(u'(Q + k)) < BS_S(u'(Q + k))$, but then the buyer would choose SS. Thus, in order to induce the buyer to choose DS firm 1 has to ensure $u'(Q + k) < p_2$. ■
Proof of Lemma 2. It is easy to see \( p_2 \leq p_1 \) under AUD, because otherwise firm 2 would have no sales under DS. Here we only show \( p_1 < u'(k) \), based on the idea that if \( u'(k) \leq p_1 \), then \( BS_S(w) \geq BS_D(w) \), which implies \( BS_S(p_2) \geq BS_D(p_2) \) for \( p_2 \leq w \), because from (4) and (6), \( \partial BS_S/\partial p_2 \leq \partial BS_D/\partial p_2 \leq 0 \). That is, if \( u'(k) \leq p_1 \), then the buyer always chooses SS when \( p_2 \leq w \).

Suppose \( u'(k) \leq p_1 \). When \( q(w) \leq k \), \( BS_S(w) = v(w) \), \( BS_D(w) = v(w) + (w - p_1) \cdot Q \). Because \( w \leq p_1 \), we have \( BS_S(w) \geq BS_D(w) \). When \( k < q(w) \), we must have \( w = u'(Q) \) because our supposition \( u'(k) \leq p_1 \), thereby \( k < Q \).

\[ BS_S(w) = u(k) - u'(Q) \cdot k \]
\[ > u(Q) + u'(k) \cdot (k - Q) - u'(Q) \cdot k \]
\[ = u(Q) + [u'(k) - u'(Q)] \cdot k - u'(k) \cdot Q \]
\[ > u(Q) - p_1 Q \]
\[ = BS_D(w), \]

where the first inequality follows from \( u''(\cdot) < 0 \) and the second inequality follows from \( k < Q \) and \( u'(k) \leq p_1 \). □

Proof of Lemma 3. First, we show that \( BS_D(w) > BS_S(w) \). Suppose not. Then \( BS_D(w) \leq BS_S(w) \) implies \( BS_D(p_2) \leq BS_S(p_2), \forall p_2 \leq w \), because \( \partial BS_S/\partial p_2 \leq \partial BS_D/\partial p_2 \leq 0 \). We have \( p_2 \leq w \) in equilibrium because otherwise firm 2 would have no sales. It follows that the buyer would always choose SS from firm 2 when \( BS_D(w) \leq BS_S(w) \). Thus, in order to induce DS we must have \( BS_D(w) > BS_S(w) \).

Recall from the proof of Lemma 1 that \( BS_D(u'(Q + k)) < BS_S(u'(Q + k)) \). Combining it and \( BS_D(w) > BS_S(w) \) with \( \partial BS_S/\partial p_2 \leq \partial BS_D/\partial p_2 \leq 0 \), the unique intersection follows.

In the AUD equilibrium, we must have \( x < p_2 \), because otherwise firm 1 would have no sales. \( x < p_2 \), (C2) and (C1) together yield \( x < p_2 \leq p_1 < u'(k) \). Because now \( x < u'(k), \bar{q}(k, x) = k \) all the time. So the determination (9) follows. □

Proof of Lemma 4. First, in the AUD equilibrium, \( x \leq p_2 \leq w \). The first inequality holds because otherwise the buyer would SS and firm 1 would have no sale. The second inequality follows because otherwise firm 2 would have no sales.

To ensure firm 2 chooses \( p_2 \) s.t. \( x \leq p_2 \leq w \), we must have (11). Note that firm 2’s profit has a drop at \( p_2 = x \), i.e., \( (x - c) \cdot k > (x - c) \cdot [q(x) - Q] \), because \( u'(Q + k) < x \). In order to have (11), we must have the optimal \( p_2 \) to \( \max_{x \leq p_2 \leq w}(p_2 - c) \cdot [q(p_2) - Q] \) as an interior solution, i.e., \( x < p_2 \leq w \). The first-order condition for an interior solution satisfies (13). Clearly, \( \max_{p_2 \leq x}(p_2 - c) \cdot k = (x - c) \cdot k \).

Next, we show that (12) holds in equilibrium. Using (10),

\[ \pi_1 = (p_1 - c) \cdot Q = (x - c)Q + v(x) - [u(k) - x \cdot k], \]

\[ \frac{\partial \pi_1}{\partial x} = Q + k - q(x) > 0, \]

where the inequality follows from \( u'(Q + k) < x \) and \( Q > 0 \). Consequently, as long as (11) is not binding, \( \pi_1 \) can always be increased by increasing \( x \). □
To prove Proposition 3, we first establish two lemmas. Lemma A.1 shows that, ignoring \( \mathbb{C}_1 \) and \( \mathbb{C}_2 \), \( \pi_1^{AUD}(Q) \) is single-peaked in \( Q \), and the peak satisfies all other constraints.

**Lemma A.1** For any \( k \in (0, q(c)) \), there exists a unique \( Q(k) \) that satisfies (FOC), (9), (12), (13), and (14). Moreover, \( Q(k) \) is strictly decreasing in \( k \).

Thus, when neither \( \mathbb{C}_1 \) nor \( \mathbb{C}_2 \) is binding, such a peak maximizes \( \pi_1^{AUD}(Q) \). So the key question is when and which of the constraints \( \mathbb{C}_1 \) and \( \mathbb{C}_2 \) will be binding. Lemma A.2 offers an answer to it.

**Lemma A.2 (When \( \mathbb{C}_2 \) is binding)** Given \( (Q, p_1, p_2, x) \) jointly determined by (9), (12), (13), and (FOC),

(i) \( u'(Q + k) < x < u'(k) \) implies \( \mathbb{C}_1 \);

(ii) there exists a unique \( \hat{k} \in (0, q(c)) \) such that \( \mathbb{C}_2 \) is binding if and only if \( k \geq \hat{k} \).

Part (i) of the lemma says that, given other constraints hold, \( \mathbb{C}_1 \) is redundant. So the only possible binding constraint is \( \mathbb{C}_2 \). Part (ii) tells us that \( \mathbb{C}_2 \) is binding only for \( k \) above \( \hat{k} \).

**Proof of Lemma A.1** Here we first show the existence, uniqueness and sufficiency of (FOC), and then we prove the solution to it satisfies the inequality constraint (14). Last, we show \( Q'(k) < 0 \).

**Step 1: Existence, Uniqueness and Sufficiency of (FOC)**

From (12), \( x(Q) = \frac{\pi^s(Q)}{k} + c \) for all \( Q \in [0, q(c)] \). Note that

\[
\frac{d\pi_1^{AUD}}{dQ} = \frac{p_2 - c}{k} \cdot \varphi(Q),
\]

where \( \varphi(Q) \equiv q(x) + q(p_2) - 2Q - k \).

Let \( Q_k \equiv \pi'(p_k) \) with \( p_k \) satisfying \( k + (p_k - c) \cdot q'(p_k) = 0 \). From (12), it follows that \( x < p_2 \) for \( Q > Q_k \).

Next we show that \( \varphi'(Q) < 0 \) for \( Q \in [Q_k, q(c)] \) and there exists a unique \( Q(k) \in [Q_k, q(c)] \) s.t. \( \varphi(Q(k)) = 0 \). These together imply that \( \pi_1^{AUD} \) is single-peaked in \( Q \), and \( Q(k) \) is such a unique peak.

\[
\varphi'(Q) = q'(x) \cdot x'(Q) + q'(p_2) \cdot p_2'(Q) - 2
= q'(x) \cdot \left(-\frac{p_2 - c}{k}\right) + q'(p_2) \cdot \frac{1}{\pi''(p_2)} - 2
= \left[ q'(p_2) \frac{1}{\pi''(p_2)} - 1 \right] - k + (p_2 - c) \cdot q'(x)
\]

where the second equality follows from (12) and (13).

From \( q''(\cdot) \leq 0, \pi''(p_2) = 2q''(p_2) + (p_2 - c)q''(p_2) < q'(p_2) < 0 \). Hence, \( 0 < \frac{q'(p_2)}{\pi''(p_2)} < 1 \). For \( Q > Q_k \), we have \( p_2 = p_2(Q) Q_2(Q_k) = p_k \), thereby \( k + (p_2 - c) \cdot q'(p_2) > 0 \). Hence, \( k + (p_2 - c) \cdot q'(x) \geq 0 \) for \( Q > Q_k \) follows from \( x < p_2 \) and \( q'' \leq 0 \). As a result, we have \( \varphi'(Q) < 0 \) for \( Q \in [Q_k, q(c)] \).

Last, we show that \( \varphi(Q) \) does cross zero from above. At \( Q = q(c), p_2(q(c)) = x(q(c)) = c, \) thus \( \varphi(q(c)) = -k < 0 \). At \( Q = Q_k, x = p_2 = p_k \), thus \( \varphi(Q_k) = 2 \cdot |q(p_k) - Q_k| - k = 0 \) with "=" only if \( k = 0 \). So there exists a unique \( Q(k) \in [Q_k, q(c)] \) s.t. \( \varphi(Q(k)) = 0 \).
Step 2: Check Constraints \( u'(Q + k) < x < p_2 < u'(Q) \)

Note that \( x < p_2 \) has been shown in Step 1 for \( Q(k) > Q_k \), and that \( p_2 < u'(Q(k)) \) follows from (13). Moreover, \( u'(Q(k) + k) < x \) follows from (FOC), because \( Q(k) + q(x) = q(p_2) - Q(k) > 0 \) due to \( p_2 < u'(Q(k)) \).

Step 3: \( Q'(k) < 0 \)

Total differentiate (FOC) w.r.t. \( k \), we have
\[
\varphi'(Q) \cdot Q'(k) + \frac{\partial \varphi}{\partial x} \cdot \frac{\partial x}{\partial k} + \frac{\partial \varphi}{\partial k} = 0
\]
\[
\therefore \varphi'(Q) \cdot Q'(k) = 1 - q'(x) \cdot \frac{\partial x}{\partial k}
\]
\[
= \frac{k + (x - c)q(x)}{k},
\]
where the last equality follows from (12). Recall that in Step 1, we have shown that \( \varphi'(Q) < 0 \) for \( Q \in [Q_k, q(c)] \). Moreover, \( k + (x - c)q(x) > k + (p_2 - c)q(p_2) > 0 \) for all \( Q \in [Q_k, q(c)] \), where the first inequality follows from \( q'' \leq 0 \) and \( x < p_2 \), and the second one follows from \( Q(k) \geq Q_k \) and the definition of \( Q_k \) in Step 1. Thus, \( Q'(k) < 0 \).

Proof of Lemma A.2 (i) From (10), \( p_1 \cdot Q = x \cdot Q + v(x) - [u(k) - x \cdot k] \), we have
\[
[p_1 - u'(k)] \cdot Q = [x - u'(k)] \cdot Q + v(x) - [u(k) - x \cdot k]
\]
\[
< [x - u'(k)] \cdot Q + [u'(k) - x] \cdot [q(x) - k]
\]
\[
= [u'(k) - x] \cdot [q(x) - k - Q]
\]
\[
< 0,
\]
where the concavity of \( u(q) \) leads to the first inequality, and the second inequality follows from \( u'(Q + k) < x < u'(k) \).

(ii) \( C2 \) \( p_2 \leq p_1 \) can be rewritten as \( (p_1 - c) \cdot Q - (p_2 - c) \cdot Q \geq 0 \). Let
\[
D(k) \equiv \pi_1^{AUD}(Q) - (p_2 - c) \cdot Q
\]
\[
= v(x) - [u(k) - x \cdot k] - (p_2 - x) \cdot Q,
\]
where \( (Q(k), p_2(k), x(k)) \) is jointly determined by (12), (13), and (FOC). Then, \( C2 \) becomes \( D(k) \geq 0 \).

Here we first show that \( D(0) > 0 \) and \( D(k) < 0 \) for \( \gamma \leq k \), then we prove \( D(k) \) decreases with \( k \) for \( k \leq \alpha \). Hence, we conclude with the existence of \( \hat{k} \in (0, \alpha) \) s.t. \( D(k) \geq 0 \) for \( k \leq \hat{k} \).

Step 1: \( D(0) > 0 \) and \( D(k) < 0 \) for \( \alpha \leq k \).

When \( k = 0 \), \( x = p_2 = c \). \( D(0) = v(c) > 0 \).

From (13), \( p_2 \) decreases with \( Q \). Combining with \( Q'(k) < 0 \) from Lemma A.1, we have \( p_2(k) \) increases with \( k \). From the concavity of \( u(q) \), \( u'(k) \) decreases with \( k \), \( p_2(0) = c < u'(0), p_2(q(c)) > c = u'(q(c)) \).

Thus, there exists a unique \( \alpha \in (0, q(c)) \) s.t. \( p_2(k) \leq u'(k) \) for \( k \leq \alpha \).
For $k \geq \alpha$, when $x \leq u'(k)$,

$$D(k) = v(x) - [u(k) - x \cdot k] - (p_2 - x) \cdot Q$$

$$< [u'(k) - x] \cdot [q(x) - k] - (p_2 - x) \cdot \pi'(p_2) \quad (\therefore u''(q) < 0 \text{ and } x \leq u'(k))$$

$$= [u'(k) - x] \cdot [\pi'(p_2) + (p_2 - c) \cdot q'(p_2)] - (p_2 - x) \cdot \pi'(p_2) \quad \text{(by FOC)}$$

$$= [u'(k) - p_2] \cdot \pi'(p_2) + [u'(k) - x] \cdot (p_2 - c) \cdot q'(p_2) \quad (\therefore x \leq u'(k) \leq p_2)$$

$$< 0$$

For $k \geq \alpha$, when $x > u'(k)$,

$$D(k) < [u'(k) - p_2] \cdot \pi'(p_2) + [u'(k) - x] \cdot (p_2 - c) \cdot q'(p_2)$$

$$= -\{[p_2 - u'(k)] \cdot \pi'(p_2) - [x - u'(k)] \cdot [-\pi'(p_2)]\}.$$

If we can show that $[p_2 - u'(k)]/[x - u'(k)] > -\pi'(p_2)$, then $D(k) < 0$.

Because $u'(k) > c$ and $p_2 > x$, $[p_2 - u'(k)]/[x - u'(k)] > (p_2 - c)/(x - c)$. Thus, to show $D(k) > 0$, it suffices to show

$$\frac{p_2 - c}{x - c} \geq \frac{-\pi'(p_2)}{\pi'(p_2)},$$

which is reduced to

$$\pi'(p_2) + (x - c) \cdot q'(p_2) \geq 0.$$

$$\pi'(p_2) + (x - c) \cdot q'(p_2) = q(x) - k - (p_2 - x) \cdot q'(p_2) \quad \text{(by FOC)}$$

$$\geq (p_2 - x) \cdot [-q'(p_2)] - [x - u'(k)] \cdot [-q'(x)] \quad (\therefore q''(p) \leq 0)$$

Because $q''(p) \leq 0$ and $p_2 > x$, $-q'(p_2) \geq -q'(x)$. If we can show that $p_2 - x > x - u'(k)$, then

$$\pi'(p_2) + (x - c) \cdot q'(p_2) \geq 0.$$

Because $u'(k) > c$,

$$p_2 - x > x - c \quad (19)$$

would imply $p_2 - x > x - u'(k)$.

(12) and (FOC) together imply (19) as follows. (12) gives

$$\frac{p_2 - c}{x - c} = \frac{k}{-\pi'(p_2)}$$

$$= \frac{q(x) - q(p_2) - 2 \cdot (p_2 - c) \cdot q'(p_2)}{-\pi'(p_2)} \quad \text{(by FOC)}$$

$$= 2 + \frac{q(x) - q(p_2)}{-\pi'(p_2)} \quad \text{(by (FOC))}$$

$$> 2 \quad (\therefore x < p_2)$$

Hence, $D(k) < 0$ for $k \geq \alpha$, when $x > u'(k)$. This completes Step 1.
Step 2: $D'(k) < 0$ for $k \leq \alpha$.

Using (FOC),

$$D'(k) = \frac{\partial D}{\partial k} + \frac{\partial D}{\partial x} \cdot \frac{\partial x}{\partial k} + \frac{\partial D}{\partial p_2} \cdot p'_2(k)$$

$$= x - u'(k) + [k + \pi'(p_2) - q(x)] \cdot \left(\frac{x - c}{k}\right) - [\pi'(p_2) + p_2\pi''(p_2)] \cdot p'_2(k)$$

$$= x - u'(k) + [k - q(x)] \cdot p'_2(k) + (p_2 - c)q'(p_2) \cdot \frac{x - c}{k}$$

$$+ (p_2 - c) \cdot \left[\frac{p_2 + c}{p_2 - c}q'(p_2) - p_2q''(p_2)\right] \cdot p'_2(k) \quad \text{(by (FOC))}$$

$$\leq [k - q(x)] \cdot p'_2(k) + (x - p_2) + (p_2 - c)q'(p_2) \cdot \frac{x - c}{k}$$

$$+ (p_2 - c) \cdot \left[\frac{p_2 + c}{p_2 - c}q'(p_2) - p_2q''(p_2)\right] \cdot p'_2(k), \quad \text{\textit{(1)}}$$

$$\leq [k - q(x)] \cdot p'_2(k) + (x - p_2) + (p_2 - c)q'(p_2) \cdot \frac{x - c}{k}$$

$$+ (p_2 - c) \cdot \left[\frac{p_2 + c}{p_2 - c}q'(p_2) - p_2q''(p_2)\right] \cdot p'_2(k), \quad \text{\textit{(2)}}$$

$$\leq [k - q(x)] \cdot p'_2(k) + (x - p_2) + (p_2 - c)q'(p_2) \cdot \frac{x - c}{k}$$

$$+ (p_2 - c) \cdot \left[\frac{p_2 + c}{p_2 - c}q'(p_2) - p_2q''(p_2)\right] \cdot p'_2(k), \quad \text{\textit{(3)}}$$

where the inequality follows from $p_2 \leq u'(k)$ for $k \leq \alpha$. Note that $[k - q(x)] \cdot p'_2(k)$ is negative, because $x < u'(k)$ for $k \leq \alpha$ and $p'_2(k) > 0$. So if we can show \textit{(1)} + \textit{(2)} + \textit{(3)} is negative, then this part is complete.

From (FOC),

$$p'_2(k) = \frac{k + (x - c) \cdot q'(x)}{-\pi''(p_2) \cdot [k + (p_2 - c)q'(x)] + k \cdot [-q'(p_2) - (p_2 - c)q''(p_2)]}$$

$$< \frac{k + (x - c)q'(x)}{2k + (p_2 - c)q'(x)} \cdot \frac{1}{-q'(p_2) - (p_2 - c)q''(p_2)}, \quad \text{(20)}$$

where the inequality follows from $-\pi''(p) < -q'(p) - (p - c)q''(p)$. It is easy to verify that

$$-\frac{p_2 + c}{p_2 - c}q'(p_2) - p_2q''(p_2) < -q'(p_2) - (p_2 - c)q''(p_2), \quad \text{(21)}$$

as it is equivalent to $\pi''(p) < 0$. Hence,

\textit{(2)} + \textit{(3)}

$$< (p_2 - c)q'(p_2) \cdot \frac{x - c}{k} + (p_2 - c) \cdot [-q'(p_2) - (p_2 - c)q''(p_2)] \cdot p'_2(k)$$

$$< (p_2 - c)q'(p_2) \cdot \frac{x - c}{k} + (p_2 - c) \cdot \frac{k + (x - c)q'(x)}{2k + (p_2 - c)q'(x)}$$

$$= \frac{(p_2 - c)q'(p_2) \cdot (x - c)2k - (x - c)q'(x) \cdot (x - c)k + k \cdot (p_2 - c) \cdot [k + (x - c) \cdot q'(x)]}{2k + (p_2 - c)q'(x)}$$

$$= \frac{(p_2 - c) \cdot [k + 2(x - c)q'(p_2)] + (x - c)q'(x) \cdot (p_2 - x)}{2k + (p_2 - c)q'(x)},$$

where the first inequality follows from (21), the second follows by (20), and the last equality is from (9).
Therefore,

\[
\langle 1 \rangle + \langle 2 \rangle + \langle 3 \rangle \\
< (x - p_2) + \frac{(p_2 - c) \cdot [k + 2(x - c)q'(p_2)] + (x - c)q'(x) \cdot (p_2 - x)}{2k + (p_2 - c)q'(x)}
\]

\[
= (p_2 + c - 2x)[q(p_2) - q(x)] + 2(p_2 - x) \cdot (p_2 - c)q'(p_2) - (p_2 - x)^2 \cdot q'(x)
\]

\[
\leq (p_2 + c - 2x)q'(x)(p_2 - x) + 2(p_2 - x) \cdot (p_2 - c)q'(p_2) - (p_2 - x)^2 \cdot q'(x)
\]

\[
= \frac{p_2 - x}{2k + (p_2 - c)q'(x)} \cdot [(p_2 - c)q'(p_2) + (p_2 - c)q'(p_2) - (x - c)q'(x)],
\]

where the first equality follows from \(\text{(FOC)}\), and the second inequality is due to \(q''(p) \leq 0\) and \(x < p_2\). Indeed, \(2k + (p_2 - c)q'(x) > k + (p_2 - c)q'(x) > k + (p_2 - c)q'(p_2) > 0\), where the second inequality follows from \(q''(p) \leq 0\) and \(x < p_2\). Because \((p - c)q'(p)\) is decreasing in \(p\), \(\forall p > c\), \((p_2 - c)q'(p_2) - (x - c)q'(x) < 0\) as \(x < p_2\). Thus, \((p_2 - c)q'(p_2) + (p_2 - c)q'(p_2) - (x - c)q'(x) < 0\), thereby Step 2 is completed.

Step 3: There exists a unique \(\hat{k} \in (0, \alpha)\) s.t. \(D(k) \geq 0\) for \(k \leq \hat{k}\).

This follows directly from Steps 1 and 2. ■

Proof of Proposition 3

With Lemmas 2 and A.2, we know that when \(k < \hat{k}\), the equilibrium outcome \((Q, p_1, p_2)\) along with threat price \(x\) is jointly determined by \((9), (12), (13),\) and \(\text{(FOC)}\), with \(\text{(FOC)}\) being replaced by \(\text{(C2)}\) when \(\hat{k} \leq k\). The sufficiency of \(\text{(FOC)}\) has already been shown in the proof of Lemma A.1.

In the proof of Lemma A.2, \(u'(Q + k) < x < u'(k)\) ensures that \(p_1 < u'(k)\). And we know that \(u'(Q + k) < x < u'(k)\) is true under \(\text{(FOC)}\) for \(k \leq \hat{k}\). So here we only need to show the existence of equilibrium when \(\hat{k} \leq k\), and check the constraint \(u'(Q + k) < x < u'(k)\).

Step 1: Existence of the Solution to \(p_2 = p_1\) when \(\hat{k} \leq k\).

Similar to \(D(k)\) but without using the equilibrium \(Q(k)\) from \(\text{(FOC)}\), we can define

\[
d(Q, k) \equiv \pi_1^{A,U,D}(Q) - (p_2 - c) \cdot Q
\]

\[
= v(x) - [u(k) - x \cdot k] - (p_2 - x) \cdot Q,
\]

where \((p_2(Q), x(Q))\) is jointly determined by (12) and (13). So \(D(k) = d(Q(k), k)\), and the constraint \((\text{C2})\) \(p_2 \leq p_1\) is \(d(Q, k) \geq 0\).

When \(\hat{k} \leq k\), \(d(Q(k), k) = D(k) < 0\). At \(Q = q(c), (12)\) and (13) lead to \(x = p_2 = c\). So \(d(q(c), k) = v(c) - [u(k) - c \cdot k] > 0\). From the continuity of \(d(Q, k)\), there must exist a \(\overline{Q}(k) \in (Q(k), q(c))\) s.t. \(d(\overline{Q}(k), k) = 0\), i.e., \(p_2(\overline{Q}(k)) = p_1(\overline{Q}(k))\).

Step 2: Check Constraints \(u'(Q + k) < x < p_2 < u'(Q)\) and \(p_1 < u'(k)\)

Because \(d(\overline{Q}(k), k) = 0\), we have \((p_2 - x) \cdot \overline{Q} = v(x) - [u(k) - x \cdot k] > 0\). So \(p_2 > x\).

In Step 1 of the proof of Lemma A.1, when showing the sufficiency of \(\text{(FOC)}\), we proved \(\phi'(Q) < 0\) for any \(Q > Q_k\). Because \(Q_k < Q(k) < \overline{Q}(k)\), we have \(\phi'(Q) = q(x) + q(p_2) - 2\overline{Q} - k < 0\), which is equivalent to \(q(p_2) - \overline{Q} < k + \overline{Q} - q(x)\). Hence, for \(u'(\overline{Q} + k) < x\) and \(p_2 < u'(\overline{Q})\), it suffices to show that \(q(p_2) < \overline{Q}\), which follows from (13) \(\pi'(p_2) = \overline{Q}\).
Proof of Corollary 2. When $k < \hat{k}$,

$$Q + k = q(x) + q(p_2) - Q \quad \text{(By \text{FOC})}$$

$$= q(x) - (p_2 - c)q'(p_2) \quad \text{(By \ref{eq:13})}$$

$$\geq q(x) - (x - c)q'(x)$$

$$\geq q(c),$$

where the first inequality is from $(p_2 - c)q'(p_2)$ decreases with $p_2$ for $c \leq p_2$ and $x \leq p_2$, and the second inequality follows from $q(x) - (x - c)q'(x)$ is weakly increasing in $x$ for $c \leq x$. Note that “=” occurs only when $x = p_2 = c$, that is, only when $k = 0$.

When $k \geq \hat{k}$, \text{FOC} is replaced by \text{C2}. So the equilibrium solution $Q(k) > Q(k)$, where $Q(k)$ is characterized by \text{FOC}.

$$Q + k > q(x) + q(p_2) - Q \quad \text{(By $\varphi'(Q) < 0$ and $Q(k) > Q(k)$)}$$

$$= q(x) - (p_2 - c)q'(p_2) \quad \text{(By \ref{eq:13})}$$

$$\geq q(x) - (x - c)q'(x)$$

$$\geq q(c),$$

where the first inequality is from $(p_2 - c)q'(p_2)$ decreases with $p_2$ for $c \leq p_2$ and $x \leq p_2$, and the second inequality follows from $q(x) - (x - c)q'(x)$ is weakly increasing in $x$ for $c \leq x$. 

Proof of Corollary 3. (i) From Lemma \ref{lem:A.1} we have $Q'(k) < 0$ for $k \leq \hat{k}$.

Total output is $q(p_2)$. $p_2(Q)$ decreases with $Q$ from \ref{eq:13}. Because $Q(k)$ decreases with $k$, $p_2(k)$ must increase with $k$ for $k \leq \hat{k}$. So total output $q(p_2)$ decreases with $k$ for $k \leq \hat{k}$.

(ii) The comparative statics on $p_2(k)$ is shown in Part (i). Similarly, we can derive the same result for $x(k)$ from \ref{eq:12}.

(iii) For $k \leq \hat{k}$, $\pi_{1}\text{AUD} = v(x) - [u(k) - xk] + (x - c)Q$, and we have $u'(q + k) < x < u'(k)$. Thus

$$\frac{d\pi_{1}\text{AUD}}{dk} = [k + Q - q(x)] \cdot \frac{\partial x}{\partial k} - [u'(k) - x]$$

$$= [k + Q - q(x)] \cdot \left(\frac{1}{k} \cdot \left(\frac{x - c}{k}\right)\right) - [u'(k) - x] \quad \text{(By \ref{eq:12})}$$

$$< 0.$$ 

Because $\pi_{2}\text{AUD} = \pi_{R}(Q)$ decreases with $Q$, this part follows from Part (i).

(iv) $TS\text{AUD} = u(q(p_2)) - c \cdot q(p_2)$. $\frac{dTS\text{AUD}}{dp_2} = [u'(q(p_2)) - c] \cdot q'(p_2) < 0$. Then this part follows from the result on $p_2(k)$ of Part (ii).

Proof of Corollary 4. (i) Note that $p_2(0) = c < p^m = p^R(0)$, and $p_2(\alpha) = u'(\alpha) > p^R(\alpha) \cdot \pi'(p) < q(p))$. Moreover, $p_2'(k) > 0$ and $p^R(k) < 0$. Hence, $3$ a unique $k_0 \in (0, \alpha)$ s.t. $p_2(k) \leq p^R(k)$, $\forall k \leq k_0$.

(ii) $q_1\text{AUD} = Q > q(c) - k > q(p^R) - k = q_{1}\text{LP}$ follows from Corollary 2. $q_2\text{AUD} = q(p_2) - Q < k = q_{2}\text{LP}$.
follows from the fact that \( u'(Q + k) < p_2 \).

(iii) It is obvious that \( \pi_1^{AUD} > \pi_1^{LP} \), \( \pi_2^{AUD} = (p_2 - c) \cdot [q(p_2) - Q] < (p_2 - c) \cdot k < (p_2^{LP} - c) \cdot k = \pi_2^{LP} \), where the first inequality follows from the fact that \( u'(Q + k) < p_2 \) and the second one follows from \( p_2^{AUD} < p_2^{LP} \), \( \forall k < \min\{k_0, \hat{k}\} \).

(iv) When \( k \to 0 \), \( BS^{AUD} \to 0 \), and \( BS^{LP} \to v(p_m) > 0 \) as \( p \to p_m \). From continuity of \( BS^{AUD} \) and \( BS^{LP} \), this part follows.

(v) \( TS^{LP} = u(q(p^R)) - c \cdot q(p^R) \). Because \( p^R(k) < 0 \) and \( q(p^R) < c \), \( \frac{dT^S^{LP}}{dk} = [u'(q(p^R)) - c] \cdot q'(p^R) \cdot p^R(k) > 0 \).

Thus the sufficiency of (17) follows.

Proof of Proposition 4. First, the buyer’s problem in the last stage is the same as when

\[
(37)\]

Proof of Corollary 5. Given the equilibrium characterizations in Propositions 2 and 3, the comparisons on profits and total surpluses are straightforward. Here we only compare the buyer’s surpluses.

Note that \( BS^{2PT} = u(k) - ck \).

\[
BS^{AUD} = u(q(p_2)) - p_1 \cdot Q - p_2 \cdot [q(p_2) - Q] \\
= [v(p_2) + p_2 \cdot Q] - [v(x) + x \cdot Q] + [u(k) - x \cdot k] \\
\leq u(k) - x \cdot k \\
\leq u(k) - c \cdot k = BS^{2PT},
\]

where the first inequality is from the fact that \( v(p) + p \cdot Q \) decreases with \( p \) for any \( q(p) \geq Q \). Note that “=” occurs only when \( k = 0 \).

Proof of Proposition 4. First, the buyer’s problem in the last stage is the same as when \( c_1 = c_2 = c \), because the buyer does not face those costs.

Lemmas (11) and (13) can be applied with adaptations of \( c \) replaced by \( c_2 \) in (12) and (13), because those conditions are for firm 2. Firm 1’s profit is the same as before with replacement of \( c \) by \( c_1 \), so is for its first-order derivative (15), which becomes

\[
\frac{d\pi_1}{dQ} = x - c_1 + \{k - [q(x) - Q]\} \cdot x'(Q) \\
= x - c_1 - [k + Q - q(x)] \cdot \frac{p_2 - c_2}{k} \quad \text{(from (12) with } c = c_2). \]

Thus, (17) follows.

Because \( k < q(x) \), \( \pi_1 = (x - c_1) \cdot Q + v(x) - [u(k) - x \cdot k] \). Note that when \( x = c_1 \), \( \pi_1 = v(c_1) - [u(k) - c_1 \cdot k] \), which is firm 1’s profit under a 2PT. It is easy to verify that \( x = p_2 = c_1 \), \( Q = q(c_1) + (c_1 - c_2) \cdot q'(c_1) \) and \( k = -(c_1 - c_2) \cdot q'(c_1) \) satisfy (17) and \( k < q(x) = q(c_1) \). (16) ensures that \( x > c_1 \) so that the AUDs improve firm 1’s profit over a 2PT. Moreover, (16) implies that \( \varphi(0) > 0 \). Thus the sufficiency of (17) follows.
The existence and uniqueness of the equilibrium follow from the same analysis in Proposition 3.

<table>
<thead>
<tr>
<th></th>
<th>LP</th>
<th>2PT</th>
<th>AUD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Fee</td>
<td>N/A</td>
<td>(\frac{(10-k)^2}{2})</td>
<td>N/A</td>
</tr>
<tr>
<td>Quantity Threshold</td>
<td>N/A</td>
<td>N/A</td>
<td>(10 - (3 - \sqrt{5})k) when (k &lt; \tilde{k})(\frac{10 - 2a}{a}) when (k \geq \tilde{k})</td>
</tr>
<tr>
<td>Firm 1’s Per-Unit Price</td>
<td>(\frac{10-k}{2})</td>
<td>0</td>
<td>(\frac{50 - 10 \cdot k + \frac{\sqrt{5} - 9}{4} \cdot k^2}{10 - (3 - \sqrt{5})k} \frac{a}{a}) when (k &lt; \tilde{k}) when (k \geq \tilde{k})</td>
</tr>
<tr>
<td>Firm 2’s Per-Unit Price</td>
<td>(\frac{10-k}{2})</td>
<td>0</td>
<td>(\frac{3-\sqrt{5}}{2} \cdot k) when (k &lt; \tilde{k}) (\frac{a}{a}) when (k \geq \tilde{k})</td>
</tr>
</tbody>
</table>

Table A2: Equilibrium Surpluses for Linear Demand

<table>
<thead>
<tr>
<th></th>
<th>LP</th>
<th>2PT</th>
<th>AUD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1’s Profit</td>
<td>(\frac{(10-k)^2}{4})</td>
<td>(\frac{(10-k)^2}{2})</td>
<td>(50 - 10 \cdot k + \frac{\sqrt{5} - 9}{4} \cdot k^2) when (k &lt; \tilde{k}) (\frac{a(10 - 2a)}{a}) when (k \geq \tilde{k})</td>
</tr>
<tr>
<td>Firm 2’s Profit</td>
<td>(\frac{k(10-k)}{2})</td>
<td>0</td>
<td>(\frac{7 - 3\sqrt{5}}{2} \cdot k^2) when (k &lt; \tilde{k}) (\frac{a^2}{a^2}) when (k \geq \tilde{k})</td>
</tr>
<tr>
<td>Buyer’s Surplus</td>
<td>(\frac{(10+k)^2}{8})</td>
<td>(\frac{k(20-k)}{2})</td>
<td>(k \cdot \left[10 - (3 - \sqrt{5})k\right] \frac{(10-a)^2}{10-a}) when (k &lt; \tilde{k}) when (k \geq \tilde{k})</td>
</tr>
<tr>
<td>Total Surplus</td>
<td>(\frac{(10+k)(30-k)}{8})</td>
<td>50</td>
<td>(50 - \frac{7 - 3\sqrt{5}}{4} \cdot k^2) when (k &lt; \tilde{k}) (\frac{100 - a^2}{100 - a^2}) when (k \geq \tilde{k})</td>
</tr>
</tbody>
</table>

Note: In Tables A1 and A2, \(\tilde{k} = \frac{7}{25} \cdot (30 + 8\sqrt{5} - \sqrt{118 + 74\sqrt{5}}) \simeq 5.3538\). \(a\) is determined by \(a(a^3 - 4k \cdot a^2 + 6k^2 \cdot a - 20k^2) + k^2(10 - k)^2 = 0 (a < k)\).
References


