A Probability-Based Stress Test of Federal Reserve Assets and Income

Jens H. E. Christensen

Jose A. Lopez

and

Glenn D. Rudebusch

Federal Reserve Bank of San Francisco
101 Market Street, Mailstop 1130
San Francisco, CA 94105

Abstract

To support the economy, the Federal Reserve amassed a large portfolio of long-term bonds. We assess the Fed’s associated interest rate risk—including potential losses to the Fed’s Treasury securities holdings and declines in the Fed’s remittances to the Treasury—across various interest rate scenarios. Importantly, unlike past examinations of interest rate risk, we attach probabilities to these scenarios. Specifically, our probability-based stress test incorporates a dynamic term structure model that respects the zero lower bound on yields. We find that the Fed’s losses are unlikely to be large, and remittances are unlikely to show more than a brief cessation.

JEL Classification: G12, E43, E52, E58.

Keywords: term structure modeling, zero lower bound, monetary policy, quantitative easing.
1 Introduction

In late 2008, in response to a severe financial crisis and recession, the Federal Reserve reduced its key policy rate—the overnight federal funds rate—to its effective lower bound between 0 and 25 basis points. To provide additional monetary stimulus to spur economic growth and avoid deflation, the Fed also conducted several rounds of large-scale asset purchases—commonly referred to as quantitative easing (QE). These actions left the Federal Reserve’s portfolio of longer-term securities several times larger than its pre-crisis level. Although the Fed’s securities portfolio carries essentially no credit risk, its market value can vary over time, and the greater size of the Fed’s portfolio does potentially expose it to unusually large financial gains and losses from interest rate fluctuations. Furthermore, the Fed’s purchases have shifted the composition of the portfolio toward longer-maturity securities, which increases its sensitivity to interest rates changes. This larger exposure to interest rate risk has raised certain policy concerns. For example, former Fed Governor Frederic Mishkin (2010) has argued that “major holdings of long-term securities expose the Fed’s balance sheet to potentially large losses if interest rates rise. Such losses would result in severe criticism of the Fed and a weakening of its independence.”

In fact, there are two types of interest rate risk that the Fed faces. First, there is the risk that increases in longer-term interest rates will erode the market value of the Fed’s portfolio—that is, balance sheet risk. Such declines could lead to capital losses on the portfolio (which would be realized if the Fed sold the undervalued securities). Second, there is also the risk that increases in short-term interest rates, notably the interest rate that the Fed pays on bank reserves, will greatly increase the funding cost of the Fed’s securities portfolio—that is, income risk. Because the Fed’s interest income is generated from fixed coupon payments on longer-maturity securities, rising short-term interest rates would squeeze the Fed’s net interest income, which in turn would lower the Fed’s remittances to the U.S. Treasury. Under extreme circumstances, remittances could fall to zero. While the Fed’s ability to conduct monetary policy operations under such conditions would not be directly impeded, concerns have been raised about the attendant political fallout from large capital losses (realized or unrealized) or a cessation in remittances; see Rudebusch (2011) and Dudley (2013).

In order to understand and assess the Fed’s balance sheet and income risks, it is crucial to quantify them. Two recent papers—Carpenter et al. (2013) and Greenlaw et al. (2013), henceforth GHHM—

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1In the past, the Bank of Japan has taken threats of such political backlash quite seriously and has limited its balance sheet policy actions in part because of a fear that capital losses could tarnish its credibility. The Bank of England obtained in advance an explicit government indemnity for potential future capital losses stemming from its program of quantitative easing to insulate itself from such criticism.

2Regardless of its income expenses or portfolio losses, the Fed still has operational control of short-term interest rates because the Fed’s ability to pay interest on bank reserves allows it to conduct monetary policy independently of the size of its balance sheet. In addition, unrealized portfolio losses do not affect the Fed’s reported balance sheet, which is not marked to market. Realized portfolio losses do affect the Fed’s financial position, but the Fed can maintain its capital by recognizing future remittances to the Treasury (via the creation of a so-called deferred asset).
have made great progress in doing so. Both studies generated detailed projections of the market value and cash flow of the Fed’s assets and liabilities under a few specific interest rate scenarios. In essence, their projections are akin to the “stress tests” that large financial institutions undergo to gauge whether they have enough capital to endure adverse economic scenarios. As is common, these stress tests do not place probabilities on the alternative interest rate scenarios but simply consider, say, shifting the level of the entire yield curve up or down from the baseline by 100 basis points. Clearly, it is of much greater interest to know what likelihoods to attach to the range of considered outcomes. Attaching likelihoods to the alternative scenarios—or more generally, looking at the entire distributional forecast—would result in what we term “probability-based” stress tests. The addition of information about the relevant probability distribution of interest rate scenarios allows us to provide new assessments of the likelihoods of certain interest rate risk events. In this paper, we illustrate this new probabilistic methodology by examining potential mark-to-market losses on the Fed’s Treasury holdings and the potential cessation of its remittances to the Treasury. The addition of distributional information enables us to provide the likelihoods of certain events, such as losses on the Fed’s securities exceeding a certain threshold or a negative net interest income persisting for more than one year.

The key component of our probability-based stress-testing methodology is a dynamic term structure model that generates yield curve projections consistent with historical interest rate variation. Since nominal yields on Treasury debt are very near their zero lower bound (ZLB) due to Federal Reserve monetary policy actions, we use the shadow-rate, arbitrage-free Nelson-Siegel (AFNS) model class developed by Christensen and Rudebusch (2013a,b) to generate the requisite, potentially asymmetric, distributional interest rate forecasts. Shadow-rate models are latent-factor models in which the state variables have standard Gaussian dynamics, but the standard short rate is replaced by a shadow short rate that may be negative, as in the spirit of Black (1995). The model-generated observed short rate and yield forecasts thus respect the ZLB. Despite its inherent nonlinearity, shadow-rate AFNS models remain as flexible and empirically tractable as standard AFNS models. Critically for our purposes, we demonstrate that these models are able to accurately price the Fed’s portfolio of Treasury securities.

To assess the Fed’s balance sheet risk, we examine the distributional forecast of the value of Fed’s Treasury securities across 10,000 yield curve simulations. We focus just on nominal Treasury securities because, as of year-end 2012, they represent almost 60 percent of the securities held outright and the largest share of the Fed’s assets. The next largest share is comprised of agency mortgage-backed

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3Stress testing financial institutions is a critical endeavor in the wake of the financial crisis; see Schuermann (2013) for an overview. Our analysis is directly related to interest rate risk stress testing, which is discussed by Drehmann et al. (2010) and Abdymomunov and Gerlach (2013).

4Berkowitz (2000, p. 6) makes a similar point in the context of bank stress tests, arguing that “to make stress scenarios useful, they must be assigned probabilities.” See also Pritsker (2011).
securities (MBS). However, the valuation of MBS is very difficult because it requires assessing mortgage refinancing and prepayment patterns in simulated interest rate environments without historical parallel. Thus, our focus on Treasuries securities should provide an important first step toward more detailed probability-based stress tests.

For our empirical assessment of the Fed’s balance sheet risk, we use two methods for generating Treasury yield curve projections. The first approach is based on the shadow-rate AFNS model favored by Christensen and Rudebusch (2013b, henceforth CR) in their analysis of U.S. Treasury yields near the ZLB. The second relies on a non-parametric ranking of historically observed, yield curve changes. Despite differences in methods, the results are similar and indicate that potential losses on the Fed’s Treasury securities holdings over the next three years should be modest and manageable. In particular, our simulation results show that the projected median values of the Fed’s Treasury holdings do not fall below face value over the exercise’s three-year horizon.

To assess the Fed’s income risk, we use the model-based distributional yield curve projections to generate distributional projections of the Fed’s remittances to the Treasury. Still, even at the lower fifth percentile of our simulated distribution of outcomes, the cumulative remittance shortfall (or the Fed’s “deferred assets”) peaks at less than $11.0 billion in 2017. In fact, in nearly 90% of the simulations, no remittance shortfalls are projected over the seven-year horizon of the exercise. As a consequence, our probability-based stress-testing methodology suggests that the risk of a long cessation of the Fed’s remittances to the U.S. Treasury appears to be remote. Furthermore, our approach allows us to assess the cumulative remittances to the U.S. Treasury over the entire period of unconventional monetary policy from 2008 through 2020. Our results suggest that the U.S. Treasury in all likelihood will receive additional remittances from the Fed’s purchases of additional securities well in excess of their historical trend before 2008.

Finally, an important caveat to our analysis should be noted. We are not conducting a comprehensive assessment of the costs and benefits of the Fed’s program of QE, as discussed by Rudebusch (2011). Indeed, our probability-based stress test captures only part of the financial consequences of the Fed’s securities purchases and, notably, excludes two key fiscal benefits accruing to the Treasury. First, as longer-term interest rates were pushed lower by the Fed’s securities purchases, the resulting higher output and household income boosted federal tax revenue and reduced federal outlays. Second, the lower longer-term interest rates associated with QE also helped lower the Treasury’s borrowing costs for issuing new debt. Furthermore, it is important to stress that any kind of financial or fiscal accounting is ancillary to the Fed’s mission. The Fed, of course, strives to be a cost-efficient steward of the public purse, but its statutory mandate for conducting monetary policy is to promote

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5 The other securities in the Fed’s asset portfolio are foreign assets and other claims, TIPS, and agency debt.
6 Carpenter et al. (2013) and GHHM provide initial attempts at this effort.
7 See Gagnon et al. (2011), Christensen and Rudebusch (2012), and Bauer and Rudebusch (2013) amongst many others.
maximum employment and price stability. These macroeconomic goals are the key metrics for judging monetary policy. Financial considerations—even potentially large capital losses—are secondary.

The rest of the paper is structured as follows. Section 2 describes the evolution of the Fed’s securities portfolio since the onset of the financial crisis and our data sample. Section 3 describes the shadow-rate AFNS model, while Section 4 presents our specific empirical representation. Section 5 is dedicated to detailing the generation of yield curve projections, their conversion into portfolio stress tests, and the results. Section 6 details our projections of the Fed’s remittances to the U.S. Treasury. Section 7 concludes.

2 The Fed’s Securities Portfolio

Figure 1 shows the evolution of the assets of the Federal Reserve System at a weekly frequency since the start of 2008. In the early stages of the financial crisis, the Fed’s balance sheet was expanded through various emergency lending facilities, most notably the Term Auction Facility (TAF). In the figure, this lending appears in the “Other Assets” category, which currently represents less than 10

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8See Christensen et al. (2013) for details on the functioning and effectiveness of the TAF.
percent of the Fed’s assets. The “Non-Treasury Securities” category is almost exclusively comprised of agency MBS, much of which was purchased during the Fed’s first large-scale asset purchase program (QE1), which ran from late 2008 to early 2010. More recently, in its third purchase program (QE3), the Fed has purchased additional MBS. At the start of 2013, the MBS portfolio totalled $927 billion and represented 34.7% of the securities held outright.\footnote{The MBS portfolio is quite diverse. About 5.5 percent of the portfolio is spread across 22,164 securities with a holding of each less than $10 million. Another 9.2 percent is held in 3,932 securities with a maximum face value of $50 million.} This category also contains a very small amount—about $77 billion as of the start of 2013—of federal agency debt issued by Fannie Mae and Freddie Mac.

The “Treasury Securities” category experienced a large expansion during QE2, which operated from November 2010 through June 2011. A small share of these purchases were inflation-indexed, Treasury Inflation Protected Securities (TIPS). These totaled $75 billion in principal and another $11 billion in accrued inflation compensation as of January 2, 2013.\footnote{Christensen and Gillan (2013) analyze the effects of the TIPS purchases included in QE2.} At the start of 2013, another $1.58 trillion was spread across 241 nominal Treasury securities. It is this portfolio—about 59% of the Fed’s securities held outright—that we will focus on in our analysis of the Fed’s balance sheet risk. As noted earlier, the duration of this portfolio is also relevant for assessing balance sheet risk. From September 2011 through the end of 2012, the Fed conducted a Maturity Extension Program that sold Treasury securities with remaining maturities of three years or less and purchased a similar amount of Treasury securities with remaining maturities of six to thirty years. As a result of this policy, the Fed sold almost all of its short-term Treasury securities, and Treasuries with less than three years to maturity only represented 0.25% of the Treasury securities holdings at year-end 2012.

To obtain prices of the Fed’s Treasury holdings, we employ the data set of zero-coupon Treasury yields described in Gürkaynak et al. (2007).\footnote{For each business day, a zero-coupon yield curve is fitted to price a large pool of underlying off-the-run Treasury bonds. The Federal Reserve Board of Governors frequently updates the factors and parameters of this method; see the related website http://www.federalreserve.gov/pubs/feds/2006/index.html} We use daily yields from January 2, 1986, to January 2, 2013 for the following eleven maturities: 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 15-year, 20-year, and 30-year.\footnote{The longest maturity Treasury yields are not available prior to November 25, 1985. Also, between October 2001 and February 2006 the U.S. Treasury did not issue any 30-year bonds, but this absence has only a minuscule effect on our estimation results, which are primarily determined by the yields with 10 years or less to maturity.} As shown in Figure 2, Treasury yields were historically low at all maturities towards the end of our sample and near if not at the effective ZLB on nominal yields.

\[\text{\footnotesize \textsuperscript{9}}\text{\footnotesize The MBS portfolio is quite diverse. About 5.5 percent of the portfolio is spread across 22,164 securities with a holding of each less than $10 million. Another 9.2 percent is held in 3,932 securities with a maximum face value of $50 million.}\]

\[\text{\footnotesize \textsuperscript{10}}\text{\footnotesize Christensen and Gillan (2013) analyze the effects of the TIPS purchases included in QE2.}\]

\[\text{\footnotesize \textsuperscript{11}}\text{\footnotesize Gürkaynak et al. (2007) describe the zero-coupon yield data set.}\]

\[\text{\footnotesize \textsuperscript{12}}\text{\footnotesize The longest maturity Treasury yields are not available prior to November 25, 1985. Also, between October 2001 and February 2006 the U.S. Treasury did not issue any 30-year bonds, but this absence has only a minuscule effect on our estimation results, which are primarily determined by the yields with 10 years or less to maturity.}\]
3 A Shadow-Rate Model of U.S. Treasury Yields

The key ingredient for our probability-based stress test is a data generating process for the Treasury yield curve, and in this section, we describe the term structure model that we use for this purpose. Because short-term interest rates have been near zero since 2009, the proximity of the ZLB affects the pricing of Treasuries and induces a notable asymmetry into the distributional forecasts of future yields. To respect the ZLB and account for its effects, we employ a shadow-rate term structure model.

3.1 The Option-Based Approach to the Shadow-Rate Model

The concept of a shadow interest rate as a modeling tool to account for the ZLB can be attributed to Black (1995). He noted that the observed nominal short rate will be nonnegative because currency is a readily available asset to investors that carries a nominal interest rate of zero. Therefore, the existence of currency sets a zero lower bound on yields. To account for this ZLB, Black postulated as a useful modeling tool the use of a shadow short rate, $s_t$, that is unconstrained by the ZLB. The usual observed instantaneous risk-free rate, $r_t$, which is used for discounting cash flows when valuing
securities, is then given by the greater of the shadow rate or zero:

\[ r_t = \max\{0, s_t\}. \] (1)

Accordingly, as \( s_t \) falls below zero, the observed \( r_t \) simply remains at the zero bound.

While Black (1995) described circumstances under which the zero bound on nominal yields might be relevant, he did not provide specifics for implementation. The small set of empirical research on shadow-rate models has relied on numerical methods for pricing.\(^\text{13}\) To overcome the computational burden of numerical-based estimation that limits the use of shadow-rate models, Krippner (2012) suggested an alternative option-based approach that makes shadow-rate models almost as easy to estimate as the standard model. To illustrate this approach, consider two bond-pricing situations: one without currency as an alternative asset, and the other that has a currency in circulation with a constant nominal value and no transaction costs. In the world without currency, the price of a shadow-rate zero-coupon bond, \( P_t(\tau) \), may trade above par; that is, its risk-neutral expected instantaneous return equals the risk-free shadow short rate, which may be negative. In contrast, in the world with currency, the price at time \( t \) for a zero-coupon bond that pays $1 when it matures in \( \tau \) years is given by \( P^c(t, \tau) \). This price will never rise above par, so nonnegative yields will never be observed.

Now consider the relationship between the two bond prices at time \( t \) for the shortest (say, overnight) maturity available, \( \delta \). In the presence of currency, investors can either buy the zero-coupon bond at price \( P_t(\delta) \) and receive one unit of currency the following day or just hold the currency. As a consequence, this bond price, which would equal the shadow bond price, must be capped at 1:

\[
P_t(\delta) = \min\{1, P_t(\delta)\} = P_t(\delta) - \max\{P_t(\delta) - 1, 0\}.
\]

That is, the availability of currency implies that the overnight claim has a value equal to the zero-coupon shadow bond price minus the value of a call option on the zero-coupon shadow bond with a strike price of 1. More generally, we can express the price of a bond in the presence of currency as the price of a shadow bond minus the call option on values of the bond above par:

\[
P_t(\tau, \tau) = P_t^c(\tau) - C^A_t(\tau, \tau; 1), \tag{2}
\]

where \( C^A_t(\tau, \tau; 1) \) is the value of an American call option at time \( t \) with maturity in \( \tau \) years and strike

\(^\text{13}\) For example, Kim and Singleton (2012) and Bomfim (2003) use finite-difference methods to calculate bond prices, while Ichiue and Ueno (2007) employ interest rate lattices.
price 1 written on the shadow bond maturing in \( \tau \) years. In essence, in a world with currency, the bond investor has had to sell off the possible gain from the bond rising above par at any time prior to maturity.

Unfortunately, analytically valuing this American option is complicated by the difficulty in determining the early exercise premium. However, Krippner (2012) argues that there is an analytically close approximation based on tractable European options. Specifically, Krippner (2012) shows that the ZLB instantaneous forward rate, \( f_t(\tau) \), is

\[
 f_t(\tau) = f_t(\tau) + z_t(\tau),
\]

where \( f_t(\tau) \) is the instantaneous forward rate on the shadow bond, which may go negative, while \( z_t(\tau) \) is an add-on term given by

\[
 z_t(\tau) = \lim_{\delta \to 0} \left[ \frac{d}{d\delta} \left( \frac{C_{tE}(s, s + \delta; 1)}{P_t(s + \delta)} \right) \right],
\]

where \( C_{tE}(\tau, \tau + \delta; 1) \) is the value of a European call option at time \( t \) with maturity \( t + \tau \) and strike price 1 written on the shadow discount bond maturing at \( t + \tau + \delta \). Thus, the observed yield-to-maturity is

\[
 y_t(\tau) = \frac{1}{\tau} \int_t^{t+\tau} f_s(s) \, ds + \frac{1}{\tau} \int_t^{t+\tau} \lim_{\delta \to 0} \left[ \frac{\partial}{\partial \delta} \frac{C_{tE}(s, s + \delta; 1)}{P_t(s + \delta)} \right] \, ds
 = y_t(\tau) + \frac{1}{\tau} \int_t^{t+\tau} \lim_{\delta \to 0} \left[ \frac{\partial}{\partial \delta} \frac{C_{tE}(s, s + \delta; 1)}{P_t(s + \delta)} \right] \, ds.
\]

Thus, bond yields constrained at the ZLB can be viewed as the sum of the yield on the unconstrained shadow bond, denoted \( y_t(\tau) \), which is modeled using standard tools, and an add-on correction term derived from the price formula for the option written on the shadow bond that provides an upward push to deliver the higher nonnegative yields actually observed.

As highlighted by Christensen and Rudebusch (2013a,b), the Krippner (2012) framework should be viewed as not fully internally consistent and simply an approximation to an arbitrage-free model.\textsuperscript{14}

Of course, away from the ZLB, with a negligible call option, the model will match the standard arbitrage-free term structure representation. In addition, the size of the approximation error near the ZLB has been determined via simulation in Christensen and Rudebusch (2013a,b) to be quite

\textsuperscript{14}In particular, there is no explicit PDE that bond prices must satisfy, including boundary conditions, for the absence of arbitrage as in Kim and Singleton (2012).
modest.\textsuperscript{15}

### 3.2 The Shadow-Rate AFNS Model

In theory, the option-based shadow-rate result is quite general and applies to any assumptions made about the dynamics of the shadow-rate process. However, as implementation requires the calculation of the limit term under the integral, option-based shadow-rate models are limited practically to the Gaussian model class where option prices are available in analytical form. The arbitrage-free Nelson-Siegel (AFNS) representation developed by Christensen et al. (2011, henceforth CDR) is well suited for this extension.\textsuperscript{16} Its three factors correspond to the level, slope, and curvature factors commonly observed for Treasury yields and are denoted \( L_t, S_t, \) and \( C_t, \) respectively. The state vector is thus defined as \( X_t = (L_t, S_t, C_t). \textsuperscript{17} \)

In the shadow-rate AFNS model, the instantaneous risk-free rate is the nonnegative constrained process of the shadow risk-free rate, which is defined as the sum of level and slope as in the original AFNS model class:

\[
    r_t = \max\{0, s_t\}, \quad s_t = L_t + S_t. \tag{3}
\]

Also, the dynamics of the state variables used for pricing under the \( Q \)-measure remain as in the regular AFNS model:

\[
    \begin{pmatrix}
    dL_t \\
    dS_t \\
    dC_t
    \end{pmatrix} = 
    \begin{pmatrix}
    0 & 0 & 0 \\
    0 & -\lambda & \lambda \\
    0 & 0 & -\lambda
    \end{pmatrix} 
    \begin{pmatrix}
    L_t \\
    S_t \\
    C_t
    \end{pmatrix} 
    dt + \Sigma 
    \begin{pmatrix}
    dW_t^{L,Q} \\
    dW_t^{S,Q} \\
    dW_t^{C,Q}
    \end{pmatrix}, \tag{4}
\]

where \( \Sigma \) is the constant covariance (or volatility) matrix.\textsuperscript{18}

Based on this specification of the \( Q \)-dynamics, the yield on the shadow discount bond maintains the popular Nelson and Siegel (1987) factor loading structure

\[
y_t(\tau) = L_t + \left(1 - \frac{e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(1 - \frac{e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A(\tau)}{\tau}, \tag{5}
\]

where \( A(\tau)/\tau \) is a maturity-dependent yield-adjustment term. The corresponding instantaneous

\textsuperscript{15}Christensen and Rudebusch (2013a,b) analyze how closely the option-based bond pricing from their estimated shadow-rate AFNS models matches an arbitrage-free bond pricing that is obtained from the same models using Black’s (1995) approach based on Monte Carlo simulations. They consider bonds of maturities out to 10 years. We extended these simulation results to consider bond maturities of 30 years (needed for pricing the longest bonds in the Fed’s portfolio). At the thirty-year maturity, the approximation errors are understandably larger but still do not exceed 6 basis points, which are notably smaller than the model’s fitted errors.

\textsuperscript{16}For details of the derivations, see Christensen and Rudebusch (2013a).

\textsuperscript{17}Please note that U.S. supervisory guidance on IRR stress testing, as summarized in Supervision and Regulation Letter SR 10-1 (2010), fits this structure quite well as it instructs firms to examine large changes in the level, slope, and shape of the yield curve.

\textsuperscript{18}As per CDR, \( \Sigma \) is a diagonal matrix, and \( \theta^Q \) is set to zero without loss of generality.
shadow forward rate is given by

\[ f_t(\tau) = -\frac{\partial}{\partial \tau} \ln P_t(\tau) = L_t + e^{-\lambda \tau} S_t + \lambda \tau e^{-\lambda \tau} C_t + A^f(\tau), \]  

(6)

where the final term is another maturity-dependent yield-adjustment term.

Christensen and Rudebusch (2013a) show that, within the shadow-rate AFNS model, the zero-coupon bond yields that observe the zero lower bound, denoted \( y_t(\tau) \), are readily calculated as

\[ y_t(\tau) = \frac{1}{\tau} \int_t^{t+\tau} \left[ f_t(s) \Phi \left( \frac{f_t(s)}{\omega(s)} \right) + \omega(s) \left( -\frac{1}{2} \left( \frac{f_t(s)}{\omega(s)} \right)^2 \right) \right] ds, \]  

(7)

where \( \Phi(\cdot) \) is the cumulative probability function for the standard normal distribution, \( f_t(\tau) \) is the shadow forward rate, and \( \omega(\tau) \) takes the following simple form

\[ \omega(\tau)^2 = \sigma_{11}^2 \tau + \sigma_{22}^2 \left( \frac{1 - e^{-2\lambda \tau}}{2\lambda} \right) + \sigma_{33}^2 \left( \frac{1 - e^{-2\lambda \tau}}{4\lambda} - \frac{1}{2} \tau e^{-2\lambda \tau} - \frac{1}{2} \lambda \tau^2 e^{-2\lambda \tau} \right), \]

when the volatility matrix \( \Sigma \) is assumed diagonal.

As in the affine AFNS model, the shadow-rate AFNS model is completed by specifying the price of risk using the essentially affine risk premium specification introduced by Duffee (2002), so the risk premium \( \Gamma_t \) is defined by the measure change

\[ dW_t^Q = dW_t^P + \Gamma_t dt, \]

with \( \Gamma_t = \gamma^0 + \gamma^1 X_t, \gamma^0 \in \mathbb{R}^3, \) and \( \gamma^1 \in \mathbb{R}^{3 \times 3} \). Therefore, the real-world dynamics of the state variables can be expressed as

\[ dX_t = K^P (\theta^P - X_t) dt + \Sigma dW_t^P. \]  

(8)

In the unrestricted case, both \( K^P \) and \( \theta^P \) are allowed to vary freely relative to their counterparts under the \( Q \)-measure just as in the original AFNS model.

Finally, we note that, due to the non-linear measurement equation for the yields in the shadow-rate AFNS models, their estimation is based on the extended Kalman filter as described in Christensen and Rudebusch (2013a).

### 4 Model Estimation and Yield Curve Fit

In this section, we describe the in-sample estimation results for our preferred model, its fit to the yield curve, and its ability to price the Treasury securities in the Fed’s portfolio.
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Table 1: Parameter Estimates for the B-CR Model.

The estimated parameters of the $K^P$ matrix, $\theta^P$ vector, and diagonal $\Sigma$ matrix are shown for the B-CR model. The estimated value of $\lambda$ is 0.4868 (0.0010). The numbers in parentheses are the estimated parameter standard deviations.

4.1 Estimation

In this subsection, we briefly describe the shadow-rate AFNS model used here, which is the shadow-rate equivalent of the AFNS model preferred by Christensen and Rudebusch (2012). Using both in- and out-of-sample performance measures, the authors determined that the zero-value restrictions on the $K^P$ matrix in the following dynamic system for the $P$-dynamics were empirically warranted; i.e.,

$$
\begin{pmatrix}
\frac{dL_t}{dt} \\
\frac{dS_t}{dt} \\
\frac{dC_t}{dt}
\end{pmatrix} = \begin{pmatrix}
0 & 0 & 0 \\
\kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P \\
0 & 0 & \kappa_{33}^P
\end{pmatrix} \begin{pmatrix}
\theta_2^P \\
\theta_3^P \\
\theta_3^P
\end{pmatrix} - \begin{pmatrix}
L_t \\
S_t \\
C_t
\end{pmatrix} dt + \begin{pmatrix}
\frac{dW_{L,P}^t}{dt} \\
\frac{dW_{S,P}^t}{dt} \\
\frac{dW_{C,P}^t}{dt}
\end{pmatrix},
$$

(9)

where the covariance matrix $\Sigma$ is assumed diagonal and constant. Throughout, we refer to the shadow-rate AFNS model given by equations (3), (4), and (9) as the B-CR model.\(^{19}\)

Note that the level factor is here restricted to be an independent unit-root process under both probability measures.\(^{20}\) As discussed in Christensen and Rudebusch (2012), this restriction helps improve forecast performance independent of the specification of the remaining elements of $K^P$.\(^{21}\) Second, we test the significance of the four parameter restrictions imposed on $K^P$ in the model relative to the corresponding model with an unrestricted $K^P$ matrix.\(^{22}\) The test results show that the four parameter restrictions are either statistically insignificant or at most borderline significant throughout our sample period. Thus, the B-CR model is flexible enough to capture the relevant information in the data. Third, and more importantly, CR extend the analysis of Christensen and

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\(^{19}\)Following Kim and Singleton (2012), the prefix “B-” refers to a shadow-rate model in the spirit of Black (1995).

\(^{20}\)Due to the unit-root property of the first factor, we can arbitrarily fix its mean at $\theta_1^P = 0$. We note that, in the model estimation, we handle the non-stationarity of the factor dynamics in equation (9) in the way described in Christensen and Rudebusch (2012).

\(^{21}\)As described in detail in Bauer et al. (2012), bias-corrected $K^P$ estimates are typically very close to a unit-root process, so we view the imposition of the unit-root restriction as a simple shortcut to overcome small-sample estimation bias.

\(^{22}\)That is, we test the hypotheses $\kappa_{12}^P = \kappa_{13}^P = \kappa_{31}^P = \kappa_{32}^P = 0$ jointly using a standard likelihood ratio test.
Table 2: **Summary Statistics of the Fitted Errors.**

The mean and root mean squared fitted errors (RMSE) as well as the estimated yield error standard deviations for the B-CR model are shown. All numbers are measured in basis points. The data covers the period from January 2, 1986, to January 2, 2013.

<table>
<thead>
<tr>
<th>Maturity in months</th>
<th>B-CR model</th>
<th>Mean (\hat{\sigma}(\tau_i))</th>
<th>RMSE</th>
<th>(\tilde{\sigma}_e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-2.62</td>
<td>10.28</td>
<td>10.31</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.04</td>
<td>0.17</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>2.80</td>
<td>6.63</td>
<td>6.64</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>2.94</td>
<td>5.21</td>
<td>5.29</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>0.01</td>
<td>0.74</td>
<td>1.46</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>-5.35</td>
<td>8.18</td>
<td>8.28</td>
<td></td>
</tr>
<tr>
<td>84</td>
<td>-6.61</td>
<td>11.05</td>
<td>11.08</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>-3.66</td>
<td>9.32</td>
<td>9.30</td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>1.74</td>
<td>4.70</td>
<td>4.69</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>1.49</td>
<td>11.19</td>
<td>11.22</td>
<td></td>
</tr>
<tr>
<td>360</td>
<td>-10.23</td>
<td>33.69</td>
<td>33.73</td>
<td></td>
</tr>
<tr>
<td>Max log (L)</td>
<td></td>
<td>417,381.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Rudebusch (2012) to encompass the most recent period with yields near the ZLB. They document that the B-CR model outperforms its standard AFNS model equivalent in that period in terms of forecasting future policy rates and matching the compression in yield volatility. To summarize, the B-CR model has desirable dynamic properties in the current yield environment in addition to enforcing the ZLB.

The estimated model parameters are reported in Table 1, and the summary statistics of the model fit in Table 2 indicate the very good fit to the entire maturity range up to 20 years with some deterioration in the fit for the 30-year yield. The good fit of the B-CR model is also apparent in Figure 3, which shows the fitted yield curve as of January 2, 2013, with a comparison to the 11 observed yields on that day. The main weakness of the model on this particular day is a tendency to underestimate the 15- and 20-year yields. As we will see in the following section, this converts into a slight overestimation of the market value of the Fed’s portfolio.

### 4.2 Market Value of Fed’s Treasury Securities

To further validate the performance of the B-CR model, we calculate the model-implied value of the 241 Treasury securities that were in the Fed’s portfolio as of January 2, 2013, and compare the result to the bond prices downloaded from Bloomberg on that same day.\(^{23}\) Table 3 reports the total

\(^{23}\)The Fed’s portfolio holdings are available at: [http://www.newyorkfed.org/markets/soma/sysopen_acchandle.html](http://www.newyorkfed.org/markets/soma/sysopen_acchandle.html). Please note that our calculations are based on the face value of the securities; taking account of the individual amortized costs of the securities (i.e., using the actual trade premiums and discounts) is a topic for future research.
Figure 3: **Fitted Yield Curve from the B-CR Model.**
Illustration of the fitted yield curve as of January 2, 2013, based on the B-CR model. Included are the eleven observed Treasury yields on that day. The data used in the model estimation cover the period from January 2, 1986, to January 2, 2013.

Table 3: **Value of Fed’s Treasury Securities Portfolio.**
The table reports the distribution of the Fed’s Treasury securities portfolio across maturity buckets as of January 2, 2013, using three different valuation methods. The first method is the official account based on the bonds’ principal values. The second method is to calculate the market value based on bond prices from Bloomberg. The third method is to calculate the market value based on the estimated B-CR model. The reported bond values are measured in billions of dollars.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>No.</th>
<th>Official account</th>
<th>Market value as of January 2, 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Face value</td>
<td>Percent</td>
</tr>
<tr>
<td>All</td>
<td>241</td>
<td>1,580</td>
<td>100.00</td>
</tr>
<tr>
<td>3 years or less</td>
<td>93</td>
<td>4</td>
<td>0.25</td>
</tr>
<tr>
<td>4-6 years</td>
<td>72</td>
<td>630</td>
<td>39.87</td>
</tr>
<tr>
<td>7-10 years</td>
<td>38</td>
<td>577</td>
<td>36.53</td>
</tr>
<tr>
<td>11 or more years</td>
<td>38</td>
<td>369</td>
<td>23.35</td>
</tr>
</tbody>
</table>

value of the Fed’s Treasury securities portfolio and its distribution across maturity buckets. The first column shows the number of securities in each maturity bucket. The second and third columns report the official account based on the face value of the securities. The following two columns reflect the market value of the portfolio based on bond prices from Bloomberg. The last two columns contain the market value of the portfolio implied by the B-CR model as of January 2, 2013. Please note that
the Fed’s Treasury securities holdings have a market value that is more than $250 billion above their face value according to both pricing methods.

Contrasting the two pricing methods more directly, the B-CR model’s implied portfolio value was $22 billion (or 1.27%) greater than the market value reported by Bloomberg. Overall, we consider this valuation difference to be acceptable for our purposes. Figure 4 presents the distribution of pricing errors across bond maturities. We note that nearly one-third of the bonds (i.e., 69 of the 241 bonds) have fitted prices that are within a quarter-dollar of the Bloomberg price. These bonds were either close to maturity or had been issued fairly recently. Importantly, several of these bonds are in this latter group and have up to ten years to maturity, which shows the model is able to price medium- and long-term bonds quite accurately. The figure shows that the model tends to overvalue long-maturity bonds as the fitted long-term yields are below those observed, as shown in Figure 3. We also found that the model tends to overvalue bonds with large coupons, but this could be a consequence of the fact that these bonds are very seasoned and illiquid for that reason. Furthermore, many of these bonds are close to maturity and were sold in large amounts by the Fed during its maturity extension program that ended in December 2012, which likely their prices further. Despite these pricing errors, we emphasize that this exercise is a very strong test of the B-CR model as we
are able to match data that was not used in the model estimation.

5 Stress Testing the Fed’s Portfolio of Treasuries

In this section, we describe a probability-based stress test of the Fed’s portfolio of Treasury securities using the distributional forecasts of the yield curve. Our focus is on the likely evolution of the market value of the Fed’s portfolio of Treasury securities during an illustrative three-year horizon. We use two different approaches to illustrate a range of possible probability-based stress tests. The first approach is based on yield curve projections from the B-CR model. A model-based approach captures the important dynamics of term structure in a parsimonious, theoretically consistent framework. However, any model-based approach may suffer from model mis-specification. The second approach bases its yield curve projections on historical Treasury yield curve changes. This is relatively easy to implement, but may not produce good forecasts during the recent, atypical period of near-zero yields.

5.1 Projections Based on the B-CR Model of the Treasury Yield Curve

Treasury yield curve projections based on the estimated B-CR model allow us to assign probabilities to specific yield curve outcomes. As the yield function in equation (7) is non-linear in the state variables, we use Monte Carlo simulations to generate the yield curve projections. Specifically, we simulate 10,000 sample paths of the state variables up to three years ahead, each starting from the filtered values at the end of our sample, denoted \( \hat{X}_t = (\hat{L}_t, \hat{S}_t, \hat{C}_t) \). For each projection \( N \) months ahead, the simulated state variables are converted into a full yield curve, and we calculate the corresponding portfolio values.

Figure 5 presents a summary diagram of the short-rate (i.e., overnight fed funds rate) from the simulated yield curves. In particular, the figure shows the median and the [5%, 95%] range of the B-CR model’s implied short-rate over the first ten years of the forecast horizon. For the short rate, the median simulated yield remains at the ZLB for the first two years of the forecast horizon and gradually rises to 3% at the ten-year horizon. The upper 95 percentile rises more rapidly and reaches 7% at the ten-year horizon, while the lower 5 percentile remains at the ZLB throughout.

Within the figure, the long-term projections of the federal funds rate from the Blue Chip forecasters

\footnote{To be clear, the Fed values its securities at acquisition cost and registers capital gains and losses only when securities are sold. Such historical-cost accounting is considered appropriate given the Fed’s macroeconomic policy objectives and is consistent with the buy-and-hold securities strategy the Fed has traditionally followed. However, the Fed also does report unrealized capital gains and losses on its securities portfolio, which mimics private-sector mark-to-market accounting on holdings of longer-term securities. For an example, see the unaudited financial report of the Federal Reserve Banks for the second quarter of 2013, p. 8, available at: http://www.federalreserve.gov/monetarypolicy/files/quarterly-report-20130630.pdf.}

\footnote{Please note that the connecting lines in the graph do not constitute a single simulated yield curve, but a representation of these percentile values over the simulation horizon.
are also presented; in particular, the median (or consensus) forecasts as well as the averages of the top and bottom ten forecasts at each horizon. These Blue Chip forecasts fit well within the range of our simulated projections, which provides evidence that our analysis encompasses current rate expectations. A similar pattern is observed for the other maturities generated by the Blue Chip forecasters.

Table 4 reports the lower percentiles and median of the projected portfolio values for each quarterly forecast horizon up to three years out, while Figure 6 illustrates the percentiles as a function of the forecast horizon. These results clearly suggest that the proposed value of the Fed’s Treasury holdings is not likely to decline below face value over the defined period; i.e., the median value remains above face value through 2016, and at most, losses are expected to occur only with a one-percent probability by 2015.

As shown in Figure 5, the upward trending short rate projection over the forecast horizon explains why the median portfolio value in our projections trend lower as the forecast horizon is increased. Still, when the portfolio is kept fixed as in this exercise, extending the forecast horizon has two effects that go in opposite directions. On one hand, with a longer projection horizon, a wider range

Figure 5: Comparison of Short Rate Projections.
The graph presents the median and [5%, 95%] range of the fed funds rate from the B-CR model’s simulated interest rate scenarios as of January 2, 2013. The graph also shows the consensus federal funds rate forecast as well as the average of the top and bottom ten forecasts from the Blue Chip Financial Forecasts survey released on December 1, 2012.
Table 4: Model-Based Projected Market Value of the Fed’s Treasury Securities.
The table shows percentiles ranging from 0.1% to 50% in the distribution of the market value of the Fed’s Treasury securities portfolio projected between 3 and 36 months ahead based on $N = 10,000$ Monte Carlo simulations of the B-CR model as described in the main text. All portfolio values are measured in billions of dollars. Projections with portfolio values below the face value as of January 2, 2013, are shown in bold.

<table>
<thead>
<tr>
<th>Projection in months</th>
<th>Percentiles in portfolio value distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1%</td>
</tr>
<tr>
<td>3</td>
<td>1,754</td>
</tr>
<tr>
<td>6</td>
<td>1,699</td>
</tr>
<tr>
<td>9</td>
<td>1,656</td>
</tr>
<tr>
<td>12</td>
<td>1,611</td>
</tr>
<tr>
<td>15</td>
<td>1,592</td>
</tr>
<tr>
<td>18</td>
<td>1,569</td>
</tr>
<tr>
<td>21</td>
<td>1,532</td>
</tr>
<tr>
<td>24</td>
<td>1,523</td>
</tr>
<tr>
<td>27</td>
<td>1,505</td>
</tr>
<tr>
<td>30</td>
<td>1,490</td>
</tr>
<tr>
<td>33</td>
<td>1,478</td>
</tr>
<tr>
<td>36</td>
<td>1,466</td>
</tr>
</tbody>
</table>

The table shows percentiles ranging from 0.1% to 50% in the distribution of the market value of the Fed’s Treasury securities portfolio projected between 3 and 36 months ahead based on $N = 10,000$ Monte Carlo simulations of the B-CR model as described in the main text. All portfolio values are measured in billions of dollars. Projections with portfolio values below the face value as of January 2, 2013, are shown in bold.

of outcomes are likely, and the potential yield changes are larger, in particular in the tail of the distribution. On the other hand, there is a mechanical reduction in the time to maturity on all securities in the portfolio. Bonds with shorter time to maturity have lower duration and, as a consequence, their prices are less sensitive to changes in the interest rate environment. For this exercise, the results in Figure 6 indicate that the former effect dominates at all forecast horizons considered.

To provide a sense of what kind of yield changes it would take to observe the lower tail outcomes for these portfolio values, Figure 7 shows the projected yield curves that produce the first, fifth, and fiftieth (i.e., median) percentiles of portfolio values as of the end of 2015, as reported in the bottom row of Table 4. From the figure, it is clear that the outcomes around the first percentile that push the value of the Fed’s Treasury securities below their face value are associated with a sharp jump of the federal funds rate to above 6 percent and a corresponding dramatic increase in the entire yield curve from its level as of January 2, 2013, shown in Figure 3. Thus, it requires not just a normalization of monetary policy, but a dramatic overshooting of the policy rate to generate such outcomes. On the other hand, it is also clear that the median outcome only reflects a slight change from the yield curve as of January 2, 2013. This is caused by the fact that the model matches the compression in yield volatilities near the ZLB, which reduces the magnitude of the yield curve changes and the variation in the model’s projected market valuations.
5.2 Projections Based on Historical Treasury Yield Curve Changes

In this section, we use the historical yield curve changes in our sample of daily Treasury yields since 1986 to generate Treasury yield curve projections up to three years ahead. The basic idea is to look to the past for guidance on what might happen $N$ months ahead in the current situation. The first step is to estimate the B-CR model on the full sample that ends on January 2, 2013. In the second step, for the $N$-month projection, we go through all the $N$-month yield curve changes observed in the data as measured by changes in the three state variables within the B-CR model. We denote the total number of such yield curve or, equivalently, state variable changes by $m_N$. In the third step, we take the estimated state variables as of January 2, 2013, denoted $\tilde{X}_t = (\tilde{L}_t, \tilde{S}_t, \tilde{C}_t)$, and add each of the $i = 1, \ldots, m_N$ factor shock constellations identified in the previous step. This gives us $m_N$ new state variable constellations each of which has the property that the yield curve shocks happened once before. In the fourth step, we convert each new state variable constellation into a full yield curve using the estimated yield function in equation (7) and then calculate the value of the

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26In the exercise, one month is defined as 21 observation dates, which is close to the average for the entire sample. Thus, for example, the six-month projections are based on the observed yield curve changes 126 observation dates apart.
Fed’s Treasury portfolio. In the fifth step, we rank all the estimated portfolio values and focus on the lowest percentiles as well as the median. Finally, this process is repeated for the forecast horizons $N \in \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36\}$ measured in months.

This approach has several advantages. First, the projections are predominantly data-driven, not model-based. Second, realism is embedded in the exercise by construction as these are yield curve changes that have occurred in the past. Finally, it is an exercise that other researchers can easily repeat and apply to other data samples and other models. In particular, it is an approach that can be applied to analyze potential stress of the securities portfolios of other central banks that have engaged in quantitative easing, notably the Bank of England and the Bank of Japan.

Table 5 reports percentiles in the projected distribution of the market value of the Fed’s Treasury securities portfolio, while Figure 8 provides a graphical representation of the same data. This historical simulation allows us to provide a comparison for the model-based projections discussed above. For this approach, the declines in the median and the lower 25 percentile are much more modest as the forecast horizon lengthens. In particular, projected portfolio values do not dip below face value even at the 0.1% tail. This milder outcome is due to the fact that yields have trended lower on average since 1986, as shown in Figure 2. These two approaches to generating yield curve projections are quite different. A key difference is that the historical approach reflects all yield curve
Projection in months | No. | Percentiles in portfolio value distribution (in billions of dollars)
--- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
3 | 6,675 | 1.717 | 1.744 | 1.794 | 1.813 | 1.844 | 1.873 |
6 | 6,612 | 1.653 | 1.709 | 1.759 | 1.790 | 1.832 | 1.877 |
9 | 6,550 | 1.611 | 1.668 | 1.742 | 1.781 | 1.833 | 1.876 |
12 | 6,487 | 1.617 | 1.653 | 1.726 | 1.774 | 1.828 | 1.873 |
15 | 6,425 | 1.613 | 1.647 | 1.728 | 1.766 | 1.822 | 1.872 |
18 | 6,363 | 1.622 | 1.673 | 1.725 | 1.772 | 1.818 | 1.866 |
21 | 6,300 | 1.667 | 1.690 | 1.739 | 1.770 | 1.814 | 1.860 |
24 | 6,238 | 1.657 | 1.690 | 1.748 | 1.775 | 1.816 | 1.853 |
27 | 6,176 | 1.665 | 1.691 | 1.745 | 1.781 | 1.816 | 1.849 |
30 | 6,113 | 1.664 | 1.709 | 1.744 | 1.774 | 1.811 | 1.845 |
33 | 6,051 | 1.674 | 1.699 | 1.746 | 1.770 | 1.806 | 1.836 |
36 | 5,989 | 1.658 | 1.690 | 1.746 | 1.770 | 1.797 | 1.830 |

Table 5: **History-Based Projected Market Value of the Fed’s Treasury Securities.**
The table shows percentiles ranging from 0.1% to 50% in the distribution of the market value of the Fed’s Treasury securities portfolio projected between 3 and 36 months ahead based on historical yield curve changes as described in the main text. The total number of projections for each forecast horizon is reported in the second column and declines over the forecast horizon as there are fewer non-overlapping periods within the dataset. All portfolio values in columns 3 to 8 are measured in billions of dollars.

changes in the past, but does not take the current, very unusual conditions into consideration. The advantage of the model-based approach is that it does condition its projections on where the interest rate environment was at the end of the sample period, even if it still has no more experience with exiting a ZLB period than what is reflected in the data from past tightening episodes.

## 6 Stress Testing the Fed’s Income

A key contribution of this paper is to introduce a probability-based approach to policy questions associated with the Federal Reserve’s balance sheet and particularly with respect to “stress testing” scenarios that provide insight on the range of possible adverse outcomes. The applicability of the approach is illustrated above with respect to questions on balance sheet risk; i.e., the potential future value of the Fed’s Treasury holdings. The approach can also be applied to questions regarding the Fed’s income risk; that is, the sensitivity of its net interest income to alternative interest rate scenarios. Again, the primary concern is that particular combinations of planned asset purchases (or sales) and interest rate outcomes could lead the Federal Reserve’s net interest income and balance sheet values to decline sufficiently to halt its remittances to the Treasury Department.

Carpenter et al. (2013) and GHHM directly consider the question of whether the Fed’s remittances to the Treasury (i.e., payments of excess interest income beyond expenses) would remain
positive under several scenarios. In this section, we address this policy question using our probabilistic, model-based approach to generate yield curve distributions. To translate these projections into remittances, we use the accounting framework of GHHM.\(^{27}\) In particular, as shown in Table 6, we adopt the GHHM assumptions regarding future MBS prepayment, currency (or liability) growth, capital accretion, operating expenses, and asset re-investment. However, we do update and alter several other assumptions. First, we link the expected path of Federal Reserve asset purchases through 2014 directly to the publicly-announced results of the New York Fed Primary Dealer Survey as of June 2013.\(^{28}\) This path is similar to that assumed in the GHHM baseline, with purchases ending at year-end 2014. Second, we assume that the Fed does not sell any securities through 2020. We make this change in light of the consensus at the June 2013 FOMC meeting to sell MBS as part of the policy normalization process.\(^{29}\) A final modification is that we set the path for the interest on excess reserves (IOER) interest rate (i.e., the rate the Fed pays on the reserves that banks holds and its

\(^{27}\)We greatly appreciate the authors’ sharing of their code with us for the purposes of this analysis.

\(^{28}\)The survey is publicly available at: http://www.newyorkfed.org/markets/survey/2013/June_result.pdf

\(^{29}\)See page 2 of the FOMC meeting minutes at http://www.federalreserve.gov/monetarypolicy/files/fomcminutes20130619.pdf
<table>
<thead>
<tr>
<th>Variable</th>
<th>GHHM</th>
<th>CLR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Asset purchases</td>
<td>Continue at current pace through December 2013, then slow to maintenance levels and end in 2014.</td>
<td>Purchases through 2014 match Primary Dealer Survey as of June 2013 and end in 2014.</td>
</tr>
<tr>
<td>2. Asset sales</td>
<td>No Treasury sales. MBS sales start in late 2015 and are completed in 2019.</td>
<td>No Treasury or MBS sales.</td>
</tr>
<tr>
<td>3. MBS prepayment</td>
<td>Calibrated to current market expectations.</td>
<td>Same.</td>
</tr>
<tr>
<td>4. Liabilities</td>
<td>Currency grows at 7% annual rate (2 percentage points above Blue Chip forecast for nominal GDP growth per historical experience); required reserves grow at 4% annual rate.</td>
<td>Same.</td>
</tr>
<tr>
<td>6. Fed capital</td>
<td>Grows at 10% annual rate per historical average.</td>
<td>Same.</td>
</tr>
<tr>
<td>7. Operating expenses</td>
<td>Grow on historical trend.</td>
<td>Same.</td>
</tr>
</tbody>
</table>

Table 6: Assumptions Underlying Balance Sheet and Income Projections.
The source for the GHHM assumptions is page 64.

Figure 9: Projections of Annual Fed Interest Expenses.
Illustration of the median and the 5th and 95th percentiles of the projected interest expenses based on the CLR baseline scenario combined with $N = 10,000$ Monte Carlo simulations of the B-CR model.
Figure 10: Projections of Payments to the U.S. Treasury and Deferred Asset.
Panel (a) shows the median and the 5th and 95th percentiles of the projected payments to the U.S. Treasury, while panel (b) illustrates the median and the 5th percentile of projections of the Fed’s deferred asset based on the CLR baseline scenario combined with $N = 10,000$ Monte Carlo simulations of the B-CR model.

main interest expense) equal to the overnight rate as implied by our yield curve simulations.\textsuperscript{30} Given the variation in our simulated short rates, this results in important variation in the Fed’s interest expenses going forward. Indeed, as shown in Figure 9, a 90 percent confidence interval for the annual Fed’s interest expenses ranges from $4$ to almost $80$ in 2016.

Figure 10 presents the key results of our simulation-based approach using this set of assumptions. Figure 10(a) presents the projected range of the Fed’s positive remittances to the Treasury over the period from 2013 to 2020. The median value declines over the period as both interest income declines based on assets maturing and interest rate expenses rise due to interest rates increasing from their current low values near the ZLB. Figure 10(b) shows the projected range of negative remittances or “deferred assets.”\textsuperscript{31} Our results imply zero remittances for only the lower fifth percentile of outcomes. Even in these cases, the low point is only from 2016 to 2018, and the deferred assets do not fall below minus $11.0$ billion. Our probabilistic results suggest that the Federal Reserve is unlikely to stop earning net interest income and making Treasury remittances over the next seven years under reasonable assumptions. Thus, concerns based on either unlikely or poorly designed interest rate assumptions appear to be unfounded.

\textsuperscript{30}In this exercise, the overnight rate is approximated by an instantaneous short rate given by $r_t = \max\{0.25\%, s_t\}$; i.e., we impose a minimum of 25 basis points for the IOER rate consistent with current practice.

\textsuperscript{31}The Federal Reserve, under its remittance policy, remits all net income to the U.S. Treasury—after expenses, dividends, and additions to capital. If earnings are insufficient to cover these costs, the Fed creates new reserves against a “deferred asset,” which represents a claim on future earnings and remittances to the Treasury.
To provide further insight on the probability of the Fed’s possible use of deferred assets, Figure 11 shows our simulated probability distribution of the maximum deferred asset amount over the forecast horizon. The distribution is heavily left-skewed with 89% being zero. For the remainder of the distribution, the simulated probability of observing maximum deferred assets in the ($0, $20] billion range is 7% and of greater than $20 billion is just 4%.

As a final exercise, we try to assess the net direct financial benefit to the Treasury from the Fed’s expansion of its balance sheet. Figure 12 shows our simulated probability distribution for the cumulative Fed remittances from 2008 to 2020 net of the projected linear trend based on remittances over the period from 1990 to 2007. For the 2008-2020 period, the trend indicates a total of $396.96 billion in remittances, which is then deducted from the sum of remittances (including the known 2008-2012 remittances of $322.23 billion) in each of the 10,000 simulated states to produce the shown distribution. There are two things to note in the figure. First, with a large probability, the expansion of the Fed’s balance sheet is likely to generate hundreds of billions of dollars in excess remittances to the U.S. Treasury over the entire 2008-2020 period. Thus, these unconventional monetary policies are most likely to provide a very direct financial benefit to the U.S Treasury (in addition to any indirect benefits from improved economic outcomes). Second, there is only a less than 0.1% chance that these policies will ultimately produce below-trend net remittances, as shown in the small amount of
Figure 12: **Simulated Distribution of Cumulative Remittances Net of Trend.**
The graph depicts the simulated probability distribution function for the cumulative remittances by the Fed to the U.S. Treasury from 2008 to 2020 net of the projected trend of remittances from the 1990-2007 period. About 0.1% of the values are below zero.

7 Conclusion

Recent stress test procedures, including those that have examined the Fed’s financial position, generally have used a few hand-picked scenarios, which introduces a substantial degree of arbitrariness. Our methodological contribution is to introduce a probabilistic structure into stress tests or scenario analysis of the Fed’s balance sheet and income prospects. We argue that attaching likelihoods to adverse outcomes based on interest rate fluctuations is a crucially important addition to the policy debate. Of course, our analysis relies on historical data to estimate forecast distributions, and these may not be completely appropriate in certain future circumstances. Further research can expand and refine our probabilistic structure.

In terms of substantive results, we use two different ways of generating Treasury yield curve projections. The first approach is based on the specific shadow-rate AFNS model favored by CR. The second approach relies on historical Treasury yield curve changes. Despite differences in methods, the results are similar and indicate that in all likelihood the potential losses to the Fed’s Treasury
securities holdings over the next several years are relatively modest. We also generate more comprehensive projections of the Fed’s future income and find a small chance (about 10%) of a temporary halt of the remittances to the Treasury. Our simulation results also suggest that even if remittances do cease, the magnitude of deferred assets created would be moderate at $11 billion at the lower fifth percentile of outcomes. In summary, our probabilistic scenario approach provides additional and generally reassuring guidance regarding questions related to the financial costs of the Federal Reserve’s balance sheet policy.
References


Supervision and Regulation Letter SR10-1, 2010, “Interagency Advisory on Interest Rate Risk.”