1) (15 points) Find \( \int \sqrt{1 - x^2} \, dx \)

2) (15 points) Find \( \int \frac{\arcsin(\sqrt{x})}{\sqrt{x}} \, dx \)

3) (20 points) Argue that \( \int_{2}^{\infty} \frac{x^2 - 4x - 1}{(x^2 - 1)(x^2 + 1)} \, dx \) diverges, or find its value.

4) (15 points) Argue that \( \int_{0}^{\pi} e^{-x^2} \, dx \) diverges, or find its value.

5) (20 points) This problem has three parts.
   A) Let \( R \) be the bounded region of the plane enclosed by the \( x \)-axis and graph of \( y = 3x - x^2 - 2 \). For each part below find an integral that gives the required answer: DO NOT EVALUATE THE INTEGRALS.
      i) The volume of the solid obtained by rotating \( R \) about the \( x \)-axis.
      ii) The volume of the solid obtained by rotating \( R \) about the line \( x = -3 \).
   B) A plate in the shape of a symmetric trapezoid three meters wide at the bottom, five meters wide at the top and two meters high (a rectangle with congruent triangles on each side: is submerged vertically with its top at the surface of a liquid of density \( \rho \). Find an integral that gives the hydrostatic force on one side of the plate: DO NOT EVALUATE THE INTEGRAL. Use \( g \) for the acceleration due to gravity.

6) (20 points) In each case below, the \( n \)-th term of a sequence is given. Give reasons why the sequence diverges, or why it converges and then find its limit.
   i) \( a_n = \left( \frac{4n - 3}{n} - \frac{3n}{n+1} \right) \)
   ii) \( b_n = (3 + \sin(n))^{1/n} \)
   iii) \( c_n = (-1)^n(1 - (1/n)) \)

7) (20 points) In each case give reasons why the series converges or why it diverges. Each series begins at \( n = 1 \).
   i) \( \sum \left( \frac{3}{5} \right)^{n+1} \left( \frac{7}{5} \right)^n \)
   ii) \( \sum \frac{\sin(1/n)}{\tan(1/n)} \)
   iii) \( \sum \frac{e^{1/n} - 1}{n} \) (compare to \( 1/n^2 \))
8) (20 points) Find the radius of convergence and the interval of convergence of
\[ \sum_{n=1}^{\infty} \frac{(-1)^n (x + 1)^n}{n5^n}. \] Give reasons for your answers.

9) (15 points) Find \( T_3(x) \), the third Taylor polynomial of \( g(x) = x^{4/3} \) about 8.
Use \( T_3(x) \) to approximate \( 7^{4/3} \) as a sum of fractions.
Using the Taylor remainder \( R_3 \), what substitution in it gives the best estimate for the error in the approximation above.

10) (15 points) Find the Taylor series about 0 (the Maclaurin series) for \( g(x) = xe^{-x^2} \), then find \( g^{[17]}(0) \) and \( g^{[20]}(0) \).

11) (25 points)
   i) (10 points) What integral gives the length of the graph of \( r = 1/\theta \) from \( \theta = \pi \) to \( \theta = 2\pi \).
   ii) (15 points) Integrate the function that appears in this definite integral, OR (not both)

   (10 points) Find \( \int \frac{x^5}{\sqrt{1 + x^2}} \, dx \)